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Does Risk-Taking Increase or Decrease with Higher Interest Rates?*

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Abstract

We present a framework that accounts for how interest rates affect risk-taking by borrowers indirectly, by changing the borrower's demand for credit (investment size). We find that this borrowing demand effect runs counter to the direct borrowing rate effect, and risk-taking can increase or decrease with higher rates depending on the relative strength of these effects. We show that the borrowing rate effect dominates when the borrower's share of project returns is increasing in investment, so risk-taking increases with interest rates. However, the borrowing demand effect dominates when the borrower's share of project returns is declining with investment demand, so that risk-taking decreases with higher interest rates. These results contribute to the understanding of linkages between monetary policy and financial stability. We apply our findings to study how lender competition affects risk-taking.

JEL codes: D82, G21, L13.
Keywords: borrowing rate; risk-shifting; monetary policy; financial stability; lender competition.

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1. Introduction

The global financial crisis has renewed interest in the linkages between monetary policy and financial stability. As monetary authorities adjust interest rates to stabilize economic activity, these changes also tend to affect agents' incentives for risk-taking. The financial stability implications of monetary policy have motivated a re-examination of the relation between interest rates and risk-taking. At the heart of the problem is the incentive to "reach for yield" that has been described as "the tendency to buy riskier assets to achieve higher returns" (La Spada, 2018, p.87). Economic theory has long argued that, under adverse selection or moral hazard, this risk-incentive mechanism gains strength for the borrower with an *increase* in rates (Stiglitz and Weiss, 1981). More recently, however, economists have suggested that this risk-incentive mechanism also grows stronger for lenders (financial intermediaries, in general) when rates *decrease* (Rajan, 2006; Taylor, 2013; Summers, 2016). These reflections would suggest that both high and low interest rates can lead to build-up of risks in the financial system. For this reason, and in spite of the large body of work on this topic, the linkages between interest rates and risk-taking continue to confound academics and policymakers.

In this paper, we examine how interest rates affect risk-taking. Because interest rates reflect the price of borrowing, changes in interest rates not only affect risk-taking incentives but also the amount of borrowing. We construct a theoretical model of the relationship between interest rates, the volume of borrowing, and risk-taking incentives of borrowers. In so doing, we build a framework to understand how risk-taking can be inextricably linked to not only to interest rates but also the demand for investment that is affected by the changes in rates. Using this framework allows us to determine how underlying economic conditions influence the risk-incentive mechanisms described above. This way, we add to previous work by delineating the conditions under which risk-taking incentives can increase or decrease.

Our results show that risk-taking can increase or decrease with higher interest rates and the outcome depends on the underlying productivity of investment. Using a lender-borrower framework wherein the lender faces moral hazard and the borrower has limited liability, we find that there are two ways in which interest rates affect risk-taking. Raising the interest rate on borrowed funds increases the debt burden for borrowers, inducing them to opt for projects with higher risk that yield higher return when successful. We term this direct mechanism, first developed in Stiglitz and Weiss (1981), as the "borrowing rate effect" between interest rates and risk-taking.

Our contribution to the literature is to uncover an indirect mechanism that affects risk-taking by altering the level of investment as interest rates change. This mechanism is both independent of, and works in addition to the borrowing rate effect. We argue that incentive compatible risk-shifting is increasing in the level of investment. With diminishing marginal productivity, capital becomes less productive as investment size increases and this in turn induces the borrower to take on more risk. In other words, "those who invest more will be more liable to moral hazard" (Banerjee, 2003, p. 12). Because borrower's demand for credit (size of investment) declines with the interest rates, the effect of interest rates on risk-taking through this indirect mechanism is negative. Higher interest rates lower borrower investment demand, leading to a lower level of capital investment. With diminishing returns, lower capital investment implies higher productivity of investment, which tends to reduce risk-taking incentives. We term this indirect mechanism as the "borrowing demand effect", which runs counter to the borrowing rate effect in Stiglitz and Weiss (1981).

Taken together, we find that borrower risk-shifting incentives can increase or decrease with interest rates. The overall effect of interest rates on risk-taking comprises the borrowing rate and borrowing demand effects. Risk-taking incentives increase or decrease with interest rates depending on the relative strength of these effects. The determining factor, in the end, is how marginal project returns change with changes in interest rates and how this affects the allocation between lender and borrower returns.[1] We find that the borrowing rate effect dominates when the borrower's share of project returns is increasing in investment, so that risk-taking increases with interest rates. On the other hand, the borrowing demand effect dominates when the borrower's share of project returns is declining with investment demand, so that risk-taking decreases with interest rates.[2] The intuition is straightforward: Raising rates lowers investment and if borrowers' share of returns increases (decreases) as investment decreases then borrowers have incentives to take on less (more) risk.

