A Model of Monetary Policy Shocks for Financial Crises and Normal Conditions

John W. Keating, Logan J. Kelly, Andrew Lee Smith, and Victor J. Valcarcel
October 2014
RWP 14-11
A Model of Monetary Policy Shocks
for Financial Crises and Normal Conditions*

John W. Keating\textsuperscript{1}, Logan J. Kelly\textsuperscript{2}, Andrew Lee Smith\textsuperscript{3}, and Victor J. Valcarcel\textsuperscript{4}

\textsuperscript{1}Department of Economics, The University of Kansas, Lawrence, KS
\textsuperscript{2}College of Business and Economics, University of Wisconsin, River Falls, WI
\textsuperscript{3}Research Department, Federal Reserve Bank of Kansas City, Kansas City, MO
\textsuperscript{4}Department of Economics, Texas Tech University, Lubbock, TX

This Version: October 1, 2014

Abstract

In late 2008, deteriorating economic conditions led the Federal Reserve to lower the federal funds rate to near zero and inject massive liquidity into the financial system through novel facilities. The combination of conventional and unconventional measures complicates the challenging task of characterizing the effects of monetary policy. We develop a novel method of identifying these effects that maintains the classic assumptions that a central bank reacts to output and the price level contemporaneously and may only affect these variables with a lag. A New-Keynesian DSGE model augmented with a representative financial structure motivates our empirical specification. The equilibrium model provides theoretical support for our choice of different series to replace variables that were popular in models of monetary policy but became problematic in the aftermath of the 2008 financial crisis. One of our most important innovations is to utilize the Divisia M4 index of money as the policy indicator variable. The model is bolstered by its ability to produce plausible responses to a monetary policy shock in samples that include or exclude the recent crisis period.

Keywords: Monetary policy rules, Dynamic Stochastic General Equilibrium (DSGE) models, money, output puzzle, price puzzle, liquidity puzzle, financial crisis, Divisia, Identification assumptions, Structural Vector Autoregressions (SVARs)

JEL classification codes: E3, E4, E5

*Corresponding author: Victor Valcarcel vic.valcarcel@ttu.edu. The authors wish to thank Nathan Balke, Brent Bundick, Todd Clark, Andrew Foerster, Oscar Jorda, Michael Sposi, Ellis Tallman, and Mark Wynne as well as seminar participants at the Federal Reserve Bank of Dallas, Federal Reserve Bank of Kansas City, University of Cincinnati, 2013 Western Economics Association Meetings and 2014 meetings of the International Association of Applied Econometrics for helpful comments. The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System.
1 Introduction

Economists have devoted considerable effort to understanding the role of monetary policy and how important it is for macroeconomic outcomes. This is a major concern in part because alternative theoretical frameworks can have very different implications for policymakers. In a classic paper, Monetary policy shocks: What have we learned and to what end?, Christiano, Eichenbaum, and Evans (1999) thoroughly investigate one of the most widely used methods for identifying monetary policy shocks. Their empirical findings provide strong support for the identifying assumption that the central bank considers output and prices when setting policy. This assumption is inspired by earlier work including Bernanke and Blinder (1992), Gertler and Gilchrist (1994), Eichenbaum and Evans (1995), Strongin (1995), Christiano, Eichenbaum, and Evans (1996), and Bernanke and Mihov (1998).

Unfortunately, these efforts to identify the effects of monetary policy were based on variables that lost their usefulness following the 2008 Financial Crisis. At the time of this writing, the fed funds rate has been stuck at its lower bound for over half a decade and, therefore, it is unable to provide a useful gauge of monetary stimulus arising from unconventional policies that were used to combat the 2008 financial crisis and ensuing recession. Nonborrowed reserves suffered perhaps an even more severe problem taking on negative values. This economic impossibility resulted from peculiar accounting and precludes the use of nonborrowed reserves in empirical work. We let microfoundations guide us in developing an alternative framework to identify monetary policy shocks. Our novel specification is heavily motivated by theoretical results derived from a standard New Keynesian DSGE model augmented with a financial sector. The theoretical model makes three predictions that are essential for developing our novel identification strategy.

First, the responses of variables to a monetary policy shock, when an interest rate rule is employed, are observationally equivalent to those that ensue from an appropriately parameterized Divisia money rule.1 This theoretical result motivates use of the broadest Divisia monetary aggregate currently available for the US as the policy indicator variable in our empirical work.2 Moreover,

---

1The Divisia monetary aggregates is an index number which measures the service flow that an agent receives from holding a portfolio of monetary assets over a given period of time. Practically speaking, the growth rate of the Divisia aggregate is the weighted growth rate of each monetary asset in the portfolio according to their expenditure shares. See Equation 3.3 for a concrete example of the Divisia aggregate in the context of our general equilibrium model.

2Barnett and Chauvet (2011b), Kelly, Barnett, and Keating (2011) and others argue that a narrow monetary aggregate ignores the monetary services of excluded assets.
monetary policy rules that consider monetary aggregates as the policy indicator do not suffer from the zero lower bound problem that now plagues interest rate rules for the US.

The second theoretical model prediction is that correlations between the simple sum stock of money and macroeconomic aggregates are consistent with the price, liquidity, and output puzzles that have plagued the monetary VAR literature. Sims (1992), Barth and Ramey (2002) and many others have noted that an increase in the fed funds rate was typically associated with a puzzling increase in the price level. Leeper and Gordon (1992), Strongin (1995), Bernanke, Boivin, and Eliasz (2005) and Kelly et al. (2011), among others, find a liquidity puzzle whereby, conditioning on policy shocks, the nominal interest rate and the stock of money are positively correlated. Eichenbaum (1992), Gordon and Leeper (1994) and Christiano et al. (1999) develop VAR models that include a simple sum monetary aggregate, and each of these papers finds a puzzling contraction in output in response to an expansionary monetary policy shock. In contrast to simple sum, the correlations between Divisia and the macroeconomic variables in the theoretical model are inconsistent with all of these puzzles. These results motivate the removal of simple sum money from our empirical model and suggest that these puzzles would not emerge when the Divisia quantity of money is used to indicate the stance of monetary policy.

Our third model prediction is that the user cost of money and the fed funds rate are highly correlated. This finding is strongly supported in the data as the unconditional correlation between the these two variables is nearly 66%. This correlation is even higher in the DSGE model when conditioning on monetary policy shocks. The user cost of money provides a useful source of information quite similar to the fed funds rate. Moreover, a prime advantage of the user cost is that it has not reached the zero lower bound in the US. And the empirical results show that the user cost performs better than Treasury rates in identifying the effects of monetary policy.

Our empirical investigation yields four major results that support the use of a properly measured broad monetary aggregate as the policy indicator. First, unexpected changes in monetary policy derived using our new approach do not generate the output, price, or liquidity puzzles so prevalent to this literature. This contradicts the view that using the interest rate as the policy indicator generally yields more reasonable responses than a monetary aggregate. Second, during normal conditions, policy shocks from our Divisia-based model have similar effects to those found in the fed

---

3 The user cost of money is the opportunity cost of holding the monetary aggregate following the derivation in Barnett (1978).
funds-based model of monetary policy. And, where there are differences, the Divisia model obtains results that are more consistent with economic theory. Third, our approach to identifying the effects of monetary policy shocks produces plausible responses which are robust in samples that include or exclude the 2008 financial crisis. Fourth, policy shocks have plausible and significant effects on output and on the price level.

The paper is organized as follows. In Section 2 we present a relatively standard New-Keynesian model that is augmented by a simple financial structure and allows for two different types of liquid assets: interest-bearing deposits and non-interest-yielding currency. Section 3 studies the response to a monetary policy shock in this model under a Taylor Rule and, alternatively, a Divisia monetary aggregate rule. Section 4 undertakes an empirical investigation in which, motivated by our theoretical analysis, we modify and extend the classic approach to identifying monetary policy shocks. Our investigation obtains robust impulse responses and variance decompositions over different sample periods. Section 5 concludes with an overview of the results and directions for future work.

2 A Minimal DSGE Model for Analyzing Monetary Shocks

This section develops a business cycle model with nominal rigidities and a model of how bank lending and the issuance of deposits influence the economy. The existence of the bank is motivated by an economic environment with two liquid assets: one of these assets (currency) pays no interest while the other asset (deposits) has an own gross rate of return that exceeds one but is less than the loan rate. The model follows from Belongia and Ireland (2012), however we take two major departures from their construct. First, contrary to their specification, we assume consumption and the monetary aggregate are additively separable. Thus, a monetary aggregate does not appear in either the log-linearized IS equation or the New-Keynesian Phillips Curve. Each of these exclusion restrictions is consistent with estimates based on U.S. data (see for example Ireland, 2004). Second, their assumptions regarding the timing of when the labor and goods markets open yield a non-standard Euler equation. We alter this timing-to-market in order to bring about a more typical
Euler equation that relates expected consumption growth and the real interest rate.\textsuperscript{4} Furthermore, we explore this economic environment and develop new theoretical results.

This DSGE model sets the stage for Section 3, which compares and contrasts policy rules based on a Divisia monetary aggregate with a benchmark Taylor rule. Below, all lower case variables are in real terms except for the lower case $i$’s which denote nominal interest rates. Also, tildes denote log-deviations from non-stochastic steady-states. A more detailed description of the general equilibrium model, including a derivation of the log-linear representation, is left for the appendix.

2.1 The Household

The representative household enters any period $t = 0, 1, 2, ...$ with a portfolio in which it holds: maturing bonds $(b_{t-1})$, shares of monopolistically competitive firm $j \in [0, 1]$ $(s_{t-1}(j))$, and wealth $(a_{t-1})$. This household faces a sequence of budget constraints in any given period $t$ which can be described by dividing the period into two separate sub-periods: a securities trading period and, subsequently, a bank settlement period.

In the securities trading session, the household buys and sells stocks $(s_t(j))$ and bonds $(b_t)$, receives wages $(w_t)$ for hours worked $(h_t)$ during the period, purchases consumption goods $(c_t)$, and obtains any loans $(l_t)$ needed to facilitate these transactions. Government transfers $(g_t)$ are also made at this time. Any remaining funds are allocated between currency $(n_t)$ and deposits $(d_t)$. This is summarized in the constraint below.

$$n_t + d_t = \frac{a_{t-1}}{\pi_t} + \frac{b_{t-1}}{\pi_t} - \frac{b_t}{\tilde{\pi_t}} - \int_0^1 q_t (s_t(j) - s_{t-1}(j)) \, di + w_t h_t + l_t - c_t + g_t t$$

(2.1)

At the end of the period, the household receives dividends $f_t(j)$ on shares of stock owned in period $t$, $s_t(j)$, and settles all interest payments with the bank. In particular, the household is owed interest on deposits made at the beginning of the period, $i_t^d d_t$ and owes the bank interest on loans taken out,

\textsuperscript{4}In Belongia and Ireland (2012), the bond market opens at the beginning of the period while the goods market opens at the end of the period resulting in an Euler equation relating expected consumption growth to the expected change in the nominal bond rate. If we were to implement this change in the timing of the goods market without changing the timing of the labor market, the nominal interest rate would appear in the consumption-leisure optimality condition.
Any remaining funds, $a_t$, can be carried over into period $t+1$.

$$a_t = n_t + \int_0^1 f_i(j)s_i(j)di + i_t^d - i_t^d l_t$$  \hspace{1cm} (2.2)$$

The household maximizes its lifetime utility, discounted at rate $\beta$: $\sum_{k=0}^{\infty} \beta^k \mathbb{E}_t\{u_{t+k}\}$ subject to (2.1) and (2.2) with the period flow utility of the household taking the following form:

$$u_t = \zeta_t \left[ \frac{c_t^{1-\theta_c}}{1-\theta_c} + v_t \frac{m_t^{1-\theta_m}}{1-\theta_m} + \eta (1 - h_t) \right].$$

The utility function contains two time-varying preference parameters that serve as structural shocks in the linearized model. In particular, $\zeta_t$ enters the linearized Euler equation as an IS shock while $v_t$ enters the linearized money demand equation as a money demand shock. Each structural shock process is assumed to follow an AR(1) in logs.

The true monetary aggregate, $m_t$, enters the period utility function and takes on a general CES form (following the specification in Barnett (1980)):

$$m_t = \left[ \nu \frac{1}{n_t^{\omega-1}} + (1-\nu) \frac{1}{d_t^{\omega-1}} \right]^{\frac{\omega-1}{-\omega}}.$$  \hspace{1cm} (2.3)$$

In the calibrated model, $\nu$ is derived from the relative expenditure shares on currency and deposits and $\omega$ represents the elasticity of substitution between the two monetary assets. If deposits and currency are perfect substitutes ($\omega \to \infty$) then the true monetary aggregate reduces to the simple sum monetary aggregate, $m_t = n_t + d_t$. Otherwise ($0 < \omega < \infty$) currency and deposits are imperfect substitutes, and the simple sum aggregate will not equal the true aggregate. In general, if assets pay different yields, which is the case for our model of the banking sector, they cannot be perfect substitutes. Therefore a finite value for $\omega$ is generally true in a modern economy.

### 2.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There are a unit measure of intermediate goods producing firms indexed by $j \in [0,1]$ who supply a differentiated product. The final goods firm produces $y_t$ combining inputs $y_t(j)$ using the constant returns to scale.
technology,

\[ y_t = \left( \int_0^1 y_t(j) \frac{\theta - 1}{\theta} \, dj \right)^{\frac{\theta}{\theta - 1}} \]

in which \( \theta > 1 \) governs the elasticity of substitution between inputs, \( y_t(j) \). The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem,

\[
\max_{y_t(j) \in [0,1]} \quad p_t \left( \int_0^1 y_t(j) \frac{\theta - 1}{\theta} \, dj \right)^{\frac{\theta}{\theta - 1}} - \int_0^1 p_t(j)y_t(j) \, dj.
\]

The resulting first order condition defines the demand curve for each intermediate goods producing firm’s product:

\[ y_t(j) = \left( \frac{p_t(j)}{p_t} \right)^{-\theta} y_t. \]  \hspace{1cm} (2.4)

**Intermediate Goods Producing Firm**

The representative intermediate goods producing firm \( j \) has access to the production technology,

\[ y_t(j) = z_t h_t(j), \]  \hspace{1cm} (2.5)

where \( z_t \) is an aggregate technology shock that follows an AR(1) process in logs. Given the downward sloping demand for its product in (2.4), the intermediate goods producing firm has the ability to set the price of its product above marginal cost. However, we assume the firm faces “menu cost” associated with changing prices (Rotemberg, 1982):

\[
\Phi(p_t(j), p_{t-1}(j), y_t) = \frac{\phi}{2} \left[ \frac{p_t(j)}{p_{t-1}(j)} - 1 \right]^2 y_t.
\]

The intermediate goods producing firm is assumed to maximize its period \( t \) real stock price:

\[ q_t(j) = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \mathcal{M}_{t+k} f_t+k(j) \right], \] \hspace{1cm} (2.6)
where $\mathcal{M}_{t|t+k} = (\beta^k/\ell_{t+k}) (\kappa_{t+k}/\kappa_t) (\gamma_{t+k}/\gamma_t)^{-\theta}$ is the rate at which the household discounts period $t+k$ payoffs in period $t$. Substituting the definition of dividends for $f_{t+k}$ in (2.6) the firm’s problem can be stated as:

$$\max_{\{y_{t+k}(j), h_{t+k}(j), p_{t+k}(j)\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{k=0}^{\infty} \mathcal{M}_{t|t+k} \left[ \frac{p_{t+k}(j)}{p_{t+k}} y_{t+k}(j) - w_{t+k} h_{t+k}(j) - \frac{\phi}{2} \left( \frac{p_{t+k}(j)}{p_{t+k-1}(j)} - 1 \right) \right]^2 y_{t+k}$$

subject to (2.4) and (2.5).