[1]Project returns are divided between returns to the borrower (equity) and returns to the lender (debt). Lower interest rates not only increase marginal returns for the borrower (reducing interest burden on debt) but they also decrease returns by increasing the borrowing amount and lowering the productivity of investment. In the end, the effect of lowering rates on borrower payoff is ambiguous. A similar mechanism works in reverse for the lender.

[2]Strictly speaking, optimal risk-taking is increasing (decreasing) with interest rates when the output elasticity of investment is decreasing (increasing) in investment demand. It can be shown that the borrower's share of project returns is decreasing with output elasticity of investment.

2. Review of Literature

Traditional theories developed around the era of high interest rates in the 1980s argued that raising interest rates increase riskiness because of adverse selection or moral hazard (Stiglitz and Weiss, 1981). A more recent influential strand of the literature has argued that the highly accommodative stance of policy prior to the financial crisis of 2008 promoted a credit boom, high leverage, and excessive risk-taking by economic agents (Rajan, 2006; Borio and Zhu, 2012). At the same time, exceptionally low rates following the financial crisis would create similar incentives for the next crisis (Taylor, 2013; Summers, 2016).

Even in a simple lender-borrower framework, risk-incentives can change depending on the participant under consideration. Canonical models of adverse selection and moral hazard show that higher interest rates lead to more risk-taking by the ultimate debt-holders, namely borrowing firms and households (Stiglitz and Weiss, 1981). The focus on risk-taking incentives of the borrowing household or firm also forms the bedrock for influential macroeconomic models of the financial accelerator (Bernanke and Gertler, 1989, 1995; Bernanke et al., 1999). In contrast, recent studies have argued that low rates can also increase risk-taking by examining monitoring incentives on the lender side (Dell'Ariccia and Marquez, 2013; Dell'Ariccia et al., 2014; Martinez-Miera and Repullo, 2017, 2019). By incorporating lender monitoring to the framework, these models show that reducing interest rates create "search for yield" incentives that, in turn, tend to reduce lender monitoring, thereby increasing risk-taking. Because these monitoring incentives tend to be associated with banks and other financial intermediaries, incentives for risk-taking at sufficiently low rates is arguably greater for financing that is intermediated as opposed to market-based financing.

This paper departs from existing theories in important ways: First, the borrowing amount (investment size), which changes with interest rates, is determined endogenously. In fact, we show that the findings of previous research discussed above can be obtained if we assume that all borrower investments have a fixed size. For this reason, the results of previous research are not only special cases but also consistent with predictions of our model. Second, we focus on systemic risk. Following Allen and Gale (2004) and Boyd and De Nicolo (2005), we assume that project risks are perfectly correlated across borrowers so that risk-taking by borrowers coincides with risk-taking by lenders.[3] This assumption helps focus on the systemic component of risk-taking which

[3]The risk associated with each project can in general be decomposed into a systemic and idiosyncratic

is crucial to financial stability.[4] Even in this simplified setting, we find that the relationship between interest rates and risk-taking is not one-sided. Depending on how the productivity of investment changes, risk-taking incentives can either increase or decrease with a rise in interest rates. A third way in which we depart from recent work on rates and risk-taking is abstracting from lender monitoring incentives. Doing so, makes our setting arguably more general in that the model applies not only to monitored financing (as done by banks, depository institutions and other regulated financial intermediaries) but also to unmonitored and market based financing (such as bond markets and shadow banking). As financial markets evolve these market-based and shadow banking sources of financing become more relevant to the overall intermediation landscape. In this sense, a study of the buildup of systemic risks across all sources of financing assume increased relevance for financial stability.

In what follows, Section 3 presents the model and illustrates the basic channels of risk-taking incentives. Section 4 characterizes how rates affect risk-taking incentives differently under the different production technologies. Section 5 discusses some empirical implications and a theoretical application of the model. Section 6 concludes.

3. The model

We model a credit market comprising three distinct groups of risk-neutral agents—a continuum of identical depositors and entrepreneurs (borrowers), both of measure 1, and $n \geq 2$ lenders (investors). The deposit supply function is perfectly elastic at a rate R which is normalized to zero.[5] We focus largely on the characterization of the loan market.