### 2.3 The Financial Firm

The financial intermediary produces deposits $d_t$ and loans $l_t$ for the household. Following Belongia and Ireland (2012), we assume that producing $d_t$ deposits requires $x_t d_t$ units of the final good. In this case, $x_t$ is the exogenous marginal cost of producing deposits and therefore an increase in $x_t$ can be interpreted as an adverse financial productivity shock.

In addition to this resource costs, the financial firm must satisfy the accounting identity which specifies assets (loans plus reserves) equal liabilities (deposits),

$$l_t + \tau_t d_t = d_t, \quad (2.7)$$

where $\tau_t$ denotes the fraction of deposits the bank keeps in reserves. Although changes in banking regulation have effectively eliminated reserve requirements, banks may choose to hold excess reserves in lieu of making loans. Following the 2008 financial crisis, excess reserves holdings have risen to enormous levels. We assume $\tau_t$ varies exogenously and therefore it can be interpreted as a reserves demand shock, as opposed to a change in policy.

The financial firm chooses $l_t$ and $d_t$ in order to maximize period profits:

$$\max_{l_t, d_t} (\ell_t^0 - 1) l_t - (\ell_t^0 - 1) d_t - x_t d_t.$$
subject to the balance-sheet constraint (2.7). The first order conditions from this problem define the interest rate spread between loans and deposits:

\[ i^l_t - i^d_t = \tau_t (i^l_t - 1) + x_t. \]  

(2.8)

If banks hold more reserves or become relatively less productive, consumers will have to pay higher interest rates on loans relative to the rate they receive on their deposits.

### 2.4 Market Clearing

It is now possible to define the equilibrium conditions that close the model. In this representative agent model, the equity and bond markets clear when \( s_t(j) = s_{t-1}(j) = 1 \) and \( b_t = b_{t-1} = 0 \), respectively. The asset market clearing condition, requires that at all times:

\[ a_t = \frac{a_{t-1}}{\pi_t} + w_t h_t - c_t + g t, \]  

(2.9)

which implies the household transfers wealth between periods through \( a_t \). Finally, imposing symmetry among the intermediate goods producing firms requires that in equilibrium \( y_t(j) = y_t, h_t(j) = h_t, \)

\( p_t(j) = p_t \), \( f_t(j) = f_t \) and \( q_t(j) = q_t \). The above equilibrium conditions along with the household’s budget constraints imply the goods market must clear:

\[ y_t = c_t + x_t a_t + \frac{\phi}{2} [\pi_t - 1]^2 y_t. \]  

(2.10)

### 2.5 The Log-linearized Model

The complete details of the log-linear model are left for the appendix. The New-Keynesian non-policy block includes equations (2.11), (2.12) and (2.13). These equations are respectively the

---

6The linearization and definitions of the semi-structural parameters as functions of the deep parameters can be found there.
dynamic IS equation, New-Keynesian Phillips Curve and money demand schedule:

\[ \tilde{c}_t = E_t [\tilde{c}_{t+1}] - \frac{1}{\theta_c} (\tilde{t}_t - E_t [\tilde{\tau}_{t+1}]) + \left( \frac{1 - \rho_c}{\theta_c} \right) \tilde{c}_t \]  
(2.11)

\[ \tilde{\pi}_t = \kappa [\tilde{c}_t - \tilde{z}_t] + \beta E_t [\tilde{\pi}_{t+1}] \]  
(2.12)

\[ \tilde{m}_t = \eta_c \tilde{c}_t - \eta_i \tilde{i}_t - \eta_\tau \tilde{\tau}_t - \eta_x \tilde{x}_t + \eta_v \tilde{v}_t \]  
(2.13)

The equilibrium model also includes equation (2.14) which relates output to consumption and financial resources using the income accounting identity. Equation (2.15) defines the user cost of the monetary aggregate. The user cost and the demand for deposits are defined in equations (2.16) and (2.17). The user cost and demand for currency are defined in equations (2.18) and (2.19). Finally, equation (2.20) is derived from the definition of the monetary base as the sum of currency and bank reserves.

\[ \tilde{y}_t = \frac{\tilde{c}_t}{\tilde{y}_t} (\tilde{x}_t + \tilde{d}_t) ; \]  
(2.14)

\[ \tilde{u}_t = s^n \tilde{u}_t^n + s^d \tilde{u}_t^d \]  
(2.15)

\[ \tilde{u}_t^d = \left[ \frac{\tilde{r} - \tilde{x}}{\tilde{r}(i - 1) + \tilde{x}} \right] \tilde{t}_t + \left[ \frac{\tilde{r}(i - 1)}{\tilde{r}(i - 1) + \tilde{x}} \right] \tilde{r}_t + \left[ \frac{\tilde{x}}{\tilde{r}(i - 1) + \tilde{x}} \right] \tilde{x}_t ; \]  
(2.16)

\[ \tilde{d}_t = \tilde{m}_t - \omega (\tilde{u}_t^d - \tilde{u}_t) ; \]  
(2.17)

\[ \tilde{u}_t^n = \left[ \frac{1}{(i - 1)} \right] \tilde{t}_t ; \]  
(2.18)

\[ \tilde{n}_t = \tilde{m}_t - \omega (\tilde{u}_t^n - \tilde{u}_t) \]  
(2.19)

\[ \tilde{m}_t = \tilde{n}_t + \frac{\tilde{c}_t}{\tilde{n} + \tilde{d}_t} (\tilde{t}_t + \tilde{d}_t) \]  
(2.20)

3 DSGE Model Analysis of Alternative Policy Rules

This section analyzes the DSGE model’s responses to a contractionary monetary policy disturbance. We show that if the central bank follows a Taylor rule, the responses to all key variables to a monetary policy shock can be replicated by a particular parameterization of the Divisia growth rate rule. This suggests that, from the perspective of the Lucas Critique (1976), a Divisia growth rule may not be all that different from the sort of Taylor rule that some central banks, including the Federal Reserve, are thought to have pursued at times. We also show that Divisia and simple sum
monetary aggregates exhibit divergent responses to a monetary contraction. The unusual responses economists have found in empirical models with money may be largely the consequence of using simple sum money which is in all but the most simplistic of cases an improper aggregate. We also find that the fed funds rate and Divisia user cost are highly correlated and share important similarities in the model. These results suggest useful information in the fed funds rate is also contained in the user cost. We conclude that Divisia data may prove useful in empirically identifying monetary policy shocks that are not plagued by the price, output and liquidity puzzles frequently found in empirical analysis.

3.1 Central Bank Policy

The system of equations defined in Section (2.5) is not fully determined without a specification of monetary policy. Interest rate rules are typically used to describe central bank policy since Taylor’s seminal paper (1993). As is common, we augment Taylor’s rule with a smoothing parameter, for which there is much empirical evidence, and a white noise policy disturbance, \( \varepsilon_t \):

\[
\tilde{i}_t = \rho \tilde{i}_{t-1} + \phi \tilde{n}_t + \phi \tilde{y}_t + \varepsilon_t.
\]  

(3.1)

However, we will also consider a money growth rule. In contrast to much of the research, the model contains multiple monetary assets, real currency, \( n_t \), and real deposits, \( d_t \). When considering money growth rules in this environment with multiple monetary assets, a choice of aggregation must be explicitly made. We eschew the assumption a central bank observes the true monetary aggregate, \( m_t \), as that requires knowledge of the structural parameters in the monetary aggregator, i.e. \( \omega \) and \( \nu \) from equation (2.3). If not the true aggregate, what monetary aggregate might a central bank control? One popular index produced by most central banks is the simple sum aggregate. This aggregate implicitly assumes \( n_t \) and \( d_t \) are perfect, one for one, substitutes (see e.g. our discussion of equation (2.3)).

**Definition 1.** The growth rate of the nominal simple sum monetary aggregate is defined by

\[
\ln(g_t^{SS}) = \ln \left( \frac{n_t + d_t}{n_{t-1} + d_{t-1}} \right) + \ln(\pi_t).
\]  

(3.2)
But, in general, since monetary assets have different nominal yields, they are not perfect substitutes. An alternative to the simple sum index that allows for possible imperfect substitutability is the Divisia monetary aggregate proposed by Barnett (1980).

**Definition 2.** The growth rate of the nominal Divisia monetary aggregate is defined as

\[
\ln(g^D_t) = \left(\frac{s^n_t + s^n_{t-1}}{2}\right) \ln\left(\frac{n_t}{n_{t-1}}\right) + \left(\frac{s^d_t + s^d_{t-1}}{2}\right) \ln\left(\frac{d_t}{d_{t-1}}\right) + \ln(\pi_t)
\]  

(3.3)

where \(s^n_t\) and \(s^d_t\) are the expenditure shares of currency and interest bearing deposits respectively defined by

\[
s^n_t = \frac{u^n_t n_t}{u^n_t n_t + u^d_t d_t}
\]

(3.4)

\[
s^d_t = \frac{u^d_t d_t}{u^n_t n_t + u^d_t d_t}
\]

(3.5)

in which \(u^n_t = (i^n_t - 1) / i^n_t\) and \(u^d_t = (i^d_t - i^d_{t-1}) / i^d_t\) are user costs for these two assets. This index has been extensively studied over the last 30 years and has been shown to outperform the simple sum alternative in many empirical and theoretical applications.\(^7\) By weighting the growth rate of the individual assets with time-varying share weights the Divisia monetary aggregate is able to successfully account for compositional changes which may have no impact on the overall aggregate. This superior accuracy places the Divisia index amongst Diewart’s (1976) class of superlative index numbers, meaning the Divisia index has the ability to track any linearly homogeneous function (with continuous second derivatives) up to second order accuracy.

Appendix A shows a similar result. The log-linearized Divisia monetary aggregate can track the true monetary aggregate to first-order:

\[
\tilde{g}^D_t = \tilde{m}_t - \tilde{m}_{t-1} + \tilde{\pi}_t.
\]

(3.6)

The more popular simple sum aggregate fails to attain this same level of accuracy. In particular, we show in the appendix that the log-linearized simple sum monetary aggregate differs from the true

\(^7\)For a more in-depth discussion of the research examining the Divisia monetary aggregate’s properties relative to alternative simple sum measures see Barnett and Singleton (1987), Belongia (1996), Barnett and Serletis (2000), Belongia and Binner (2000), Barnett and Chauvet (2011a), and Barnett and Chauvet (2011b).
monetary aggregate at first order:

$$\tilde{g}^{SS}_{t} = \tilde{m}_{t} - \tilde{m}_{t-1} + \tilde{\pi}_{t} + \psi^{SS}_{i} \Delta \tilde{\pi}_{t} - \psi^{SS}_{\tau} \Delta \tilde{\pi}_{t} - \psi^{SS}_{x} \Delta \tilde{x}_{t}$$  (3.7)

where each $\psi^{SS}_{j}$ for $j = i, \tau, x$ is equal to zero when currency and deposits are perfect substitutes and otherwise each of these is greater than zero. Due to its superior accuracy, we define the money growth rule in terms of the Divisia monetary aggregate\(^8\) (augmented to include a white noise monetary policy shock):

$$\tilde{g}^{D}_{t} = \rho_{g} \tilde{g}^{D}_{t-1} - \phi_{g}^{\pi} \tilde{\pi}_{t} - \phi_{g}^{y} \tilde{y}_{t} - \epsilon^{g}_{t}.$$  (3.8)

### 3.2 Equivalence between Divisia Growth and Taylor Rules

Suppose the central bank follows a Taylor rule:

$$\tilde{i}_{t} = 0.5 \tilde{i}_{t-1} + 1.5 \tilde{\pi}_{t} + 0.125 \tilde{y}_{t} + \epsilon^{i}_{t}. \quad (3.9)$$

We generate impulse responses to an increase in $\epsilon^{i}_{t}$ and then search for coefficients in an analogous Divisia rule to match these dynamics. The results are presented in Table 1.

A Divisia rule with no persistence ($\rho_{g} = 0$), no reaction to inflation ($\phi_{g}^{\pi} = 0$) and a strong reaction to output ($\phi_{g}^{y} = 2.18$) is able to match the dynamics of a monetary policy contraction under a Taylor rule regime.\(^9\) The resulting dynamics are featured in Figure 1.

We characterize this result in the following model prediction:

**Model Prediction 1:** Monetary policy shocks to an interest rate rule and to an appropriately parametrized Divisia rule have observationally equivalent effects on macroeconomic variables.

---

\(^8\)When we tried using simple sum in place of the Divisia aggregate in the money growth rule (Equation (3.8)), the economy was indeterminate.

\(^9\)Furthermore, when we set the coefficients in the Taylor rule equal to the estimates of Clarida, Gali, and Gertler (2000), $\tilde{i}_{t} = 0.76 \tilde{i}_{t} + 0.47 \tilde{\pi}_{t} + 0.13 \tilde{y}_{t} + \epsilon^{i}_{t}$, and/or alter the slope of the Phillips Curve to $\kappa = 0.25$ we are still able to match the dynamics of a monetary policy contraction with an appropriately calibrated Divisia growth rule. The form of the Divisia growth rule that matches these alternative data generating processes is robust in the sense that it is of the form of a feedback rule with $\rho_{g} = 0$, $\phi_{g}^{\pi} = 0$ and $\phi_{g}^{y} > 0$.\(^\dagger\)
3.3 Measurement Matters: Divisia Quantity vs. Simple Sum

Regardless of the policy instrument, the simple sum monetary aggregate fails to get the sign ‘right’ (see Figure 1) in response to a monetary contraction. Therefore, it is not surprising that using the simple sum aggregate in the policy-maker’s information set of an estimated VAR may be a source of output, price, or liquidity puzzles found often in VAR studies. Eichenbaum (1992) finds that increases in the simple sum aggregate produce incredulous output responses - the output puzzle. He concludes that changes in the money supply fail to properly identify monetary policy disturbances in a VAR. Using various monetary aggregates as the policy indicator in their benchmark specification, Christiano et al. (1999) also find a variety of puzzling responses depending on the choice of simple sum aggregate.\(^{10}\)

In contrast, the DSGE model implies that the Divisia monetary aggregate has the theoretically correct correlations associated with a monetary contraction. This point is clear in Table 2. When monetary policy shocks are the only driving force of this sticky price economy, interest rates are perfectly negatively correlated with inflation and output and negatively correlated with Divisia.

[Table 2 about here.]

Why do the simple sum and Divisia aggregates diverge following a monetary policy shock? From the impulse responses in Figure 1, we observe that the monetary policy contraction increases the user cost of the true monetary aggregate, \(\tilde{u}_t\). Naturally, this results in a fall in the monetary aggregate. The relative costs of both currency and deposits also rise; however, the cost of currency increases by more than the cost of deposits.\(^{11}\) This results in a substitution into deposits, out of currency. Since the Divisia monetary aggregate properly weights the component assets (by expenditure share), it is able to consistently internalize this substitution effect, hence getting the sign ‘right’ and falling in response to a monetary contraction. However, currency and deposits are equally weighted in the simple sum aggregate, even when currency makes up a substantially larger expenditure share than do deposits. Simple sum increases in response to the monetary contraction due to its failure to internalize this substitution. This apparent lack of a liquidity effect leads to simple sum growth

\(^{10}\)They find that a contractionary policy shock to money yields an output puzzle when either the monetary base or M1 are used as the policy indicator and they also find a price puzzle when M1 is the indicator. Interestingly, the common puzzles do not emerge when using M2 in the benchmark model. However, the positive and unusually large response of nonborrowed reserves after a few quarters is puzzling.

\(^{11}\)This can be seen by inserting (2.8) into the user cost of deposits expression and comparing that with the user cost of currency. Given that \(\tau\) is small, that proves that the user cost of currency is more sensitive to market rates than the user cost of deposits.
being positively correlated with interest rates and negatively correlated with output and inflation. These perverse correlations from the equilibrium model yield a second empirical implication.

**Model Prediction 2:** The use of simple sum monetary aggregates in estimated VARs may be a source of output, price and liquidity puzzles.