3.1. Entrepreneurs

Demand for credit is obtained from a simple model of borrowing under entrepreneur (borrower) moral hazard. We assume a contractual environment where entrepreneurs have access to a set of risky projects indexed by θ whose returns are random and per-

component. With a large number of projects, the idiosyncratic component can be perfectly diversified away. This assumption helps focus the baseline analysis on the common component representing systemic risks.

[4]This is not to suggest that considering imperfectly correlated risks are unlikely to change results. See Martinez-Miera and Repullo (2010) for how results on risk-taking can change when risks are imperfectly correlated.

[5]All our results are robust to assuming that the deposit supply function is sloping upward, as in Boyd and De Nicolo (2005).

fectly correlated (Allen and Gale, 2004; Boyd and De Nicolo, 2005). In general, the risk associated with the entrepreneurs' project can be decomposed into a systemic and idiosyncratic component. With a large number of projects, the idiosyncratic component can be perfectly diversified away. Therefore, the assumption of perfectly correlated returns helps focus the baseline analysis on the common component reflecting systemic risk. For this reason too, risk-taking by borrowers coincides with risk-taking by lenders.

Entrepreneurs have no assets and must borrow to invest in the project. If k dollars are invested in a given project θ, it yields

$$\tilde{y}(\theta, k) = \left\{ y(\theta) f(k) \right.$$

We assume that (i) the return, $y(\theta)$, is strictly increasing and strictly concave on $[0, \bar{\theta}]$, (ii) the probability of success, $p(\theta)$, is strictly decreasing and strictly concave on $[0, \bar{\theta}]$ with $p(0) = 1$ and $p(\bar{\theta}) = 0$, and (iii) the output function, $f(k)$, is strictly increasing and strictly concave on $[0, \bar{k}]$ with $f(0) \geq 0$. The variable θ represents the "riskiness" of the project—for a given k, the higher the θ, the higher is the return $y(\theta)$, but the lower is the probability of success, $p(\theta)$. Borrowers' choice of risk is not publicly verifiable, and therefore, not contractible.

3.2. Borrower moral hazard, investment and risk-taking

Borrowers have *limited liability* and the conjunction of moral hazard and limited liability affects terms of the loan contract. In granting loans, lenders cannot write contracts that are contingent on project riskiness θ because this is private information of the borrower. However, lenders correctly anticipate the risk-shifting incentives of borrowers, imposing a sequential rationality constraint on the equilibrium (Brander and Spencer, 1989). Given a borrowing rate $r \geq 1$, each borrower gets

$$v(r) = \max_{\theta} \left\{ p(\theta)(y(\theta) f(k) - rk) \mid \theta = \operatorname*{argmax}_{\theta'} \; p(\theta')(y(\theta') f(k) - rk) \right\} \tag{1}$$

The constraint in the maximization problem (1) is the *incentive compatibility* constraint of the borrower. Because the constraint function is strictly concave in θ, it can be replaced by the following first-order condition:

$$h(\theta) \equiv y(\theta) + y'(\theta) \cdot \frac{p(\theta)}{p'(\theta)} = \frac{r}{f(k)/k}. \tag{2}$$

Equation (2) can be expressed as the equality between the expected marginal revenue of risk-shifting and its expected marginal cost.[6] We can also restate this condition in terms of the incentive compatible level of risk-taking as a function of borrower investment and borrowing rate, $\theta(k;r)$.

Lemma 1. *The incentive compatible level of risk-shifting increases with the level of investment and borrowing rate, that is, $\theta_k(k;r) \geq 0$ and $\theta_r(k;r) \geq 0$.*

To understand the intuition behind the above result, let $V(\theta;k,r) \equiv p(\theta)(y(\theta)f(k) - rk)$ denote the expected payoff function of the entrepreneur. We find that

$$\frac{\partial}{\partial k}\left(\frac{\partial V}{\partial \theta}\right) \geq 0 \quad \text{and} \quad \frac{\partial}{\partial r}\left(\frac{\partial V}{\partial \theta}\right) \geq 0.$$

The first inequality asserts that the expected marginal payoff for the borrower with respect to risk-shifting is higher for higher k. Because $f(k)$ is concave and $f(0) \geq 0$, the average product decreases with k. From (2), the incentive compatible level of risk-shifting is increasing in investment k, and consequently, decreasing in the average product of investment, $f(k)/k$. For these reasons, we obtain that borrower's risk choice is increasing in investment, that is, they are *complementary*. As capital becomes less productive, higher investment incentivizes the borrower to take on more risk. In short, those who invest more are more liable to moral hazard (Banerjee, 2003).