### 3.4 The Relationship between User Cost and the Policy Interest Rate

Belongia and Ireland (2006) show that the user cost captures the monetary transmission mechanism in a flexible price equilibrium model and a small VAR. Their result carries over to our sticky-price model where we discover a striking relationship between the user cost and the federal funds rate. Figure 1 and Table 2 show that, for the purposes of identifying monetary policy, the user cost and the policy rate contain the same content.

Combining equations (2.15), (2.16) and (2.18) shows that without financial sector shocks the user cost of money and the policy rate move proportionally to one another. Adding financial sector shocks can drive a wedge between the user cost of money and the policy rate. Table 3 shows these variables remain highly correlated when all the shocks in our model are operational. The correlations in the model are similar to what we find in the data. Figure 2 highlights this strong co-movement in the data. The user cost of Divisia M4 has tracked movements in the fed funds rate less closely since the onset and aftermath of the 2008 financial crisis – a period during which financial shocks likely became more prevalent. The theoretical model and relevant empirical evidence from our investigation suggest the following:

[Table 3 about here.]

Combining equations (2.15), (2.16) and (2.18) shows that without financial sector shocks the user cost of money and the policy rate move proportionally to one another. Adding financial sector shocks can drive a wedge between the user cost of money and the policy rate. Table 3 shows these variables remain highly correlated when all the shocks in our model are operational. The correlations in the model are similar to what we find in the data. Figure 2 highlights this strong co-movement in the data. The user cost of Divisia M4 has tracked movements in the fed funds rate less closely since the onset and aftermath of the 2008 financial crisis – a period during which financial shocks likely became more prevalent. The theoretical model and relevant empirical evidence from our investigation suggest the following:

[Figure 2 about here.]

---

12 Although simulations for the model have only included the true price aggregates (see Eq. (2.15)), two applications of Fisher’s factor reversal test verify that in this linearized model, the Divisia price dual will track the true price dual (in growth rates) without error. First, define the nominal Divisia level by \( \text{div}_t \) where \( \frac{\text{div}_t}{\text{div}_{t-1}} = g^D_t \). Then applying Fisher’s factor reversal test implicitly defines the Divisia price dual, \( u^D_t \), by the equation \( \text{div}_t u^D_t = (p^d t \mu^d_t + p^n \mu^n_t) \). Finally, notice the true monetary aggregate also satisfies Fisher’s factor reversal test which implies that,

\[
\tilde{g}_t^D + \Delta \tilde{u}^D_t = \tilde{m}_t - \tilde{m}_{t-1} + \tilde{\pi}_t + \Delta \tilde{u}_t.
\]

Since we have previously shown the growth rate of the Divisia monetary aggregate tracks the growth rate of the true monetary aggregate to first order without error (see Eq. (3.6)), we have that \( \Delta \tilde{u}^D_t = \Delta \tilde{u}_t \).

13 We could have calibrated the financial shocks to perfectly match the correlations between the user cost and the fed funds rate in the model. Instead, we calibrate these shocks by backing out the time series for \( \tau_t \) and \( x_t \) using data on reserves and Equation (2.8). Thus, the similar correlations further bolster our model.
Model Prediction 3: Conditioning on monetary policy shocks, the Divisia user cost and the policy rate move closely under normal conditions.

The remainder of the paper develops an empirical model that intends to identify the effects of monetary policy shocks under normal and crisis conditions.

4 A Robust Model of Monetary Policy Shocks

The three theoretical model predictions laid out above are used to help formulate our new strategy for identifying monetary policy shocks based on the reduced-form vector autoregression (VAR):

\[ z_t = B_1 z_{t-1} + \ldots + B_q z_{t-q} + u_t \]  \hspace{1cm} (4.1)

where q is the number of lags and \( Eu_t u_t' = V \) is the covariance matrix for residuals. Correspondingly, a linear structural model may be written as:

\[ A_0 z_t = A_1 z_{t-1} + \ldots + A_q z_{t-q} + \varepsilon_t \]  \hspace{1cm} (4.2)

where \( E\varepsilon_t \varepsilon_t' = D \) is the diagonal covariance matrix for structural shocks.\(^{14}\) The variables in the model are sub-divided into three groups:

\[ z_t = \begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{pmatrix} \]  \hspace{1cm} (4.3)

Each group might consist of multiple variables; however in this work \( X_{2t} \) is a single policy indicator variable. The structure is assumed to take on the following form:

\[ A_0 = \begin{bmatrix} A_{11} & 0_{12} & 0_{13} \\ A_{21} & A_{22} & 0_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \]  \hspace{1cm} (4.4)

\(^{14}\)For our purposes we only require that \( D \) take on a block-diagonal form. This condition is sufficient to make the policy shock uncorrelated with the other structural shocks.
where $A_{ij}$ is an $n_i \times n_j$ matrix of parameters and $0_{ij}$ is an $n_i \times n_j$ zero matrix. The vector of structural shocks is given by: $\varepsilon_t = (\varepsilon_{1t}', \varepsilon_{2t}', \varepsilon_{3t}')'$. Christiano et al. (1999) prove that under these assumptions the Cholesky factor of $V$ will identify the effects of shocks to the structural equation for $X_{2t}$.$^{15}$

This identification assumes that the policy variable, $X_{2t}$, responds contemporaneously to $X_{1t}$ a set of important macroeconomic variables that slowly adjust to policy and a variety of other nominal shocks. It is common to identify monetary policy shocks using a block-recursive formulation in which $X_{1t}$ consists of real GDP, the GDP deflator, and a commodity price index. The first two variables are included because of the assumption that central banks consider real output and prices when determining the stance of monetary policy. If the policy variable is the fed funds rate, a reaction to these two variables is consistent with a Taylor Rule formulation which is often assumed to describe the central bank’s policy rule. The commodity price index has been included in previous work because it was thought to encapsulate current information about future price movements that forward-looking central banks tend to monitor. A key identifying assumption is that monetary policy has no contemporaneous effect on $X_{1t}$.

The third set of variables, $X_{3t}$, consists of variables that respond immediately to $X_{1t}$ or $X_{2t}$. In the benchmark specifications of Christiano et al. (1999), $X_{3t}$ includes various reserves measures along with a simple sum measure of money. These money market variables are allowed to respond immediately to macroeconomic conditions and monetary policy. Another crucial assumption is that the macroeconomic variables in $X_{1t}$ and monetary policy variable do not respond contemporaneously to money market variables. In the Fed Funds Benchmark model, $X_{3t}$ includes nonborrowed reserves. Conversely, in the Nonborrowed Reserves Benchmark model, the fed funds rate is included in $X_{3t}$. Ultimately, our DSGE model results guides us to a new method for identifying monetary policy shocks within this recursive framework.

### 4.1 Fed Funds Rate and Nonborrowed Reserves Benchmark Models

As a starting point, we want to compare our approach to identifying monetary policy shocks with the Fed Funds and Nonborrowed Reserves Benchmark specifications of policy shocks. To maintain consistency we wanted to use the same quarterly time series as in Christiano et al. (1999). However, the commodity price measure they used has been discontinued. Thus, our first task is to find

---

$^{15}$Keating (1996) generalizes Christiano et al. (1999), showing that if $n_2 > 1$ and $A_{22}$ is a lower triangular matrix, the Cholesky factor of $V$ will identify the effects of structural shocks for all $n_2$ shocks in $\varepsilon_{2t}$. Structures that take on this form are defined as partially recursive.
a substitute commodity price index that is currently being produced and that is available over a reasonably long period of time. We employ the Commodity Research Bureau (CRB) spot price index provided in the Thomson-Reuters/Jefferies CRB index report. This index comprises 19 commodities falling under four different groups (Petroleum-based products, liquid assets, highly liquid assets, and diverse commodities – each group with different weightings).

Next, we estimate these two benchmark specifications to determine whether our data yield roughly the same results as Christiano et al. (1999) obtained. Along with a different commodity price index we employ data of a more recent vintage. A third difference is that our sample must begin at a slightly later date than the benchmark models estimated in Christiano et al. (1999). This difference stems from the availability of the Divisia time series. Despite these differences, our estimates are similar to Christiano et al. (1999).

Figure 3 reports our estimated impulse responses along with 90% bootstrapped confidence intervals for these two benchmark models. The first column considers the fed funds rate as the policy indicator while the second column assumes nonborrowed reserves as the policy indicator.

Output falls with some delay and has a U-shaped response to a monetary contraction in the Fed Funds Benchmark model. The negative output response is significant after 3 quarters. In contrast to Christiano et al. (1999), we find the response of output in the Nonborrowed Reserves Benchmark model is relatively small and never statistically significant. This response is the salient difference between their benchmark models and our models.

The price level response is negative in both of our models, and like Christiano et al. (1999), the decline is more delayed in the Fed Funds Benchmark model. The price level rises initially in the Fed Funds Benchmark model though this effect is not large and is not statistically significant – similar to their findings. Christiano et al. (1999) find that the price level responses are never statistically significant. Our Fed Funds Benchmark model also obtains this result while the Nonborrowed Reserves Benchmark model finds that the decline in prices becomes significant after 9 quarters. The commodity price response is initially zero, by construction, but afterwards is consistently negative. This seems more natural than the estimates in Christiano et al. (1999) where the commodity price

---

16 We estimated but do not report a third benchmark model which is based on Strongin (1995). That model has similar problems to the ones we document for the Nonborrowed Reserves Benchmark model.

17 All impulse response functions in the paper are accompanied by 90% confidence bands. The confidence bands are calculated using a classical bootstrap.
response in the Fed Funds Benchmark model is sometimes positive and where the response in the Nonborrowed Reserves Benchmark model goes to zero very quickly.

The remaining responses are very similar to the findings of Christiano et al. (1999). Initially, the fed funds rate rises while nonborrowed reserves falls, and both effects are initially significant. Total reserves falls significantly in the Nonborrowed Reserves Benchmark model while the decline in the Fed Funds Benchmark model is not significant.\(^{18}\) M2 falls immediately and that response is always negative and frequently significant.

Most of our estimates of both Benchmark models are qualitatively and quantitatively quite similar to those reported in Christiano et al. (1999). The responses of the commodity price index are the primary exception. They find commodity prices sometimes rise in their Fed Funds Benchmark model while our estimated response is never positive. In further contrast to Christiano et al. (1999), we find that commodity prices persistently fall in the Nonborrowed Reserves Benchmark model. Otherwise, the typically modest differences between their results and ours can be attributed to using a different commodity price index, using a more recent vintage of data and selecting VAR lags based on the AIC criterion. Since our Benchmark model estimates are very similar to those estimated in Christiano et al. (1999), we conclude that a suitable substitute for their commodity price index has been found.

### 4.2 The Divisia Benchmark Model

We propose using Divisia M4 as the policy indicator variable, in place of the fed funds rate to address the zero lower bound issue.\(^{19}\) Given that the Fed has historically conducted policy with relatively little interest in Divisia, this usage of the monetary aggregate might raise Lucas-critique concerns (Lucas Jr, 1976). However, our replacement is justified by Model Prediction 1 which shows theoretically that the effects of a monetary policy shock to an interest rate rule or to an appropriately specified Divisia quantity rule are observationally equivalent. The choice to use the broadest of the Divisia aggregates available for the US (M4) is also motivated by economic theory.

A broad measure of money has a number of advantages over the more commonly used narrow measures. Especially relevant for our purpose, Divisia M4 contains: treasury bills, overnight and term repurchase agreements and commercial paper, among other assets. Conventional wisdom sug-

\(^{18}\)And this decline of total reserves in the Fed Funds Benchmark model is delayed, which is consistent with the identification assumptions in Strongin (1995).

\(^{19}\)Broad Divisia aggregates are published monthly by the Center for Financial Stability. For more information see http://www.centerforfinancialstability.org/amfm_data.php
gests that monetary policy transmission starts with the central bank open market operations on
treasuries. However, Adrian and Shin (2010) show that traditional monetary policy has consider-
able effects on the repo holdings, and ultimately the “risk-taking,” of large broker-dealers suggesting
monetary policy may transmit through various financial markets.

We argue that the inclusion of information on repo’s, t-bills and commercial paper in our policy
indicator allows us to more directly capture the effects of the Fed’s liquidity facilities. This is
particularly important for identifying monetary policy shocks in sample periods that include the
2008 financial crisis and beyond. The Primary Dealer Credit Facility (PDCF), created in March of
2008, targeted repo markets to ensure that primary dealers could obtain credit and consequently
continue to provide financing in securitized markets. In early 2008, treasury markets also experienced
disruptions as the ability to finance even safe assets became increasingly difficult. This led the Fed
to develop the Term Securities Lending Facility (TSLF) which auctioned treasuries in exchange for
collateral. Another liquidity facility introduced by the Fed came in response to Lehman’s bankruptcy.
The Commercial Paper Funding Facility (CPFF) allowed the Fed to purchase commercial paper in
an effort to revive short-term debt markets. Using a narrower aggregate which excludes these assets,
such as the Fed’s M2 MSI series, would seriously limit our ability to measure the stance of monetary
policy. This is particularly true during the 2008 financial crises during which the Fed’s policy reaction
included these lending facilities.

Impulse responses from the Divisia Benchmark model are reported in the third column of Figure
3 to facilitate comparison with the other two Benchmark models. Responses are often quite similar
to those found in the Fed Funds model, except for the different policy indicator variables. And
sometimes the Divisia Benchmark model’s responses are marginally better for a shock to monetary
policy. The output responses generally share the same dynamics and statistical significance in these
two benchmark specifications. In contrast to the Fed Funds Benchmark model, the Divisia model’s
price response to a negative money supply shock is always non-positive and becomes significantly
negative after 8 quarters. Declines in commodity prices and M2 are very similar for these two
Benchmark specifications. The Divisia Benchmark model yields a much longer period of significance
for the commodity price response than either the Fed Funds or Nonborrowed Reserves Benchmark
models. Responses of nonborrowed and total reserves in the Divisia Benchmark Model are always
negative and sometimes statistically significant making these results somewhat better than the Fed
Funds Benchmark Model. Using roughly the same sample as Christiano et al. (1999) we find that
Divisia works very well as the policy indicator variable and has similar effects on variables to a model in which the fed funds rate is used in this capacity. This finding provides empirical support for theoretical Model Prediction 1. The interchangeability of the two policy indicators suggests that when the fed funds rate hits its lower bound, Divisia may continue to perform well in measuring the stance of monetary policy.

4.3 Benchmark Models in the Pre-Crisis Period

We estimate the three Benchmark models (Fed Funds, Nonborrowed Reserves and Divisia) with more recent data to examine the extent to which our results are robust to an extended sample period. How far we are able to extend the sample is limited by the problems nonborrowed reserves and the fed funds rate incurred during the financial crisis. Therefore, our sample period goes from 1967:Q1 to 2007:Q4. We report the estimated impulse responses in Figure 4.

In this extended sample the Nonborrowed Reserves Benchmark Model performs notably worse. The weak output effect found in the shorter sample period (1967:Q1-1995:Q2) becomes a statistically significant output puzzle after 14 quarters in this longer sample (1967:Q1-2007:Q4). Also, this model now features an M2 puzzle. While the M2 response is initially negative, it turns positive after about six quarters and becomes nearly statistically significant. Otherwise, this benchmark model’s responses are much the same as those in the shorter sample.

Once again the Fed Funds and Divisia Benchmark models yield many responses to a policy shock that are qualitatively similar and theoretically plausible, consistent with Model Prediction 1. There is, however, one significant difference. The Fed Funds Benchmark model obtains a price puzzle that is large and statistically significant. On the other hand, the Divisia Benchmark model has the correct sign on each response and is the only model to have a correctly signed (negative) price level response that eventually becomes statistically significant. Our empirical evidence provides strong support for Model Prediction 1, and so for the remainder of this paper we will use Divisia M4 as the policy indicator variable in VAR models.
4.4 Modifying the Divisia Benchmark Model

We have shown theoretically that it is largely a matter of choice whether the Divisia quantity of money or the fed funds rate is used as a policy indicator. We have also verified that Divisia has certain advantages in the empirical model. However, a substitute for nonborrowed reserves in the money market block of variables is still necessary to identify monetary policy shocks following the 2008 financial crisis. A natural candidate is the monetary base. The base has the desirable feature of always being positive. And since it equals the sum of currency and total reserves, the monetary base internalizes the separation of reserves into borrowed and nonborrowed components.

Furthermore, interest rates play a major role in both theoretical and empirical models of monetary policy, and the exclusion of an interest rate would seem to be a potentially serious weakness of the Divisia Benchmark model. Therefore, we add the 10-year Treasury to the money market block. Short term-to-maturity Treasury bills are not useful given that these rates, like the fed funds rate, have been fairly close to lower bounds since the Great Recession began. Furthermore, those rates have changed very little over the last few years. Basic term structure theory can explain extremely low rates on short-term securities as a consequence of the Fed’s “Forward-Guidance” policy.

Our new specification, which replaces nonborrowed reserves with the monetary base and includes the 10-year Treasury rate, is labeled Divisia Model-A. Results for this model are reported in Figure 5. The first column reports impulse responses estimated for the 1967:Q1 - 2007:Q4 period to compare with our earlier results while the second column provides estimates for the 1967:Q1 - 2013:Q2 to determine if adding the financial crisis to the sample has any effect on our results.

When estimated over the pre-crisis sample period, the results for Divisia Model-A are largely the same as in the Divisia Benchmark model except for the new variables. Interestingly, the base response is always negative and statistically significant in the pre-crisis period. Thus, the relatively weak and insignificant nonborrowed reserves response found in the Divisia Benchmark model is not replicated by the monetary base. But the total reserves response remains insignificant. Following an unexpected decrease in the Divisia monetary aggregate, the interest rate response is positive for more than 8 quarters and this effect is statistically significant in the short-run. Afterwards this response turns negative and eventually becomes almost significant. This sign switch has an intuitive

\footnote{Using either the 3-year or the 5-year Treasury yields qualitatively the same results.}
interpretation. The initial rise in interest rates is associated with the liquidity effect, but this effect is temporary and as prices eventually begin to fall disinflation leads to a reduction in the 10-year Treasury rate through the Fisher effect.

The second column in Figure 5 extends the sample to 2013Q2 to include the crisis. Qualitatively the effects are similar to the pre-crisis sample results with a few minor differences. The price response is weaker and this seems to be reflected by the fact that the interest rate now fails to exhibit a Fisher effect, although the liquidity effect is still present. Not only is the price response weaker but the decline is no longer significant. In fact, there is a small, almost statistically significant, price puzzle for the first year and a half. The largest differences are witnessed in the responses of the monetary base and total reserves. Including the crisis period causes these responses to both become very large compared to the non-crisis sample estimates. Also, these effects are initially insignificant but eventually become significantly negative.

### 4.5 Eliminating the Simple Sum Measure of Money

Divisia Model-A shows a price puzzle in the full sample estimate. Model Prediction 2 suggests this may be related to the fact that a simple sum aggregate is in the model. Our theoretical analysis shows that simple sum money may possess conditional correlations consistent with the standard empirical puzzles found in the monetary VAR literature. Based on that result, we remove simple sum from Divisia Model-A to form Divisia Model-B. The results for this model are reported in Figure 6 where the first column contains the responses estimated for the pre-crisis sample period and the second one provides estimates for the full sample. Results in Model-B are essentially the same as Model-A for all variables common to both models, except that we no longer have a price puzzle. This finding provides empirical support for Model Prediction 2.

[Figure 6 about here.]

### 4.6 Using the User Cost of Money as a Measure of Interest Rates

Monetary aggregation theory suggests that rather than arbitrarily choosing a single interest rate, the choice of how much money to hold depends on its user cost. In the appendix, we show that the aggregate user cost is, in essence, a weighted average of interest rates spreads.\(^{21}\) Moreover, Model

\(^{21}\text{Eq. A.49 shows the user cost of money, } u_t = \left[ \nu \left( (i^L_t - 1)/i^L_t \right)^{(1-\omega)} + (1 - \nu) \left( (i^I_t - i^L_t)/i^I_t \right)^{(1-\omega)} \right]^{1/\omega}. \)
Prediction 3 suggests that the user cost contains much the same information as the fed funds rate. Previously, we showed in Table 3 that the correlation between user cost and the policy interest rate is relatively high in the data, and the theoretical model exhibits similar correlation. Figure 2 plots the fed funds rate and the user cost of Divisia M4 to better illustrate this pattern of co-movement. We are the first to theoretically show that a strong relationship between these two variables may exist and provide empirical evidence which bolsters this claim.

Given the useful information contained in the user cost we replace the Treasury rate in Divisia Model-B with the Divisia M4 user cost to form Divisia Model-C, and estimate the model over the three sample periods used earlier: 1967:Q1-1995:Q2; 1967:Q1-2007:Q4 and 1967:Q1-2013:Q2. Figure 7 reports the impulse responses.

We find a significant output response that peaks between 3 and 6 quarters following a monetary shock depending on the sample period. The price level falls gradually while commodity prices fall more rapidly consistent with the idea that commodity prices react faster to news about monetary policy. Each of these responses is somewhat attenuated when the crisis period is included in the sample, although these differences are not statistically significant. The Divisia monetary aggregate falls while the user cost initially spikes upward yielding a statistically significant short-term liquidity effect in all cases.

The responses of monetary base and total reserves to a policy shock are the only ones sensitive to the choice of sample period. While both variables have significant negative responses in each period, responses are much larger in magnitude when the financial crisis period is included in the sample.

4.7 Removing Total Reserves

When Divisia is used as the policy variable we find that the monetary base and total reserves each have had qualitatively similar responses to a policy shock. This general result suggests there might be little cost to dropping one of these variables from the model. And in that case, the VAR model would have fewer coefficients and so we would obtain more efficient estimates of impulse responses.

We choose to remove total reserves from the model for two reasons. One comes from the VAR model estimates. A somewhat larger fraction of the monetary base impulse response is statistically significant in each model. The second reason stems from a principle of monetary theory. When a
central bank implements a policy that changes the amount of inside money its effect on monetary base will be the same irrespective of the amount of currency consumers hold or the amount of excess reserves commercial banks keep on hand. On the other hand, the amount of reserves may be affected by those decisions.

Therefore, we remove total reserves from the Divisia Model-C and label it Divisia Model-D. The response to all 3 sample periods are reported in Figure 8. Comparing these results with Figure 7, the model which includes total reserves, we see that each response is nearly unaffected by the removal of total reserves.

The monetary base continues to have a substantially different response to policy shocks depending on whether or not the financial crisis is included in the sample. Our DSGE model, which is most useful for understanding the effects of monetary policy during normal times, suggests that a traditional policy disturbance tends to have an immediate effect on the monetary base. This finding is largely corroborated from our pre-crisis (1967:Q1-2007:Q4) estimates in which the monetary base significantly falls on impact. Along with the base ultimately having a much larger response when the crisis period is included, the minimum for the monetary base response occurs well after the minimum of output, commodity prices and Divisia M4. Also, when the crisis period is included, the base response becomes significantly negative much later. These full sample findings suggest the macroeconomy responds faster than the monetary base to policy shocks. At first glance this finding might seem unintuitive, but we argue the policy regime in place during the crisis can explain the lagged base response.

In its capacity as lender of last resort, the Federal Reserve created several liquidity facilities in an effort to improve the functioning of credit markets. Such actions had strong signaling effects despite the fact that the full impact on the monetary base would not occur until months or years later. Three of the more prominent actions taken by the Fed involved assuring primary dealers a source of cheap funds through repo markets (PDCF), preventing the market for commercial paper from evaporating (CPFF) and providing liquidity in the financing markets for Treasury (TLSF). Figure 9 plots the interest rates for assets, particularly targeted by these facilities, along with the monetary
A rapid increase in the base lags the large fall in rates associated with the announcement of the respective programs.

Moreover, several other studies have found the various rounds of QE had large announcement effects despite the fact that the actual purchase of securities took place far into the future (Krishnamurthy and Vissing-Jorgensen, 2011; Gagnon, Raskin, Remache, and Sack, 2011). This point can be seen in Figure 10. In each round of quantitative easing the monetary base continues to rise for an extended period after the asset purchases begin. This protracted rise in the monetary base occurs despite the fact that for both QE 1 and QE 2, the total stock of assets the Fed purchased were announced at the start of the programs.

Therefore, we interpret the lagged monetary base response in the full sample estimates as further evidence that the Fed’s announcement effects served as a powerful tool during the crisis to assure markets that credit would continue to flow. This interpretation is further supported by the variance decompositions in the following section which analyzes the share of the forecast error variance of each variable explained by monetary policy shocks.

### 4.8 Quantitatively Assessing Monetary Policy Shocks


The share of output variance that is attributed to policy shocks is in the ballpark of other VAR work in this context. Our largest point estimate is roughly 20% and is obtained in the sample ending in 2007:Q4. Notice that the variance of output attributable to policy shocks falls when the estimation includes the recent financial crisis. In fact, for that sample, this variance is always less than 10%.

The price variance attributable to policy shocks is very small for the first two years. It rises substantially at longer horizons, except when the financial crisis period is included in the sample. In that case, a small amount of the variance of the price level is explained by policy shocks even after 20 quarters.
Quantitatively, different results are consistent with the events that occurred during the financial crisis. The manner in which the Federal Reserve financed unconventional policy tools, such as Quantitative Easing, eventually resulted in massive increases in the amount of reserves and the monetary base in the banking system. However, as a result of the financial crisis, banks were making far fewer loans and holding tremendous amounts of excess reserves. And so it takes a much larger increase in total reserves to generate a given level of monetary stimulus.

In all samples, the variance of the monetary base is primarily explained by non-policy shocks. This finding can be explained by the Fed’s tendency to follow an interest rate rule. If the central bank uses the interest rate as the policy instrument, then fluctuations in the demand for highly liquid assets will tend to be the dominant source of variability for the monetary base and total reserves.

Alternatively, the relative size of the monetary base’s forecast error variance at different horizons varies significantly across samples. We find that monetary policy explains nearly ten times as much base variance 20-quarters ahead as it does 8-quarters ahead when we include the crisis period. Alternatively, in the pre-crisis estimates, monetary policy explains about the same amount of base variance 8-quarters ahead as it does 20-quarters ahead. This is consistent with the idea that during the financial crisis monetary policy had a protracted impact on the monetary base.

The user cost variance explained by policy shocks shrinks when the crisis period is included. And these shocks also explain less of the variance of the Divisia aggregate, particularly at longer horizons. Our theoretical model predicts this outcome. Equations 2.13, 2.15, 2.16 and 2.18 show the user cost of money and the Divisia aggregate are driven by interest rates and financial sector shocks. Therefore when the financial shocks are large, such as during a major financial crisis, the contribution to variance made by policy disturbances decreases. Overall, while the monetary stimulus seems to have had some beneficial effects, the responsiveness of the economy to monetary policy has attenuated as a result of the crisis.

[Table 4 about here.]

[Table 5 about here.]

[Table 6 about here.]
5 Conclusion

Our goal in this paper is to characterize the effects of monetary policy shocks not only under normal conditions, but also during financial crises. A good deal of recent work has focused exclusively on explaining, or accounting for, the aftermath of the recent US financial crisis with methodological departures from what was overwhelmingly orthodox but a few years ago. Thus, one might conclude that a consensus model might work under normal conditions, and a different rule book should be used to characterize large financial crises. While it may certainly be worthwhile to study such (prima facie) different periods separately, we do not set out to analyze these sub-periods in a vacuum. Instead we develop a new method for identifying monetary policy shocks.

We extend the identifying assumption the central bank reacts to real economics activity and prices, but only affects these variables with a lag, to a framework that that retains its suitability whether the federal funds rate is positive or stuck at zero. Our specification draws heavily from the implications of a relatively standard New-Keynesian theoretical framework. One of our major innovations is to use a broad Divisia measure of money as the policy indicator variable. By relaxing the common assumption that the federal funds rate is a sufficient statistic to indicate the stance of monetary policy, we are able to elicit the response of the macroeconomy to policy shocks while the zero interest-rate policy is in effect.

This zero lower bound problem has led to a concerted search for other options. Our own search here focuses on alternative data measures that are officially released. Other work fills in the void with model-based proxies. Krippner (2013), Christensen and Rudebusch (2014), Lombardi and Zhu (2014), and Wu and Xia (2014) use factor models to estimate 'shadow' measures of the fed funds rate that extend beyond 2008. These shadow rates allow economists to continue to use macroeconomic models that assign important roles to the fed funds rate. This innovative approach is not without serious concerns.

Shadow rates are derived from empirical models and, thus, subject to estimation error. The potential for significant model error is perhaps even more problematic. For example, Christensen and Rudebusch (2014) warn against using the shadow rate to measure the stance of monetary policy based on their finding that the estimated shadow rates are very sensitive to the number of factors. Compounding these sampling and modeling uncertainties will likely leave little statistical
confidence in the effects of monetary shocks. In contrast, Divisia measures of money and user costs are nonparametric and so our approach is insulated from these predicaments.

Abandoning the fed funds rate as the policy indicator may raise concerns that we reach fundamentally different conclusions about the effects of monetary policy. However, our results show that when the zero lower bound does not hold the dynamic responses are very similar whether the Divisia aggregate or the fed funds rate is used. Importantly, the Divisia aggregate is able to measure the effects of monetary policy whether or not the fed funds rate is stuck at zero. However, completely eliminating interest rates from the model overlooks the important informative role they play in economic theory and empirical analysis of monetary policy. We show that the user cost contains much the same information as the fed funds rate – with the major advantage that it has not bumped up against a zero lower bound. Nonborrowed reserves have also behaved strangely since 2008, taking on theoretically impossible negative values. The monetary base is not subject to this problem and encompasses nonborrowed reserves. We also show that simple sum measures of money exhibit peculiar correlations in the theoretical model consistent with various puzzles economists have often in VARs. Therefore, we add the user cost of money, employ the monetary base in place of nonborrowed reserves and remove simple sum in our identification strategy. Our theoretically motivated identification strategy produces economically plausible responses of all variables to a monetary policy shock.

Given the considerable line of work investigating the effects of monetary policy, our novel identification scheme opens the door for extensive future research.\textsuperscript{22} For example, we have focused exclusively on the effects of monetary policy on macroeconomic aggregates but do not attempt to disentangle the various transmission mechanisms of monetary policy. Attempts to measure the existence and strength of the credit channel of monetary policy by Bernanke and Blinder (1992) and Gertler and Gilchrist (1994) should be re-examined to quantify the extent to which unconventional policy is transmitted through this channel. Along these same lines, McCallum (2000, 2006) argues that once interest rates reach zero monetary policy can gain traction through an exchange rate channel. The relative strength of this channel at the zero lower bound could be measured using our identification scheme in the model developed by Eichenbaum and Evans (1995).

\textsuperscript{22}See for example the many applications surveyed in Christiano et al. (1999). These applications could be reexamined over a much larger sample period using our identification strategy.
Turning to more theoretically oriented work, our approach could be used to estimate general equilibrium models using an impulse response matching approach. This estimation strategy, originally popularized by Rotemberg and Woodford (1997), seeks to minimize the difference between monetary policy shocks in the equilibrium model and those from an identified vector autoregression. Ideally, the policy indicator will be consistent across the theoretical and empirical models of monetary policy shocks. However, as implemented in Christiano, Eichenbaum, and Evans (2005), the fed funds rate is the policy indicator in the VAR while the DSGE model specifies an exogenous growth rule for money.23 Our empirical model of monetary policy shocks eliminates this inconsistency by providing researchers a method of identifying monetary policy shocks using a monetary aggregate as the policy indicator in the vector autoregression.