The second inequality implies that borrower's expected marginal payoff from increased risk-taking is higher at higher borrowing rates, and therefore, risk shifting and the borrowing rate are also *complementary*. Increases in borrowing rate, r, raises the cost of investment and reduces borrowers' margin. This, in turn, increases the entrepreneur's incentives for risk-taking.[7]

The optimal investment, $k(r)$, is characterized by the following first-order condition

$$y(\theta(k;r))f'(k) = r. \tag{3}$$

[6] Condition (2) can be written as

$$[p'(\theta)y(\theta) + p(\theta)y'(\theta)]f(k) = p'(\theta)rk.$$

The left-hand-side and the right-hand side of this equation are the expected marginal revenue and the expected marginal cost of risk-shifting, respectively.

[7] The maximization problem (1) should also take into account the participation constraints of the lender and the borrower which are given by $p(\theta)rk \geq 0$ and $p(\theta)(y(\theta)f(k) - rk) \geq 0$, respectively. The first constraint holds because $p(\theta) \geq 0$, $k \geq 0$, and $r \geq 1$. The second constraint follows from (2) and the fact that $y(\theta) > h(\theta)$.

Condition (3) is the equality between the risk-adjusted marginal product of investment and the borrowing rate—the marginal cost of investment. The second-order necessary condition implies that $k'(r) \leq 0$, that is, each borrower's demand (for credit) function is *downward-sloping*. The optimal risk-shifting is given by $\tilde{\theta}(r) \equiv \theta(k(r); r)$.

3.3. Output elasticity of investment and risk-taking

Following the arguments above, we obtain a relationship between risk-shifting and productivity of investment. Dividing (2) by (3), we obtain

$$\frac{h(\theta)}{y(\theta)} = \frac{kf'(k)}{f(k)} \equiv \varepsilon(k), \tag{4}$$

where $\varepsilon(k)$ is the *output elasticity of investment*. Because $h'(\theta) > y'(\theta)$, the left-hand-side of (4) is increasing in θ, and as a result, the optimal risk-shifting is monotonically increasing in $\varepsilon(k)$.

Proposition 1. *Optimal risk-shifting is higher for the project with the higher output elasticity of investment*

To capture the intuition behind the above result, let $s(k)$ denote entrepreneur's payoff as a share of project returns when the project succeeds. We obtain the following:

$$s(k) = \frac{y(\theta)f(k) - rk}{y(\theta)f(k)} = 1 - \frac{h(\theta)f(k)}{y(\theta)f(k)} = 1 - \varepsilon(k). \tag{5}$$

The second equality follows from (2), while the last equality is obtained from (4). A higher elasticity of output implies that the entreprenuers' payoff as a share of project returns (when successful) is lower. With limited liability, lowering entrepreneurs' payoff as a share of total project returns (raising the output elasticity) incentivizes them to seek projects less likely to succeed but with higher returns when successful.

3.4. Effect of borrowing rates and risk-taking

There are two ways in which borrowing rates affect risk-taking in this framework. We define the direct effect of changes in borrowing rate on the entrepreneurs' optimal choice of risk as the *borrowing rate channel.* In addition, borrowing rates affect risk-taking indirectly by altering borrower's credit demand, k. This indirect effect of borrowing rate changes on risk-taking is defined as the *investment channel.* Taken together,

$$\frac{d\tilde{\theta}}{dr} = \underbrace{\theta_r(k;r)}_{\text{direct effect}} + \underbrace{\theta_k(k;r) \cdot k'(r)}_{\text{indirect effect}}. \tag{6}$$

Because $\theta_r(k;r) \geq 0$, the direct effect of an increase in borrowing rate is *positive*. So, the borrowing rate channel increases risk-taking. In contrast, we have $\theta_k(k;r) \geq 0$ and $k'(r) \leq 0$, and the indirect effect of an increase in borrowing rate is *negative*. Therefore, the investment channel decreases risk-taking. As a result, how borrowing rate changes affect optimal risk-shifting depends on the relative strengths of the two opposing effects. We summarize these results in terms of the following proposition.

Proposition 2. *Entrepreneurs' optimal risk choice, $\tilde{\theta}(r)$, is increasing (decreasing) in r according as the output elasticity of investment, $\varepsilon(k)$, is decreasing (increasing) in k.*

This result follows directly from Proposition 1. An increase in the borrowing rate, r, decreases borrower's investment, k because $k'(r) \leq 0$. As k decreases, it follows from (5) that entrepreneurs' payoffs as a share of project returns increases (decreases) according as $\varepsilon'(k) > (<)\, 0$. As entreprenuers' payoffs (as a share of returns) increase (decrease) it incentivizes them to choose projects with lower (higher) risk. The mechanisms work in reverse with decreases in borrowing rate r. In other words, the effect of changes in borrowing rates on risk-shifting is not unconditional, but depends on the productivity of investment for the entreprenuers' project. In the knife-edge case, when $\varepsilon'(k) = 0$, a decrease in k following an increase in r does not alter the entrepreneur's share of project returns, and consequently, optimal risk-shifting does not change with the borrowing rate.