Due to multiple changes in stewardship and operating procedures of the Fed during our sample of study, an obvious direction for future research is to estimate, rather than impose, the structural break dates or allow for time-varying parameters. However, we find that across various sample periods monetary policy has qualitatively similar effects on the economy and therefore suspect these efforts would only lead to quantitative differences. This prediction is consistent with the well-known contribution of Sims and Zha (2006) who find that US monetary policy has undergone changes in regime, but the differences among them are not large enough to explain the Great Inflation nor the Great Moderation. Studying the effects of policy over the 2008 financial crisis, we find further qualitative robustness but attenuated quantitative effects of monetary policy on the economy.

Overall, our results suggest that during the 2008 financial crisis the Fed had to inject a larger amount of base money into the banking system to achieve a given amount of liquidity. We also find that when the crisis period is added to the sample, the effects on price and output of monetary shocks are more subtle. These findings are consistent with Friedman and Schwartz’s (1963) view that monetary policy becomes less effective during a financial crisis, and therefore policymakers must respond more aggressively to achieve the same effect on macroeconomic targets. The slow recovery from the Great Recession can also be understood to be due to this attenuation in the effectiveness of policy, a key factor determining the speed of recovery from financial crises according to Reinhart and Rogoff (2008).

23 The use of an exogenous money growth rule is necessary in the DSGE model due to the result that exogenous interest rate rules lead to multiple equilibria (Gali, 2008).
The implication, then, is that unconventional monetary policy is less effective than conventional policy. One explanation could be that the recent shocks arising from sources outside of monetary policy have been large, making it seem as if policy exerts less influence on output and prices. For example Jermann and Quadrini (2012) and Christiano, Motto, and Rostagno (2014) argue that exogenous financial shocks played an important role during the crisis. Other reasons may be uncertainty resulting from, amongst other factors, strident political disagreement as argued by Baker, Bloom, Canes-Wrone, Davis, and Rodden (2014). Differentiating between these possibilities and other explanations remains an open question for future research.
References


Table 1: Observationally Equivalent Monetary Policy Regimes

<table>
<thead>
<tr>
<th>Taylor Rule</th>
<th>Divisia Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_i = 0.0025 )</td>
<td>( \sigma_g = 0.0107 )</td>
</tr>
<tr>
<td>( \rho_i = 0.500 )</td>
<td>( \rho_g = 0 )</td>
</tr>
<tr>
<td>( \phi_{\pi}^i = 1.500 )</td>
<td>( \phi_{\pi}^g = 0 )</td>
</tr>
<tr>
<td>( \phi_y^i = 0.125 )</td>
<td>( \phi_y^g = 2.180 )</td>
</tr>
</tbody>
</table>

The coefficients on the Divisia instrument rules are found by minimizing the distance between IRFs to a policy shock in the Taylor rule (3.9) and the Divisia rule (3.8) in the DSGE model.
<table>
<thead>
<tr>
<th></th>
<th>$\hat{u}_t$</th>
<th>$\hat{y}_t$</th>
<th>$\hat{π}_t$</th>
<th>$\hat{g}^D_t$</th>
<th>$\hat{g}^{NS}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_t$</td>
<td>1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.5729</td>
<td>0.5386</td>
</tr>
<tr>
<td>$\hat{u}_t$</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.5729</td>
<td>0.5386</td>
</tr>
<tr>
<td>$\hat{y}_t$</td>
<td>1.00</td>
<td>0.5729</td>
<td>-0.5386</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{π}_t$</td>
<td>0.5729</td>
<td>-0.5386</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3: Correlations in the Model and the Data

<table>
<thead>
<tr>
<th></th>
<th>$\rho_{\tilde{u}, \tilde{i}}$</th>
<th>$\rho_{\Delta \tilde{u}, \Delta \tilde{i}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.497</td>
<td>0.473</td>
</tr>
<tr>
<td>Data</td>
<td>0.659</td>
<td>0.380</td>
</tr>
</tbody>
</table>

The moments in the model are computed assuming the central bank follows a Taylor rule specified in Equation (3.9).
### Table 4: Percentage Variance Due to Monetary Policy Shocks: Divisia Model-B (1967:Q1-1995:Q2)

<table>
<thead>
<tr>
<th></th>
<th>4 Quarters Ahead</th>
<th>8 Quarters Ahead</th>
<th>20 Quarters Ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>9.08</td>
<td>11.38</td>
<td>8.75</td>
</tr>
<tr>
<td></td>
<td>(1.37, 20.80)</td>
<td>(2.01, 23.94)</td>
<td>(2.43, 21.74)</td>
</tr>
<tr>
<td>GDP Deflator</td>
<td>1.17</td>
<td>4.13</td>
<td>55.01</td>
</tr>
<tr>
<td></td>
<td>(0.09, 7.21)</td>
<td>(0.25, 12.75)</td>
<td>(10.77, 58.16)</td>
</tr>
<tr>
<td>Commodity Prices</td>
<td>8.66</td>
<td>31.73</td>
<td>48.30</td>
</tr>
<tr>
<td></td>
<td>(0.85, 17.50)</td>
<td>(4.65, 41.88)</td>
<td>(8.08, 47.67)</td>
</tr>
<tr>
<td>Divisia M4</td>
<td>76.84</td>
<td>63.68</td>
<td>45.98</td>
</tr>
<tr>
<td></td>
<td>(45.68, 81.05)</td>
<td>(24.73, 68.90)</td>
<td>(14.88, 55.94)</td>
</tr>
<tr>
<td>Monetary Base</td>
<td>7.22</td>
<td>7.95</td>
<td>7.34</td>
</tr>
<tr>
<td></td>
<td>(1.29, 18.71)</td>
<td>(0.94, 19.62)</td>
<td>(0.66, 20.39)</td>
</tr>
<tr>
<td>User Cost (M4)</td>
<td>2.52</td>
<td>1.92</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>(1.05, 8.49)</td>
<td>(1.53, 11.39)</td>
<td>(1.96, 13.05)</td>
</tr>
</tbody>
</table>

Numbers in parentheses are the boundaries of the associated 90% confidence interval.
Table 5: Percentage Variance Due to Monetary Policy Shocks: Divisia Model-B (1967:Q1-2007:Q4)

<table>
<thead>
<tr>
<th></th>
<th>4 Quarters Ahead</th>
<th>8 Quarters Ahead</th>
<th>20 Quarters Ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>12.65 (4.81, 23.73)</td>
<td>20.03 (8.53, 33.42)</td>
<td>20.27 (7.68, 32.32)</td>
</tr>
<tr>
<td>GDP Deflator</td>
<td>0.05 (0.03, 2.24)</td>
<td>0.37 (0.09, 3.28)</td>
<td>20.64 (7.29, 29.03)</td>
</tr>
<tr>
<td>Commodity Prices</td>
<td>0.23 (0.08, 2.65)</td>
<td>5.26 (1.03, 9.06)</td>
<td>18.88 (6.30, 24.25)</td>
</tr>
<tr>
<td>Divisia M4</td>
<td>77.81 (57.22, 87.25)</td>
<td>66.04 (39.68, 78.75)</td>
<td>55.65 (29.23, 66.63)</td>
</tr>
<tr>
<td>Monetary Base</td>
<td>9.85 (1.53, 18.82)</td>
<td>15.81 (2.82, 24.23)</td>
<td>16.42 (3.20, 29.19)</td>
</tr>
<tr>
<td>User Cost (M4)</td>
<td>3.24 (1.45, 9.00)</td>
<td>2.42 (1.48, 8.96)</td>
<td>5.35 (3.00, 12.04)</td>
</tr>
</tbody>
</table>

Numbers in parentheses are the boundaries of the associated 90% confidence interval.
Table 6: Percentage Variance Due to Monetary Policy Shocks: Divisia Model-B (1967:Q1-2013:Q2)

<table>
<thead>
<tr>
<th></th>
<th>4 Quarters Ahead</th>
<th>8 Quarters Ahead</th>
<th>20 Quarters Ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>6.79</td>
<td>8.07</td>
<td>5.67</td>
</tr>
<tr>
<td></td>
<td>(1.82, 13.97)</td>
<td>(1.92, 17.30)</td>
<td>(1.36, 13.20)</td>
</tr>
<tr>
<td>GDP Deflator</td>
<td>0.06</td>
<td>0.44</td>
<td>3.82</td>
</tr>
<tr>
<td></td>
<td>(0.03, 1.77)</td>
<td>(0.06, 3.84)</td>
<td>(0.55, 15.39)</td>
</tr>
<tr>
<td>Commodity Prices</td>
<td>3.93</td>
<td>10.59</td>
<td>18.70</td>
</tr>
<tr>
<td></td>
<td>(0.92, 10.50)</td>
<td>(3.37, 20.75)</td>
<td>(8.56, 31.49)</td>
</tr>
<tr>
<td>Divisia M4</td>
<td>63.31</td>
<td>39.12</td>
<td>28.23</td>
</tr>
<tr>
<td></td>
<td>(42.55, 79.94)</td>
<td>(20.81, 64.89)</td>
<td>(14.05, 50.02)</td>
</tr>
<tr>
<td>Monetary Base</td>
<td>0.67</td>
<td>1.17</td>
<td>9.50</td>
</tr>
<tr>
<td></td>
<td>(0.48, 6.33)</td>
<td>(0.55, 6.62)</td>
<td>(3.32, 23.05)</td>
</tr>
<tr>
<td>User Cost (M4)</td>
<td>1.90</td>
<td>1.82</td>
<td>2.36</td>
</tr>
<tr>
<td></td>
<td>(1.05, 6.48)</td>
<td>(1.09, 6.48)</td>
<td>(1.28, 8.14)</td>
</tr>
</tbody>
</table>

Numbers in parentheses are the boundaries of the associated 90% confidence interval.
Figure 1: Contractionary Monetary Policy Shocks under alternative monetary policy regimes. Taylor Rule denotes the dynamics to a contractionary monetary policy shock under the policy rule in (3.9) and Divisia Growth Rule denotes the dynamics to a contractionary policy shock monetary policy shock under the rule defined in (3.8).
Figure 2: Divisia (M4) User Cost and the Fed Funds Rate
Figure 3: Benchmark Specifications (1967:Q1 - 1995:Q2)
The Fed Funds Benchmark model uses the fed funds rate as the policy indicator variable. The Nonborrowed Reserves Benchmark model uses nonborrowed reserves as the policy indicator. The Divisia Benchmark model replaces the fed funds rate in the Fed Funds Benchmark model with Divisia (M4).
**Figure 4: Benchmark Specifications (1967:Q1-2007:Q4)**
Same models as in Figure 3 except that we extend the sample to just before the onset of the 2008 financial crisis.
Figure 5: Divisia Model-A (Various Sample Periods)
Divisia Model-A replaces nonborrowed reserves in the Divisia Benchmark model with the monetary base and adds the 10-Year Treasury Constant Maturity Rate.
Figure 6: Divisia Model-B (Various Sample Periods))

Divisia Model-B removes simple sum money from Divisia Model-A.
Figure 7: Divisia Model-C (Various Sample Periods)
Divisia Model-C replaces the 10-Year Treasury Constant Maturity Rate in Divisia Model-B with the user cost of Divisia (M4).
Divisia Model-D removes Total Reserves from Divisia Model-C.
Figure 9: Announcement Effects
The data in all the subfigures are monthly. Each figure begins with the month before the announce-
ment of the respective liquidity facility. The TLSF was announced on March 11, 2008, and the
first auction was conducted on March 27, 2008. The PDCF was announced on March 16, 2008 was
available for business on March 17, 2008. The CPFF was announced on October 7, 2008, began
purchases of commercial paper on October 27, 2008. The monetary base is plotted in billions of $.
Figure 10: **Quantitative Easing**
This figure plots the monetary base (in billions of $). Within each round of QE, the monetary base peaks near the end of the round.
A DSGE Model

This appendix describes the DSGE model used in the paper in detail. The model follows from Belongia and Ireland (2012), however there are some adjustments and we develop additional results from the linearized model which are not in their paper. For this reason, we describe the model in detail. Below, $\Delta$ denotes the change over two consecutive time periods (in lag operator notation $\Delta = (1 - L)$). Finally, variables with a tilde denote log-deviations from steady-state.

A.1 The Household

The representative household enters any period $t = 0, 1, 2, ...$ with a portfolio consisting of 3 assets. The household holds maturing bonds $b_{t-1}$, shares of monopolistically competitive firm $j \in [0, 1] s_{t-1}(j)$, and wealth totaling $a_{t-1}$. The household faces a sequence of budget constraints in any given period. This budgeting can be described by dividing period $t$ into 2 separate periods: first a securities trading session and then bank settlement period.

In the securities trading session the household can buy and sell stocks and bonds. It also receives wages $w_t$ for hours worked $h_t$ during the period, purchases consumption goods $c_t$ and obtains any loans $l_t$ needed to facilitate these transactions. Any government transfers are also made at this time, denoted by $g_t$. Any remaining funds can be allocated between currency $n_t$ and deposits $d_t$. This is summarized in the constraint below.

$$n_t + d_t = \frac{a_{t-1}}{\pi_t} + \frac{b_{t-1}}{\pi_t} - \frac{1}{\delta_t} \int_0^1 g_t (s_t(j) - s_{t-1}(j)) \, di + w_t h_t + l_t - c_t + g_t$$

(A.1)

At the end of the period, the household receives dividends $f_t(j)$ on shares of stock owned in period $t$, $s_t(j)$, and settles all interest payments with the bank. In particular, the household is owed interest on deposits made at the beginning of the period, $i_t^d d_t$, and owes the bank interest on loans taken out, $i_t^l l_t$. Any remaining funds can be carried over into period $t + 1$ in the form of wealth, $a_t$.

$$a_t = n_t + \int_0^1 f_t(j) s_t(j) \, di + i_t^d d_t - i_t^l l_t$$

(A.2)
The household seeks to maximize their lifetime utility, discounted at rate $\beta$, subject to these constraints. The period flow utility of the household takes the following form.

$$u_t = \zeta_t \left[ \frac{c_t^{(1-\theta_c)}}{1-\theta_c} + v_t \frac{m_t^{(1-\theta_m)}}{1-\theta_m} + \eta (1 - h_t) \right]$$

The time-varying preference parameter $\zeta_t$ enters the linearized Euler equation as an IS shock and similarly, $v_t$ enters the linearized money demand equation as a money demand shock. Both of these processes are assumed to follow an AR(1) (in logs).

$$\ln(\zeta_t) = \rho \ln(\zeta_{t-1}) + \varepsilon^\zeta_t \quad \varepsilon^\zeta_t \sim N(0, \sigma^\zeta) \quad (A.3)$$

$$\ln(v_t) = (1 - \rho_v) \ln(\bar{v}) + \rho_v \ln(v_{t-1}) + \varepsilon^v_t \quad \varepsilon^v_t \sim N(0, \sigma^v) \quad (A.4)$$

The true monetary aggregate, $m_t$, which enters the period utility function takes a rather general CES form,

$$m_t = \left[ \nu \frac{n_t^{\frac{1-\omega}{\omega}} + (1 - \nu) \frac{1}{2} d_t^{\frac{1-\omega}{\omega}}}{\omega} \right]^{\frac{\omega}{1-\omega}} \quad (A.5)$$

where $\nu$ calibrates the relative expenditure shares on currency and deposits and $\omega$ calibrates the elasticity of substitution between the two monetary assets.

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. Letting $\Omega_t = [c_t, h_t, m_t, n_t, d_t, b_t, a_t, s_t(j)]$ denote the vector of choice variables, the household’s optimization problem can be recursively defined using Bellman’s method.

$$V_t \left( b_{t-1}, a_{t-1}, s_{t-1}(j) \right) = \max_{\Omega_t} \left\{ \zeta_t \left[ \frac{c_t^{(1-\theta_c)}}{1-\theta_c} + v_t \frac{m_t^{(1-\theta_m)}}{1-\theta_m} + \eta (1 - h_t) \right] \right. \quad$$

$$-\lambda_t^1 \left( d_t + n_t + c_t - w_t h_t - l_t - a_{t-1} - g_t b_t - b_{t-1} + \int_0^1 q_t(j)(s_t(j) - s_{t-1}(j))di + b_t \right) \quad$$

$$-\lambda_t^2 \left( m_t - \left[ \nu \frac{n_t^{\frac{1-\omega}{\omega}} + (1 - \nu) \frac{1}{2} d_t^{\frac{1-\omega}{\omega}}}{\omega} \right]^{\frac{\omega}{1-\omega}} \right) \quad$$

$$-\lambda_t^3 \left( a_t - n_t - \int_0^1 f_t(j)s_t(j)di - \nu d_t + \nu l_t \right) + \beta \mathbb{E}_t \left[ V_{t+1} \left( b_t, a_t, s_t(j) \right) \right] \}$$

The first order necessary conditions are given by the following equations. The system of equations (A.6)-(A.14) is under-determined in the sense that we have introduced various derivatives of the value function. However, we can complement these first order necessary conditions with the Bienveniste-
Scheinkman Envelope Conditions to eliminate the value function from the system above. These envelope conditions are given in equations (A.15)-(A.17) below.

\[
\begin{align*}
\zeta_t e^{-\beta t} - \lambda^1_t &= 0 \quad (A.6) \\
-\zeta_t \eta + \lambda^1_t w_t &= 0 \quad (A.7) \\
\zeta_t \nu_t m_t - \lambda^2_t &= 0 \quad (A.8) \\
n_t - \nu m_t \left[ \frac{\lambda^2_t}{\lambda^1_t - \lambda^2_t} \right]^\omega &= 0 \quad (A.9) \\
d_t - (1 - \nu) m_t \left[ \frac{\lambda^2_t}{\lambda^1_t - \lambda^2_t} \right]^\omega &= 0 \quad (A.10) \\
\lambda^1_t - \lambda^3_t &= 0 \quad (A.11) \\
-\lambda^1_t + \beta \mathbb{E}_t [V'_{t+1,b_t}(b_t, a_t, s_t(j))] &= 0 \quad (A.12) \\
-\lambda^3_t + \beta \mathbb{E}_t [V'_{t+1,a_t}(b_t, a_t, s_t(j))] &= 0 \quad (A.13) \\
-\lambda^1_t q_t(j) + \lambda^3_t f_t(j) + \beta \mathbb{E}_t [V'_{t+1,s_t(j)}(b_t, a_t, s_t(j))] &= 0 \quad (A.14)
\end{align*}
\]

Envelope Conditions:

\[
\begin{align*}
V'_{t,b_{t-1}}(b_{t-1}, a_{t-1}, s_t(j)) &= \frac{\lambda^1_t}{\pi_t} \quad (A.15) \\
V'_{t,a_{t-1}}(b_{t-1}, a_{t-1}, s_t(j)) &= \frac{\lambda^1_t}{\pi_t} \quad (A.16) \\
V'_{t,s_{t-1}}(j)(b_{t-1}, a_{t-1}, s_t(j)) &= \lambda^1_t q_t(j) \quad (A.17)
\end{align*}
\]

Now update (A.15)-(A.17) and substitute the resulting equations into (A.12)-(A.14) yielding:

\[
\begin{align*}
-\lambda^1_t + \beta \mathbb{E}_t \left[ \frac{\lambda^1_{t+1}}{\pi_{t+1}} \right] &= 0 \quad (A.18) \\
-\lambda^3_t + \beta \mathbb{E}_t \left[ \frac{\lambda^1_{t+1}}{\pi_{t+1}} \right] &= 0 \quad (A.19) \\
-\lambda^1_t q_t(j) + \lambda^3_t f_t(j) + \beta \mathbb{E}_t \left[ \lambda^1_{t+1} q_{t+1}(j) \right] &= 0 \quad (A.20)
\end{align*}
\]

The conditions (A.6)-(A.11) and (A.18)-(A.20) define the consumers optimal behavior.
A.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There are a unit measure of intermediate goods producing firms indexed by \( j \in [0, 1] \) who produce a differentiated product. The final goods firm produces \( y_t \) combining inputs \( y_t(j) \) using the constant returns to scale technology,

\[
y_t = \left( \int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta - 1}}
\]

in which \( \theta > 1 \) governs the elasticity of substitution between inputs, \( y_t(j) \). The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem,

\[
\max_{y_t(j), \, j \in [0, 1]} \left( \int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta - 1}} - \int_0^1 p_t(j)y_t(j) \, di.
\]

The resulting first order condition defines the demand curve for each intermediate goods producing firm’s product.

\[
y_t(j) = \left( \frac{p_t(j)}{p_t} \right)^{-\theta} y_t
\]

Intermediate Goods Producing Firm

Given the downward sloping demand for its product in (A.21), the intermediate goods producing firm has the ability to set the price of its product above marginal cost. Unlike the final goods market, the intermediate goods market is not purely competitive as evident by the downward sloping demand for its product in equation (A.21). To permit aggregation and allow for the consideration of a representative intermediate goods producing firm \( i \), we assume all such firms have the same constant returns to scale technology which implies linearity in the single input labor \( h_t(j) \),

\[
y_t(j) = z_t h_t(j).
\]

The \( z_t \) term in (A.22) is an aggregate technology shock that follows an AR(1) (in logs),

\[
\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_z).
\]
The price setting ability of each firm is constrained in two ways. First, each intermediate goods producing firm faces a demand for its product from the representative final goods producing firm defined in (A.21). Second, each intermediate goods producing firm faces a convex cost of price adjustment proportional to one unit of the final good defined by Rotemberg (1982) to take the form,

$$\Phi(p_t(j), p_{t-1}(j), y_t) = \frac{\phi}{2} \left[ \frac{p_t(j)}{p_{t-1}(j)} - 1 \right]^2 y_t.$$  

The intermediate goods producing firm maximizes its period $t$ real stock price, $q_t(j)$. Using the representative household's demand for firm $i$'s stock (A.20) and iterating forward defines the real (no-bubbles) share price as the discounted sum of future dividend payments.

$$q_t(j) = E_t \left[ \sum_{k=0}^{\infty} \beta^k \frac{\lambda^j_{t+k}}{\lambda^j_t} \hat{f}_{t+k}(j) \right]. \quad (A.24)$$

Though the firm maximizes period $t$ share price, the costly price adjustment constraint makes the intermediate goods producing firm's problem dynamic. Mathematically summarizing, each intermediate goods producing firm solves the following dynamic problem.

$$\max_{\{y_{t+k}(j), h_{t+k}(j), p_{t+k}(j)\}} E_t \left[ \sum_{k=0}^{\infty} \beta^k \frac{\lambda^j_{t+k}}{\lambda^j_t} \left[ \frac{p_{t+k}(j)}{p_{t+k-1}(j)} y_t - \frac{\phi}{2} \left[ \frac{p_{t+k}(j)}{p_{t+k-1}(j)} - 1 \right]^2 y_{t+k} \right] \right]$$

subject to

$$y_t(j) = \left[ \frac{p_t(j)}{p_t} \right]^{-\theta} y_t$$

$$y_t(j) = z_t h_t(j)$$

The problem can be simplified by substituting the inverse of the technology constraint for $h_t(j)$ and then substituting the factor demand into the resulting expression for $y_t(j)$ so that now the representative intermediate goods producing firm solves the following recursive problem defined by Bellman’s equation.

$$V_t(p_{t-1}(j)) = \max_{p_t(j)} \left\{ \lambda^j_t y_t \left[ \left[ \frac{p_t(j)}{p_t} \right]^{1-\theta} - \frac{w_t}{z_t} \left[ \frac{p_t(j)}{p_t} \right]^{-\theta} \right] - \frac{\phi}{2} \left[ \frac{p_t(j)}{p_{t-1}(j)} - 1 \right]^2 \right\}$$

$$+ \beta E_t \left[ V_{t+1}(p_t(j)) \right]$$

57
The first order necessary condition for the problem is given by

\[
(1 - \theta) \frac{\lambda y_t}{\lambda^t} \left[ \frac{p_t(j)}{p_t} \right]^{-\theta} + \theta \frac{\lambda y_t}{\lambda^t} \frac{w_t}{w_t} \left[ \frac{p_t(j)}{p_t} \right]^{-1-\theta} - \phi \frac{\lambda y_t}{\lambda^t} \left[ \frac{p_t(j)}{p_t-1(j)} \right] - 1 \left[ \frac{1}{p_t-1(j)} \right] + \beta E_t [V'_{t+1}(p_t(j))] = 0
\]  

(A.25)

Once again invoking the Bienveniste-Scheinkman Envelope Condition we have

\[
V'_t(p_{t-1}(j)) = \frac{\lambda y_t}{\lambda^t} \phi \left[ \frac{p_t(j)}{p_t-1(j)} - 1 \right] \frac{p_t(j)}{(p_t-1(j))^2}.
\]  

(A.26)

Updating (A.26) one period and substituting into (A.25) and then multiplying the resulting equation by \(\lambda^t p_t\) yields

\[
(1 - \theta) \left[ \frac{p_t(j)}{p_t} \right]^{-\theta} + \theta \frac{w_t}{w_t} \left[ \frac{p_t(j)}{p_t} \right]^{-1-\theta} - \phi \left[ \frac{p_t(j)}{p_t-1(j)} - 1 \right] \frac{p_t}{p_t-1(j)} + \beta \phi E_t \left[ \frac{\lambda y_{t+1} y_{t+1} + 1}{\lambda^t y_t} \left[ \frac{p_{t+1}(j)}{p_t(j)} - 1 \right] \frac{p_t p_{t+1}(j)}{(p_t(j))^2} \right] = 0
\]  

(A.27)

In a symmetric equilibrium where \(p_t(j) = p_t \forall j \in [0, 1]\), (A.27) can be log-linearized in which case it takes the form of a New-Keynesian Phillips Curve relating current inflation to the average real marginal cost and expected future inflation.

A.3 The Financial Firm

The financial firm produces deposits \(d_t\) and loans \(l_t\) for its client, the household. Following Belongia and Ireland (2012), assume that producing \(d_t\) deposits requires \(x_t d_t\) units of the final good. In this case, \(x_t\) is the marginal cost of producing deposits and varies according to the AR(1) process (in logs),

\[
ln(x_t) = (1 - \rho_x)ln(\bar{x}) + \rho_x ln(x_{t-1}) + \varepsilon_t^x \quad \varepsilon_t^x \sim N(0, \sigma_x).
\]  

(A.28)

Therefore an increase in \(x_t\) can be interpreted as an adverse financial productivity shock.

In addition to this resource costs, the financial firm must satisfy the accounting identity which specifies assets (loans plus reserves) equal liabilities (deposits),

\[
l_t + \tau_t d_t = d_t.
\]  

(A.29)
Although changes in banking regulation have effectively eliminated reserve requirements, banks may often choose to hold reserves in lieu of making loans - a flight to quality of sorts. Therefore, instead of assuming that the central bank controls the reserve ratio $\tau_t$, we assume that it varies exogenously according to the AR(1) process (in logs),

$$\ln(\tau_t) = (1 - \rho_r)\ln(\bar{\tau}) + \rho_r \ln(\tau_{t-1}) + \varepsilon_t^\tau \quad \varepsilon_t^\tau \sim N(0, \sigma_\tau).$$

(A.30)

An increase in $\tau_t$ can therefore be interpreted as a reserves demand shock, as opposed to a change in policy.

The financial firm chooses $l_t$ and $d_t$ in order to maximize period profits

$$\max_{l_t, d_t} (i_l^t - 1)l_t - (i_d^t - 1)d_t - x_t d_t$$

subject to the balance sheet constraint (A.29). Since the loan and deposits markets are perfectly competitive, substituting the balance-sheet constraint into the profit function and imposing zero results in the loan-deposit spread,

$$i_l^t - i_d^t = \tau_t(i_l^t - 1) + x_t.$$  

(A.31)

This confirms that if banks choose to hold more reserves or become relatively less productive, the consumer will have to pay a higher interest rate on their loans relative to the rate they receive on their deposits.

### A.4 Central Bank Policy

As is standard in New-Keynesian economies, the system of equations is under determined without a specification of monetary policy. Interest-rate rules are typically used to describe central bank policy since Taylor’s (1993) seminal paper,

$$\tilde{i}_t = \rho_r \tilde{i}_{t-1} + \phi_r \tilde{\pi} + \phi_y \tilde{y} + \varepsilon_t^i$$

(A.32)

where we have augmented the rule slightly to include a policy disturbance (which is white noise) and allow for the possibility that the central bank chooses to smooth movements in interest rates.
However, we may also describe central bank policy by a money growth rule. In this economy, there are multiple monetary assets, \( n_t \) and \( d_t \). This raises the question as to which monetary aggregate to control. One popular index produced by most central banks is the simple sum aggregate.

**Definition 3.** The growth rate of the nominal Simple-Sum monetary aggregate is defined by

\[
\ln(g_t^{SS}) = \ln\left(\frac{n_t + d_t}{n_{t-1} + d_{t-1}}\right) + \ln(\pi_t) \tag{A.33}
\]

However, this popular aggregate lacks many desirable features one would want in an index number. In particular, the simple sum aggregate implicitly assumes \( n_t \) and \( d_t \) are perfect, one for one, substitutes. This is only true in the extreme cases that the true aggregate has only one asset or these assets enter the blockwise separable sub-utility function in a linear fashion.

An alternative to the simple sum index is the Divisia Monetary Aggregate proposed by Barnett (1980). This index has been extensively studied over the last 30 years and has been shown to outperform the simple sum alternative in both empirical and theoretical applications. Simply put, the Divisia index number is better suited to track the growth rate of the monetary service flow than its simple sum counterpart.

**Definition 4.** The growth rate of the nominal Divisia monetary aggregate is defined by

\[
\ln(g_t^D) = \left(\frac{s_t^n + s_{t-1}^n}{2}\right) \ln\left(\frac{n_t}{n_{t-1}}\right) + \left(\frac{s_t^d + s_{t-1}^d}{2}\right) \ln\left(\frac{d_t}{d_{t-1}}\right) + \ln(\pi_t) \tag{A.34}
\]

where \( s_t^n \) and \( s_t^d \) are the expenditure shares of currency and interest bearing deposits respectively defined by

\[
s_t^n = \frac{u_t^n n_t}{u_t^n n_t + u_t^d d_t} = \frac{(i_t^c - 1)n_t}{(i_t^c - 1)n_t + (i_t^d - i_t^c)d_t} \tag{A.35}
\]

\[
s_t^d = \frac{u_t^d d_t}{u_t^n n_t + u_t^d d_t} = \frac{(i_t^c - i_t^d)d_t}{(i_t^c - 1)n_t + (i_t^d - i_t^c)d_t}. \tag{A.36}
\]

By weighting the growth rate of the individual assets (with time-varying weights) the Divisia monetary aggregate is able to successfully account for changes in the composition of the aggregate.

\[\text{As specified in equation (A.5), these respective conditions are equivalent to: } \nu = 1 \text{ or } \nu = 0, \text{ or } \omega \to \infty.\]
which may have no impact on the overall aggregate. This accuracy places the Divisia index amongst Diewart’s (1976) class of superlative index numbers, meaning the Divisia index has the ability track any linearly homogeneous function (with continuous second-derivatives) up to second-order accuracy. Due to this superior accuracy, we choose to define the money growth rule in terms of the Divisia monetary aggregate (augmented to include a white noise monetary policy shock).

\[ \tilde{g}_t^D = \rho g_{t-1}^D - \phi_g \tilde{g}_t - \phi_y \tilde{y}_t - \varepsilon_t^g \]  

(A.37)

### A.5 Market Clearing

It is now possible to define the equilibrium conditions that close the model. Equilibrium in the final goods market requires that the following accounting identity holds

\[ y_t = c_t + x_t d_t + \frac{\phi}{2} [\pi_t - 1]^2 y_t \]  

(A.38)

Equilibrium in the asset market, equity market and bond market requires that at all times

\[ a_t = \frac{a_{t-1}}{\pi_t} + w_t h_t - c_t + g t \]  

(A.39)

\[ s_t(j) = s_{t-1}(j) = 1 \]  

(A.40)

\[ b_t = b_{t-1} = 0 \]  

(A.41)

respectively. Finally, imposing the symmetry among the intermediate goods producing firms requires that in equilibrium

\[ y_t(j) = y_t \]  

(A.42)

\[ h_t(j) = h_t \]  

(A.43)

\[ p_t(j) = p_t \]  

(A.44)

\[ f_t(j) = f_t \]  

(A.45)

\[ q_t(j) = q_t \]  

(A.46)
A.6 The Log-Linear System

The large system of 17 variables\(^n\) can be condensed down to a much smaller 5-equation New-Keynesian model (when appended with a money demand equation and the definition of the Simple-Sum aggregate) focusing on the typical macro time-series of interest. In what follows, variables with a bar denote steady-state values and variables with a tilde denote log-deviations from steady-state.