Our results depart from conventional theories of moral hazard with limited liability which predict an unambiguously positive relation between borrowing rates and entrepreneurial risk-taking (Stiglitz and Weiss, 1981; Boyd and De Nicolo, 2005; Martinez-Miera and Repullo, 2010). The distinguishing feature in our analysis is that investment size is determined endogenously. In contrast, investment size is fixed in conventional theories.[8] In terms of our model, the optimality condition (2) for a fixed level of investment, k_0, is given by

$$h(\theta) = \frac{r}{f(k_0)/k_0}.$$

[8]The traditional models of borrower moral hazard (e.g. Stiglitz and Weiss, 1981) assume linear output function with fixed investment size, that is, $f(k_0) = k_0$. In this case, the optimality condition (2) boils down to $h(\theta) = r$.

For a fixed k_0, θ increases with r because $h'(\theta) > 0$. By fixing the level of investment, one can shut down the investment channel, and consequently, optimal risk-taking depends only on the borrowing rate and this relation is unambiguously positive. Put differently, the predictions of conventional theories are obtained as a special case of our general model, namely, the case where investment size is fixed. The novelty of our approach lies in accounting for borrowing demand (investment size), which changes with the borrowing rate, in an environment wherein the productivity of investment exhibits diminishing returns. In this setting, we find that risk-taking can increase or decrease with borrowing rates–this relationship is not one-sided.

4. Elasticity of Investment and Elasticity of Substitution

4.1. Output Elasticity of Investment

The effect of loan rate changes on risk-taking, $d\tilde{\theta}/dr_j$, depends on the relative magnitudes of the direct and indirect effects because they point in opposite directions. We show that the output elasticity of investment, $\varepsilon(k)$, is decreasing (increasing) in k according as the average productivity is more (less) responsive to changes in investment (alternatively, the production function is more (less) concave). When average productivity is more responsive, the indirect effect in (6) is likely to outweigh the direct effect. The converse is more likely when average productivity is less responsive. In sum, the relation between loan rates and risk-taking depends on the responsiveness of average productivity to changes in investment (the concavity of the production function). Table 1 illustrates how different functional forms of $f(k)$ yield differences in the relation between loan rates and risk-taking. Loan rates affect risk-taking through two channels—a positive direct effect, $\theta_r(k;\, r_j)$, and a negative indirect effect, $\theta_k(k;\, r_j)k'(r_j)$, which works through changes in optimal investment, $k(r_j)$.

	Functional form of $f(k)$	$\varepsilon'(k)$	Risk-taking ___ with loan rate
Example 1	$f(k) = k(1-k);\ 0 \le k \le 1/2$	negative	increases
Example 2	$f(k) = \sqrt{k_0 + k};\ \ k,\, k_0 > 0$	positive	decreases
Example 3	$f(k) = k^{\delta};\ k > 0, 0 < \delta < 1$	zero	does not change

Table 1: *The relationship between loan rate and the optimal risk-taking under different functional forms with $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$.*

4.2. A Constant Elasticity of Substitution (CES) Production Function

We can summarize our findings above using the following constant elasticity-of-substitution (CES) production technology:

$$f(k) = A(\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha))^{\frac{\sigma}{\sigma-1}} \tag{7}$$

where $A > 0$ is the Hicks-neutral technological change, $\alpha \in (0, 1)$ is the share parameter, and σ is the *elasticity of substitution* between capital and labor. The entreprenuer's production technology are expressed as a function of capital-labor ratio (or capital intensity), k, by normalizing labor inputs in the production process to 1 (see Klump and de La Grandville, 2000). Given the production function in (7), the output elasticity of investment is given by

$$\varepsilon(k) \equiv \frac{kf'(k)}{f(k)} = \frac{1}{1 + \frac{1-\alpha}{\alpha}k^{-\frac{\sigma-1}{\sigma}}}.$$

Using this, we obtain the following results.

1. The technology with the higher elasticity of substitution also has the higher elasticity of output. For any two technologies, $f_1(k)$ and $f_2(k)$, we have $\varepsilon_1(k) > \varepsilon_2(k)$ if and only if $\sigma_1(k) > \sigma_2(k)$. Recall from (5) that $\varepsilon_1(k) > \varepsilon_2(k)$ implies $s_1(k) < s_2(k)$. Intuitively, the greater the substitutability of capital for labor in the production technology, the lower is the capital's (entrepreneur's) share of project returns (Klump and de La Grandville, 2000).

2. The output elasticity $\varepsilon(k)$ is increasing (decreasing) in k, and hence, entrepreneur's profit share $s(k)$ decreasing (increasing) in k according as $\sigma(k) > (<)1$. For a Cobb-Douglas production function, $\sigma(k) = 1$, the output elasticity of investment is constant. It follows that $\varepsilon'(k) = 0$, and hence, the entrepreneur's share of project returns is fixed.

We obtain the following result.

Corollary 1. *Optimal risk-taking $\tilde{\theta}(r)$ is increasing (decreasing) in r in economies where capital and labor are complements (substitutes), that is, the elasticity of substitution is less (greater) than 1.*

A testable implication of Corollary 1 is that differences in the equilibrium association between rates and risk can be explained by differences in the degree of technological substitution between capital and labor in the production process. Production technologies that exhibit a positive association between investment demand and the entrepreneur's share of project returns yield a positive association between rates and risk-taking. Raising rates lowers investment demand as well as the entreprenuer's share of returns which incentivizes shifting to projects with higher returns and higher risk, as described in previous studies (Stiglitz and Weiss, 1981; Boyd and De Nicolo, 2005). The opposite holds for technologies that exhibit a negative association between investment demand and the entrepreneur's share of project returns. For these technologies, lowering rates increases investment demand but *reduces* the entreprenuer's share of returns which incentivizes shifting to projects with higher returns and higher risk. In contrast to previous work, the association between rates and risk-taking is negative.

5. Application: Loan market competition and risk-taking

We use the results of the model and apply them to analyze the effect of lender competition on entrepreneurial risk-taking. Project risks are perfectly correlated across borrowers so that risk-taking by borrowers coincides with risk-taking by lenders. Although returns are perfectly correlated, lender risk-taking is determined by an optimal contracting problem as discussed in Boyd and De Nicolo (2005) instead of a portfolio choice problem as modeled in Allen and Gale (2004).[9] We assume that borrowers are heterogeneous with respect to reservation utility, v. Let $H(v)$ be the cumulative distribution function of u, the fraction of borrowers with reservation utility less than u. Then, a borrower participates in the loan market only if $v(r) \geq v$. Thus, the loan demand in the market is given by $L(r) = H(v(r))$. The envelope condition for the maximization problem (1) implies that $v'(r) = -p(\tilde{\theta}(r))k(r) < 0$, that is, the loan demand function is downward-sloping. The inverse loan demand function is given by

$$r = r(L) \quad \text{with} \quad r'(L) \leq 0. \tag{8}$$

We assume $r(0) > 0$ and $r''(L) \leq 0$. We also assume that lenders have no equity, and hence, all of lender i's deposits are invested in loans, that is, $D_i = L_i$. Consequently, aggregate deposits at the market level are equal to aggregate loans, that is, $D = \sum_i^n D_i = \sum_i^n L_i = L$. Because lenders compete in a Cournot fashion in that each

[9] Martinez-Miera and Repullo (2010) show how risk-taking can change when project risks are imperfectly correlated.

lender i solves

$$\max_{L_i} \; P(L)r(L)L_i, \tag{9}$$

where $P(L) \equiv p(\tilde{\theta}(r(L)))$. The following lemma characterizes the unique symmetric Cournot equilibrium of the loan market.

Lemma 2. *(Boyd and De Nicolo, 2005) The unique symmetric equilibrium borrowing rate is given by*

$$r(L) = -\frac{r'(L)P(L)L}{P'(L)L + nP(L)}\,.$$

Moreover, the equilibrium borrowing rate, $r(L)$ decreases with competition as measured by the number of lenders, n.