The IS Curve

The dynamic IS curve is the log-linearized version of the household’s Euler equation. In particular, combine equations (A.6) and (A.18) and take note of the stochastic process defined for \(\zeta_t\) in (A.3).

\[
\tilde{c}_t \approx E_t [\tilde{c}_{t+1}] - \frac{1}{\theta_c} (\tilde{i}_t - E_t [\tilde{\pi}_{t+1}]) + \left(1 - \rho \zeta \theta_c \right) \tilde{\zeta}_t
\]  

(A.47)

This equation relates consumption to the real interest rate, similar to a typical IS curve. However, the appearance of expected future consumption makes this equation dynamic.

The Demand for Money and Monetary Aggregation

Now I seek a log-linear demand for the monetary aggregate. Two points are worth noting before proceeding to the linearization of the household’s first order conditions. First, define the user costs of monetary assets, \(n_t\) and \(d_t\). Specifically, the user costs appear naturally as the price of monetary assets according to the familiar optimality condition from microeconomics which dictates, at an optimum, equating the marginal rate of substitution of currency for deposits to the ratio of the price of currency to the price of deposits.

\[
\frac{\partial u_t}{\partial n_t} = \frac{\partial u_t}{\partial m_t} \frac{\partial m_t}{\partial n_t} = \frac{\lambda_i^1 - \lambda_i^3}{\lambda_i^2} = \frac{\lambda_i^3 \tilde{i}_t^d - \lambda_i^3}{\lambda_i^2} = \frac{\tilde{i}_t^d - 1}{\tilde{i}_t^d} \equiv \frac{u_t}{u_t^d}
\]  

(A.48)

Second, we can also derive the exact price dual to the true quantity aggregate in a similar fashion. Instead of considering that the price of each monetary assets individually, consider the optimality condition the monetary aggregate must satisfy. For simplicity, consider the marginal rate of sub-

\textsuperscript{25}And 17 equations, (A.6)-(A.11), (A.18)-(A.20), (A.22), (A.27), (A.29), (A.31), (A.33), (A.34), (A.38), and either (A.32) or (A.37).
stitution of the monetary aggregate for consumption. Since the price of the consumption good is normalized to 1, the result will denote the price of the monetary aggregate.

\[ \frac{\partial u_t}{\partial m_t} = \lambda_t^2 \left[ \nu(u_t^n) (1-\omega) + (1-\nu)(u_t^d) (1-\omega) \right] = \frac{i_t^l - i_t^A}{i_t^l} = u_t \]  \hspace{1cm} (A.49)

The first equality follows from equations (A.6) and (A.8), the household’s first order conditions for \( c_t \) and \( m_t \) respectively. The second equality follows from solving equation (A.9) for \( d_t \), equation (A.10) for \( n_t \) and substituting the resulting expressions into equation (A.5). We define this expression to be \( u_t \), or the opportunity cost of holding the aggregate monetary asset \( m_t \).

The resulting aggregate user cost (A.49) as defined by Barnett (1978) is of the same form as the individual component user costs in equation (A.48). It can be verified that \( u_t \) is in fact the true price dual to the true monetary aggregate in equation (A.5) since it satisfies Fisher’s factor reversal test.

\[ m_t u_t = u_t^n n_t + u_t^d d_t \]  \hspace{1cm} (A.50)

Equation (A.50) states that the true quantity index times the true price index equals total expenditures, in this sense, (A.49) is the exact price dual to (A.5).

**Demand for the Monetary Aggregate**

Using the relationships defined in the above equations allows us to easily define a typical log-linear demand curve for the monetary aggregate. Combining (A.6) and (A.8) and taking logs,

\[ \tilde{m}_t = \frac{\theta_c}{\theta_m} \tilde{c}_t - \frac{1}{\theta_m} \tilde{u}_t + \frac{1}{\theta_m} \tilde{v}_t. \]  \hspace{1cm} (A.51)

Although this expression is compact, it is useful to further expand \( \tilde{u}_t \) in terms of \( \tilde{i}_t \).

\[ \tilde{u}_t \approx \left[ \frac{\partial \ln(u_t)}{\partial \ln(u_t^n)} \right] \tilde{i}_t + \left[ \frac{\partial \ln(u_t)}{\partial \ln(u_t^d)} \right] \tilde{i}_t + \left[ \frac{\partial \ln(u_t^d)}{\partial \ln(u_t^d)} \right] \tilde{d}_t + \left[ \frac{\partial \ln(u_t^d)}{\partial \ln(u_t^d)} \right] \tilde{d}_t \]

\[ = \left[ \tilde{s}^n \frac{1}{i - 1} + \tilde{s}^d \frac{(\bar{i} - \bar{x})}{(i - 1) + \bar{x}} \right] \tilde{i}_t + \tilde{s}^d \frac{(\bar{i} - 1)}{(i - 1) + \bar{x}} \tilde{d}_t + \tilde{s}^d \frac{\bar{x}}{(i - 1) + \bar{x}} \tilde{x}_t \]  \hspace{1cm} (A.52)

where

\[ \tilde{s}^n = \frac{\nu u N^{(1-\omega)}}{\nu u N^{(1-\omega)} + (1-\nu)u D^{(1-\omega)}} \quad \text{and} \quad \tilde{s}^d = 1 - \tilde{s}^n. \]
Combining (A.51) and (A.52) we have the following demand curve for the monetary aggregate:

\[ \tilde{m}_t \approx \eta_c \tilde{c}_t - \eta_i \tilde{i}_t - \eta_r \tilde{r}_t - \eta_x \tilde{x}_t + \eta_v \tilde{v}_t, \]

where

\[ \eta_c = \frac{\theta_c}{\theta_m}, \]
\[ \eta_i = \frac{1}{\theta_m} \left[ \frac{\bar{s}}{\bar{i}} \frac{1}{\bar{i} - 1} + \frac{\bar{g}d}{\bar{r}(\bar{i} - 1) + \bar{x}} \right], \]
\[ \eta_r = \frac{1}{\theta_m} \left[ \frac{\bar{g}d}{\bar{r}(\bar{i} - 1)} \right], \]
\[ \eta_x = \frac{1}{\theta_m} \left[ \frac{\bar{g}d}{\bar{r}(\bar{i} - 1) + \bar{x}} \right]. \]
\[ \eta_v = \frac{1}{\theta_m}. \]

**Accuracy Properties of Divisia and Simple-Sum Aggregates**

It is useful at this point to define the accuracy properties of the Divisia and Simple-Sum monetary aggregates in the context of the linearized economy. Two results emerge from this section. First, a log-linear approximation of the Divisia aggregate track the growth rate of the true monetary aggregate without error. The same can not be said for the simple sum aggregate. The simple sum aggregate and the growth rate of the true monetary aggregate differ endogenously, at first order, due to the loan rate and exogenously due to financial sector disturbances.

**Divisia Monetary Aggregate**

First notice that if we combine the component user costs and the aggregate user costs, defined in (A.48) and (A.49) respectively, with the demand for the component assets from the household's first order condition, equations (A.9) and (A.10), we can specify the factor demands for the components of the CES aggregate.

\[ n_t = \nu m_t \left( \frac{u_t}{u^t_t} \right)^\omega \]  
\[ (A.53) \]
\[ d_t = (1 - \nu) m_t \left( \frac{u_t}{u^t_t} \right)^\omega \]  
\[ (A.54) \]
Finally, notice that to these variables are zero since ‘ss’.

A first order Taylor approximation around the deterministic steady state (denoted by a subscript ‘ss’).

\[
\begin{align*}
\ln (g_t^D) - \ln \left( \frac{m_t}{m_{t-1}} \right) - \ln (\pi_t) \\
= \omega \left[ \Delta \ln \left( u_t \left( u_t^n, u_t^d \right) \right) - \frac{1}{2} \left( s_t^n + s_{t-1}^n \right) \Delta \ln \left( u_t^n \right) - \frac{1}{2} \left( s_t^d + s_{t-1}^d \right) \Delta \ln \left( u_t^d \right) \right] \\
= E^D \left( \ln(u_t^n), \ln(u_{t-1}^n), \ln(u_t^d), \ln(u_{t-1}^d) \right)
\end{align*}
\]

Now using these in the definition of the Divisia monetary aggregate we have the following useful expression for the difference between the growth rates of the Divisia monetary aggregate and the true monetary aggregate.

\[
\begin{align*}
\ln (g_t^D) - \Delta \ln (m_t) - \ln (\pi_t) \\
\approx \left[ \frac{\partial E^D}{\partial \ln(u_t^n)} \right]_{ss} \tilde{u}_t^n + \left[ \frac{\partial E^D}{\partial \ln(u_t^d)} \right]_{ss} \tilde{u}_t^d + \left[ \frac{\partial E^D}{\partial \ln(u_{t-1}^n)} \right]_{ss} \tilde{u}_{t-1}^n + \left[ \frac{\partial E^D}{\partial \ln(u_{t-1}^d)} \right]_{ss} \tilde{u}_{t-1}^d
\end{align*}
\]

Finally, notice that

\[
\begin{align*}
\left[ \frac{\partial E^D}{\partial \ln(u_t^n)} \right]_{ss} &= \omega \left[ \frac{\partial \ln(u_t)}{\partial \ln(u_t^n)} - \frac{1}{2} \left( s_t^n + s_{t-1}^n \right) \right]_{ss} \\
&= \omega \left[ \frac{\nu (\tilde{u}^n)^{1-\omega}}{\nu (\bar{u}^n)^{1-\omega} + (1 - \nu) (\tilde{u}^d)^{1-\omega}} - \frac{\nu (\tilde{u}^n)^{1-\omega}}{(\bar{u}^A)^{1-\omega}} \right] = 0
\end{align*}
\]

and

\[
\begin{align*}
\left[ \frac{\partial E^D}{\partial \ln(u_t^d)} \right]_{ss} &= \omega \left[ \frac{\partial \ln(u_t)}{\partial \ln(u_t^d)} - \frac{1}{2} \left( s_t^d + s_{t-1}^d \right) \right]_{ss} \\
&= \omega \left[ \frac{(1 - \nu) (\tilde{u}^d)^{1-\omega}}{(\bar{u}^n)^{1-\omega} + (1 - \nu) (\tilde{u}^d)^{1-\omega}} - \frac{(1 - \nu) (\tilde{u}^d)^{1-\omega}}{(\bar{u}^A)^{1-\omega}} \right] = 0.
\end{align*}
\]

\[\text{26Notice we do not include any terms of } s_t^n, s_{t-1}^n, s_t^d \text{ nor } s_{t-1}^d \text{ because derivative of the error term, } E^D, \text{ with respect to these variables are zero since } \Delta \ln (\tilde{u}^n) = \Delta \ln (\tilde{u}^d) = 0.\]
Summarizing this more compactly, we have that up to first order,

\[ \ln \left( g'^D_t \right) \approx \ln \left( \frac{m_t}{m_{t-1}} \right) + \ln(\pi_t). \]  

(A.55)

Therefore, in all the simulations, the growth rate of the Divisia monetary aggregate equals that of the true monetary aggregate.

**Simple-Sum Monetary Aggregate**

Proceeding in a similar fashion as we did for the Divisia monetary aggregate, substitute the component factor demands into the definition of the simple sum monetary aggregate.

\[ \ln(\text{g}^\text{SS}_t) - \Delta \ln(m_t) - \ln(\pi_t) = \omega \Delta \ln(u_t(u^n_t, u^d_t)) + \Delta \ln(\nu(u^n_t)^{-\omega} + (1 - \nu)(u^d_t)^{-\omega}) \]

\[ \equiv E^\text{SS}(\ln(u^n_t), \ln(u^d_t), \ln(u^n_{t-1}), \ln(u^d_{t-1})). \]

Now take a first-order Taylor expansion of the right hand side around the deterministic steady-state, denoted with a subscript ‘ss’.
Let \( \alpha = \bar{u}/\bar{n} \in (0, 1) \), then we have

\[
E_{SS}(\ln(u_n^l), \ln(u_l^d), \ln(u_{n-1}^l), \ln(u_{l-1}^d))
\]

\[
\approx \omega \left[ \frac{\nu(\alpha)^{\omega-1}}{\nu(\alpha)^{\omega-1} + (1 - \nu)} - \frac{\nu(\alpha)^{\omega}}{\nu(\alpha)^{\omega} + (1 - \nu)} \right] \Delta \bar{u}_t^p
+ \omega \left[ \frac{(1 - \nu)}{\nu(\alpha)^{\omega-1} + (1 - \nu)} - \frac{1}{\nu(\alpha)^{\omega} + (1 - \nu)} \right] \Delta \bar{u}_t^d
\]

\[
= \omega \left[ \frac{(1 - \nu)(\nu(\alpha)^{\omega} + 1)}{(\nu(\alpha)^{\omega})^{\omega-1} + (1 - \nu)} - \frac{(1 - \nu)(\nu(\alpha)^{\omega} + 1)}{(\nu(\alpha)^{\omega})^{\omega-1} + (1 - \nu)} \right] \Delta \bar{u}_t^p
+ \omega \left[ \frac{(1 - \nu)(\nu(\alpha)^{\omega} + 1)}{(\nu(\alpha)^{\omega})^{\omega-1} + (1 - \nu)} - \frac{(1 - \nu)(\nu(\alpha)^{\omega} + 1)}{(\nu(\alpha)^{\omega})^{\omega-1} + (1 - \nu)} \right] \Delta \bar{u}_t^d
\]

Summarizing this error term more compactly, we have:

\[
\Delta \bar{u}_t^p
\]

\[
\Delta \bar{u}_t^d
\]

\[
\Delta \bar{u}_t^p
\]

\[
\Delta \bar{u}_t^d
\]
\[ \ln \left( g_{\ln SS}^2 \right) - \ln \left( \frac{m_t}{m_{t-1}} \right) = E^{SS}(\ln(u^n_t), \ln(u^d_t), \ln(u^n_{t-1}), \ln(u^d_{t-1})) \]

\[ \approx \psi(\omega) \left[ \frac{\bar{i}\bar{x}}{(i-1)(i(i-1) + \bar{x})} \right] \Delta \tilde{t}_t \]

\[ - \psi(\omega) \left[ \frac{\bar{\tau}(i-1)}{\bar{\tau}(i-1) + \bar{x}} \right] \Delta \tilde{\tau}_t \]

\[ - \psi(\omega) \left[ \frac{\bar{x}}{\bar{\tau}(i-1) + \bar{x}} \right] \Delta \tilde{x}_t \]

\[ \approx \psi^{SS}_i \Delta \tilde{t}_t - \psi^{SS}_\tau \Delta \tilde{\tau}_t - \psi^{SS}_x \Delta \tilde{x}_t, \]

where:

\[ \psi(\omega) = \omega \left[ \frac{\alpha^\omega [(\alpha)^{-1} - (1-\nu)]}{(\nu^\omega (1-\nu))} \right] \]

\[ \psi^{SS}_i = \psi(\omega) \left[ \frac{\bar{i}\bar{x}}{(i-1)(i(i-1) + \bar{x})} \right] \]

\[ \psi^{SS}_\tau = \psi(\omega) \left[ \frac{\bar{\tau}(i-1)}{\bar{\tau}(i-1) + \bar{x}} \right] \]

\[ \psi^{SS}_x = \psi(\omega) \left[ \frac{\bar{x}}{\bar{\tau}(i-1) + \bar{x}} \right]. \]

Notice that if \( \omega \to \infty, \nu \to 1 \) or \( \nu \to 0 \) then \( \psi(\omega) \to 0 \). Except for these extreme cases, the simple sum monetary aggregate will fail to accurately track the true monetary aggregate.

**New-Keynesian Phillips Curve**

The New-Keynesian Phillips Curve, relating inflation to real marginal cost and expected future inflation, emerges from log-linearizing the intermediate goods producing firm’s optimal pricing rule in a symmetric equilibrium (where \( p_t(j) = p_t \quad \forall j \in [0,1] \) and \( \pi_t = p_t/p_{t-1} \)), (A.27), around the deterministic steady-state.

\[ 0 = \theta e^{\ln(w_t) - \ln(z_t)} - \phi \left( e^{2\ln(\pi_t)} - e^{\ln(\pi_t)} \right) \]

\[ + \beta \phi \tilde{E}_t \left[ e^{\Delta \ln(\lambda_t)} \left( e^{2\ln(\pi_{t+1})} - e^{\ln(\pi_{t+1})} \right) \right] \]

\[ \approx \theta \bar{\bar{w}} (\bar{w}_t - \bar{z}_t) - \phi \bar{\pi}_t + \beta \phi \tilde{E}_t [\bar{\pi}_{t+1}] \]
Now substitute the household’s consumption/leisure rule to eliminate the real wage. Also, make use of the fact that (A.27) evaluated in steady-state implies that \( \bar{w} = \frac{\theta - 1}{\theta} \).

\[
\tilde{\pi}_t \approx \frac{(\theta - 1) \theta_c}{\phi} [\tilde{z}_t - \tilde{z}_t] + \beta E_t [\tilde{\pi}_{t+1}]
= \kappa [\tilde{c}_t - \tilde{z}_t] + \beta E_t [\tilde{\pi}_{t+1}],
\]

where \( \kappa = (\theta - 1) \theta_c / \phi \).

**Output**

The goods market clearing condition is useful for defining output in this economy so that we have a relevant counterpart for the variable in the VAR. Any reasonable calibration will imply in steady-state that output is largely equal to consumption, to be precise however, log-linearizing (A.38) implies,

\[
\tilde{y}_t \approx \bar{c} \bar{y} \tilde{c}_t + (1 - \bar{c} \bar{y}) (\tilde{x}_t + \tilde{d}_t).
\]

(A.60)

**A.7 Calibration**

Before dynamics of monetary policy shocks can be studied, values must be assigned to the model’s parameters. In what follows, we describe the approach to choosing values for the parameters, following Kydland and Prescott’s (1982) calibration strategy. The choice of parameters is summarized in Table 7 below.

**Steady-State Parameters**

The model has 11 deep structural parameters, \( \beta, \eta, \theta_c, \theta_m, \bar{v}, \omega, \nu, \theta, \phi, \bar{\tau} \), and \( \bar{x} \), which pin down the steady-state. Values for these parameters are chosen to match U.S. quarterly data running from 1967:Q1 to 2007:Q4. Beginning with the household’s preference parameters, the model implies the benchmark interest rate equals the rate on one-period bonds. Using data from the Center for Financial Stability (CFS), the average quarterly gross benchmark interest rate equals 1.02, implying \( \beta = .98 \). Calibrating \( \eta = 1.7 \) implies the steady-state share of time spent working equals 1/3. Setting \( \theta_c = 1 \) implies log-utility. The average ratio of simple sum M2 to personal consumption expenditures, \( \frac{N+D}{C} = 3.3 \), the average value in the data when we set \( \bar{v} = .04 \). The CRRA parameter for monetary
services pins down the interest semi-elasticity. There is a long-standing literature estimating this parameter across various money-demand specifications, however a more recent estimate is provided by Ireland (2009) who finds that $\eta = 1.9$. Matching this estimate, we set $\theta_m = 10$.

An estimate for $\omega$, the elasticity of substitution between currency and deposits, can be found using OLS to estimate the following regression model.

$$\ln \left( \frac{n_t}{m_t} \right) = \beta_0 + \beta_1 \ln \left( \frac{u^n_t}{u_t} \right) + \varepsilon_t$$

Just as for the VAR model, the data for components and user costs are from the CFS. The resulting estimates are $\hat{\beta}_0 = -0.17$ and $\hat{\beta}_1 = -0.4956$ which implies that $\omega = .5$. Setting $\nu = .2566$ calibrates the steady state share $\frac{N}{N+D} = .164$, the average of currency to simple sum M2 over 1967 to 2007.

On the production side of the economy, the model can not identify $\theta$ and $\phi$ independently, but instead we set the ratio $\frac{(\theta - 1)\phi}{\phi} = .025 = \kappa$, the estimated slope of the NKPC from Rotemberg and Woodford (1997) who match the dynamic responses from an equilibrium model to an estimated VAR monetary policy IRF. As for the production of financial assets, $\bar{\tau} = .02$ matches the average reserves ratio from 1967 to 2007 using data from the Federal Reserve Bank of St.Louis on bank reserves and non-currency components of M2. Finally, setting $\bar{x} = 0.006$ implies the annualized steady-state spread, $\bar{i} - \bar{i}^D = .027$, the average annualized spread between the benchmark interest rate and the own-rate of return on the non-currency component of M2 using data from the CFS.

To verify the logic of the calibration, the coefficient determining the Simple-Sum’s error term, $\psi^{SS}_t$ (See Equation (A.56)), is estimated in Keating and Smith (2014), where they find that $\hat{\psi}^{SS}_t = 5.6362$ with standard error of $S.E. = .6467$. The implied value of $\psi^{SS}_t$ in the steady state of this model, given the above calibration, is slightly larger than 5 confirming the above parameterization of the steady-state is consistent with the data along this additional dimension.

[Table 7 about here.]

**Dynamic Parameters**

The model has a remaining set of parameters which does not enter into the steady-state of the model’s equations, but instead describes the central bank’s behavior and the statistical properties of the exogenous state variables, $\tilde{\zeta}_t$, $\tilde{v}_t$, $\tilde{z}_t$, $\tilde{\tau}_t$, $\tilde{x}_t$ and $\varepsilon_t^i$.  

70
The IS, money-demand, technology and monetary policy shocks are standard in the DSGE literature and have been estimated in similar models. In particular, Ireland (2004) model’s these variables as stationary AR(1) processes and estimates their autocorrelation parameters and standard deviations using maximum likelihood methods. Although the policy parameters are also estimated, the purpose of this study is largely to focus on the dynamics of monetary policy shocks under various rules. Hence in the paper we vary policy rules and the parameters characterizing those rules.

The financial shock processes have not been estimated in the context of a monetary New-Keynesian model. Fortunately though, both series can be recovered using observable variable and the model’s equations. Specifically, the ratio of deposits (non-currency components of M2) held as reserves can be recovered using the St. Louis Fed’s adjusted reserves series. The demeaned logged series has estimated parameters, $\rho_r = 0.98$ and $\sigma_r = 0.0038$. Then using this series along with the benchmark interest rate and the own rate of return on non-currency components of M2 and equation (A.31) allows us to back-out a time series for $x_t$. The demeaned logged series has estimated parameters, $\rho_x = 0.95$ and $\sigma_x = 0.1549$. 
A.8 The Complete Log-Linearized Model

The semi-structural parameters along with the complete log-linearized model are defined in this section. Below, we utilize the fact that \( \tilde{\eta}_t = \tilde{\eta}_t \).

### Endogenous Variables

\[
\begin{align*}
\tilde{c}_t &= E_t [c_{t+1}] - \frac{1}{\theta_c} (\tilde{g}_t - E_t [\tilde{g}_{t+1}]) + \left( \frac{1 - \rho_c}{\theta_c} \right) \tilde{c}_t \quad (A.8.1) \\
\tilde{m}_t &= \eta_c \tilde{c}_t - \eta_x \tilde{x}_t - \eta_r \tilde{r}_t + \bar{\eta} \tilde{\pi}_t \\
\tilde{\pi}_t &= \kappa [\tilde{c}_t - \tilde{z}_t] + \beta E_t [\tilde{\pi}_{t+1}] \quad (A.8.3) \\
\tilde{\pi}_t &= \rho \tilde{\pi}_{t-1} + \phi_i \tilde{\pi}_t + \phi_y \tilde{y}_t + \varepsilon_t \\
\end{align*}
\]

or

\[
\begin{align*}
\tilde{g}_t^D &= \rho \tilde{g}_{t-1}^D - \phi_x^g \tilde{x}_t - \phi_y^g \tilde{y}_t - \varepsilon^g_t \\
\tilde{y}_t &= \tilde{c}_t + \left( 1 - \frac{\tilde{c}}{\tilde{y}} \right) \tilde{y}_t \\
\tilde{u}_t &= s^n \tilde{u}_t^n + s^d \tilde{u}_t^d \\
\tilde{u}_t^d &= \left[ \frac{\tilde{\pi} - \tilde{x}}{\tilde{\pi}(i-1) + \tilde{x}} \right] \tilde{i}_t + \left[ \frac{\tilde{\pi}(i-1) - \tilde{x}}{\tilde{\pi}(i-1) + \tilde{x}} \right] \tilde{\pi}_t + \left[ \frac{\tilde{\pi}(i-1) - \tilde{x}}{\tilde{\pi}(i-1) + \tilde{x}} \right] \tilde{\pi}_t \\
\tilde{d}_t &= \tilde{m}_t - \omega (\tilde{u}_t^d - \tilde{u}_t) \\
\tilde{u}_t^n &= \frac{1}{i-1} \tilde{\pi}_t \\
\tilde{n}_t &= \tilde{m}_t - \omega (\tilde{u}_t^n - \tilde{u}_t) \\
\tilde{m}_t &= \tilde{m}_t - \tilde{m}_{t-1} + \tilde{\pi}_t \\
\tilde{g}_t^S &= \tilde{m}_t - \tilde{m}_{t-1} + \tilde{\pi}_t + \psi_i^S \Delta \tilde{i}_t - \psi_{\tilde{x}}^S \Delta \tilde{x}_t - \psi_{\tilde{\pi}}^S \Delta \tilde{\pi}_t \\
\end{align*}
\]

### Exogenous Variables

\[
\begin{align*}
\tilde{c}_t &= \rho \tilde{c}_{t-1} + \varepsilon_t^c \\
\tilde{v}_t &= \rho \tilde{v}_{t-1} + \varepsilon_t^v \\
\tilde{z}_t &= \rho \tilde{z}_{t-1} + \varepsilon_t^z \\
\tilde{\pi}_t &= \rho \tilde{\pi}_{t-1} + \varepsilon_t^\pi \\
\tilde{x}_t &= \rho \tilde{x}_{t-1} + \varepsilon_t^x \\
\end{align*}
\]
Combining equations (A.8.1) - (A.8.4), depending on the choice of policy rules (rules that don’t react to output) is an identified system in the sense that there are as many equations as endogenous variables. If output is included, then the system grows by 1 equation. We analyze the larger system so that auxiliary variables such as the simple sum monetary aggregate and the monetary base can be analyzed in the context of the model.

The semi-structural parameters $\kappa$, $\eta_c$, $\eta_i$, $\eta_r$, $\eta_\nu$ are defined as functions of the deep-parameters as follows:

\[
\begin{align*}
\kappa &= \frac{(\theta - 1) \theta_c}{\phi} \\
\eta_i &= \frac{1}{\theta_m} \left[ \frac{\bar{s}^{n} \bar{s}^{d}}{\bar{i} - 1} + \frac{\bar{s}^{d} (\bar{i} - 1)}{\bar{i} + \bar{x}} \right] \\
\eta_x &= \frac{1}{\theta_m} \left[ \frac{\bar{s}^{d} \bar{i}}{\bar{i} + \bar{r} (\bar{i} - 1) + \bar{x}} \right] \\
\eta_n &= \frac{1}{\theta_m} \left[ \frac{\bar{s}^{d} \bar{r} (\bar{i} - 1)}{\bar{i} + \bar{r} (\bar{i} - 1) + \bar{x}} \right] \\
\eta_\nu &= \frac{1}{\theta_m} \left[ \frac{\bar{s}^{d} \bar{r} (\bar{i} - 1)}{\bar{i} + \bar{r} (\bar{i} - 1) + \bar{x}} \right]
\end{align*}
\]

\[
\begin{align*}
\eta_c &= \frac{\theta_c}{\theta_m} \\
\eta_x &= \frac{1}{\theta_m} \left[ \frac{\bar{s}^{d} \bar{r} (\bar{i} - 1)}{\bar{i} + \bar{r} (\bar{i} - 1) + \bar{x}} \right] \\
\eta_\nu &= \frac{1}{\theta_m} \left[ \frac{\bar{s}^{d} \bar{r} (\bar{i} - 1)}{\bar{i} + \bar{r} (\bar{i} - 1) + \bar{x}} \right]
\end{align*}
\]

\[
\begin{align*}
\bar{s}^{d} &= 1 - \bar{s}^{n}.
\end{align*}
\]
Table 7: Calibration Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate:</td>
<td>$\beta$ 0.98</td>
</tr>
<tr>
<td>Disutility of Work:</td>
<td>$\eta$ 1.70</td>
</tr>
<tr>
<td>Consuption CRRA:</td>
<td>$\theta_c$ 1.00</td>
</tr>
<tr>
<td>Monetary Services CRRA:</td>
<td>$\theta_m$ 10</td>
</tr>
<tr>
<td>Money Demand Shock:</td>
<td>$\nu$ 0.04</td>
</tr>
<tr>
<td>CES Monetary Aggregate:</td>
<td>$\nu$ 0.25</td>
</tr>
<tr>
<td>CES Monetary Aggregate:</td>
<td>$\omega$ 0.5</td>
</tr>
<tr>
<td>Final Goods CES:</td>
<td>$\theta$ 2.25</td>
</tr>
<tr>
<td>Cost of Price Adjustment:</td>
<td>$\phi$ 50</td>
</tr>
<tr>
<td>Reserves Demand Shock:</td>
<td>$\bar{\tau}$ 0.02</td>
</tr>
<tr>
<td>Banking Productivity Shock:</td>
<td>$\bar{x}$ 0.0062</td>
</tr>
<tr>
<td>Preference Shock:</td>
<td>$\rho_\zeta$ 0.9579</td>
</tr>
<tr>
<td>Money Demand Shock:</td>
<td>$\rho_\nu$ 0.9853</td>
</tr>
<tr>
<td>Technology Shock:</td>
<td>$\rho_z$ 0.9853</td>
</tr>
<tr>
<td>Reserves Demand Shock:</td>
<td>$\rho_r$ 0.9843</td>
</tr>
<tr>
<td>Banking Productivity Shock:</td>
<td>$\rho_x$ 0.9535</td>
</tr>
<tr>
<td>Preference Shock:</td>
<td>$\sigma_\zeta$ 0.0188</td>
</tr>
<tr>
<td>Money Demand Shock:</td>
<td>$\sigma_\nu$ 0.0088</td>
</tr>
<tr>
<td>Technology Shock:</td>
<td>$\sigma_z$ 0.0098</td>
</tr>
<tr>
<td>Reserves Demand Shock:</td>
<td>$\sigma_r$ 0.0319</td>
</tr>
<tr>
<td>Banking Productivity Shock:</td>
<td>$\sigma_x$ 0.1549</td>
</tr>
<tr>
<td>Monetary Policy Shock:</td>
<td>$\sigma_i$ 0.0025</td>
</tr>
</tbody>
</table>