The proof follows from Boyd and De Nicolo (2005). More lenders in the loan market implies greater aggregate loan volume which decreases equilibrium borrowing rates. However, unlike Boyd and De Nicolo (2005), the effect of increased market competition on borrowing rates is not straight forward. From Proposition 2, it follows that

Proposition 3. *Equilibrium risk-shifting increases (decreases) with loan market competition according as the output elasticity of investment, $\varepsilon(k)$ is increasing (decreasing) in k.*

While increased competition in the loan market unambiguously decreases borrowing rates, the effect on risk-taking is not unambiguous. Increased competition from an increase in the number of lenders decreases risk-taking if and only if risk-taking increases with borrowing rates. Proposition 2 asserts that this risk-incentive mechanism, first shown in Boyd and De Nicolo (2005), is obtained in the loan market when the production technology exhibits decreasing elasticity of investment. However, in situations where the production technology exhibits increasing elasticity of investment, we find that increased competition can increase risk-taking by borrowers. Using a natural experiment, Carlson, Correia, and Luck (2022) find that lenders operating in markets with lower entry barriers in the National Banking Era increased riskiness in lending and were more likely to default.[10] This empirical finding lends support for our result that increased competition within segmented markets can also increase risk-taking.

Martinez-Miera and Repullo (2010) analyze a loan market with imperfectly correlated default risk. Apart from the standard risk-shifting effect (Boyd and De Nicolo, 2005),

[10] **?**, p. 464 study competition *within* segmented markets: "The National Banking Era constitutes a close-to-ideal empirical laboratory to study the causal effects of banking competition . . . the prevalence of unit banking ensures that banking markets are local and well defined, which allows us to compare different, arguably independent markets."

they identify a countervailing *margin effect* which contributes to a non-monotonic relation between risk-taking and borrowing rates. Competition decreases borrowing rates, which work as a buffer against non-performing loans, which induce lenders to invest in riskier assets. Unlike Martinez-Miera and Repullo (2010), our risk-incentive mechanism under perfectly correlated returns operates via the investment channel. As more intense competition implies greater investment demand by individual borrowers, it may increase risk-shifting.

6. Conclusion

Research on the effect of interest rates and risk-taking remains inconclusive. We argue that one gap in this literature is not considering the effect that interest rates have on borrower demand, and consequently, the volume of investment. With diminishing returns to investment, borrowers investing more when rates are lower are more liable to moral hazard. Meanwhile higher rates also create incentives for borrowers to choose riskier projects that have higher returns when successful. These two opposing risk-incentive mechanisms are at play when rates change and the final outcome of rates on risk depends on which mechanism dominates. As a result the effect of rates on risk-taking is not unambiguous and depends on the underlying investment returns.

The finding that the effects of interest rates on risk-taking can vary depending on investment technologies informs the discussion on financial stability. This understanding can help guide considerations when adjusting rates. Depending on how this relationship between rates and risk-taking changes across firms and households, further research could explore how policies and regulations might address risk-taking incentives.

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Appendix A: Proofs

Proof of Proposition 2. Let $\phi(k) \equiv f(k)/k$ be the average product of investment. The first-order condition of (2) is given by:

$$\underbrace{\left\{y(\theta) + y'(\theta) \cdot \frac{p(\theta)}{p'(\theta)}\right\}}_{h(\theta)} f(k) - rk = 0 \quad \Longleftrightarrow \quad h(\theta) = \frac{r}{\phi(k)}, \tag{10}$$

which defines $\theta = \theta(k; r)$. Note that, because $y''(\theta) \leq 0 < y'(\theta)$, $p'(\theta) < 0$ and $p''(\theta) \leq 0$,

$$h'(\theta) = 2y'(\theta) + \frac{p(\theta)}{p'(\theta)} \left\{y''(\theta) - \frac{y'(\theta)p''(\theta)}{p'(\theta)}\right\} \geq 2y'(\theta) > 0.$$

Differentiating (10) with respect to k and r, respectively we obtain

$$\theta_k(k; r) = \frac{h(\theta)[1-\varepsilon(k)]}{h'(\theta)k} = \underbrace{\frac{rk}{f(k)}}_{h(\theta) \text{ from } (10)} \cdot \frac{1-\varepsilon(k)}{h'(\theta)k} > 0, \quad \text{and} \quad \theta_r(k; r) = \frac{k}{h'(\theta)f(k)} > 0, \tag{11}$$

where $\varepsilon(k) \equiv kf'(k)/f(k)$ is the output elasticity of investment. Because $f(k)$ is strictly concave and $f(0) \geq 0$, the average product of investment, $\phi(k)$ is strictly decreasing in k which is equivalent to $\varepsilon(k) < 1$.[11] Therefore, $\theta_k(k; r) > 0$. Note that the objective function of the maximization problem (2) is strictly concave in θ because $p(\theta) \geq 0$, $p'(\theta) < 0$, $p''(\theta) < 0$, $y(\theta) \geq 0$, $y'(\theta) > 0$ and $y''(\theta) < 0$ for all $\theta \in [0, \bar{\theta}]$, and hence, $\theta(k; r)$ is unique. Let $V(k; r)$ be the value function of the maximization problem (2). Then, by the Envelope theorem, we have $U_k(k; r) = p(\theta)\{y(\theta)f'(k) - r\}$, and hence, the first-order condition is given by:

$$V_k(k; r) = 0 \quad \Longrightarrow \quad y(\theta)f'(k) = r. \tag{12}$$

The second-order necessary condition is given by:

$$\begin{aligned} & p(\theta)\{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)\} + p'(\theta)\theta_k \{\underbrace{y(\theta)f'(k) - r}_{=0 \text{ by } (12)}\} \leq 0 \\ \Longrightarrow \quad & \underbrace{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)}_{\Omega(k,r)} \leq 0. \end{aligned} \tag{13}$$

Differentiating (12) with respect to r we obtain

$$k'(r) = \frac{1 - (y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k, r)}.$$

Observe that $h'(\theta) \geq 2y'(\theta)$ implies that $y'(\theta)/h'(\theta) \leq 1/2$. Therefore, the numerator of the last expression is strictly positive because $\varepsilon(k) < 1$. On the other hand, the denominator is negative by the second-order condition. Consequently, $k'(r) \leq 0$. Because $d\theta^*/dr = \theta_k \cdot k'(r) + \theta_r$ with θ_k, $\theta_r > 0$ and $k'(r) \leq 0$, the sign of $d\theta^*/dr$ is indeterminate.

We prove the final part of Proposition 2 that $d\theta^*/dr > (<)\, 0$ according as $\varepsilon'(k) < (>)\, 0$. Note that

$$\varepsilon'(k) = \frac{d}{dk}\left(\frac{kf'(k)}{f(k)}\right) = \frac{kf''(k) + f'(k)[1 - \varepsilon(k)]}{f(k)} \tag{14}$$

[11] To see this, take a twice differentiable function $f(k)$ that is strictly concave with $f(0) \geq 0$. Note that $\phi'(k) < 0$ is equivalent to $\phi(k) > f'(k) \iff \varepsilon(k) < 1$. Take any point $(k_0, f(k_0))$ on the graph of $f(k)$. Then, there is $\kappa \in (0, k_0)$ such that

$$\phi(k_0) \equiv \frac{f(k_0)}{k_0} \geq \frac{f(k_0) - f(0)}{k_0} = f'(\kappa) > f'(k_0).$$

The first (weak) inequality follows from the fact that $f(0) \geq 0$, the second equality holds for some $\kappa \in (0, k_0)$ which follows from the Mean Value theorem, and the last (strict) inequality is implied by $f''(k) < 0$ and $\kappa < k_0$. This proves that $\phi'(k) < 0$ as k_0 has been chosen arbitrary.

Now,

$$\frac{d\theta^*}{dr} = \underbrace{\frac{h(\theta)[1-\varepsilon(k)]}{h'(\theta)k}}_{\theta_k} \cdot \underbrace{\frac{1-(y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,r)}}_{k'(r)} + \underbrace{\frac{k}{h'(\theta)f(k)}}_{\theta_r}$$

$$= \underbrace{\frac{rk}{f(k)}}_{h(\theta)\text{ from (10)}} \cdot \frac{1-\varepsilon(k)}{h'(\theta)k} \cdot \frac{1-(y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,r)} + \frac{k}{h'(\theta)f(k)}$$

$$\iff \frac{d\theta^*}{dr} = \frac{h'(\theta)k\Omega(k,r) + r[1-\varepsilon(k)][h'(\theta)-y'(\theta)\varepsilon(k)]}{[h'(\theta)]^2 f(k)\Omega(k,r)} \equiv \frac{Q(k,r)}{[h'(\theta)]^2 f(k)\Omega(k,r)}. \tag{15}$$

Using the expression of θ_k from (11) and that of $\Omega(k, r)$ from (13), we obtain

$$Q(k,r) = \frac{rh'(\theta)}{f'(k)} \cdot \underbrace{\{kf''(k) + f'(k)[1-\varepsilon(k)]\}}_{f(k)\varepsilon'(k)\text{ from (14)}}.$$

Therefore,

$$\frac{d\theta^*}{dr} = \frac{r\varepsilon'(k)}{h'(\theta)f'(k)\Omega(k,r)},$$

which implies that $\text{sign}[d\theta^*/dr] = -\text{sign}[\varepsilon'(k)]$ because r, $h'(\theta)$, $f'(k) > 0$ and $\Omega(k,r) \leq 0$. This completes the proof of the proposition.

Examples in Table 1.

For all the examples below, we assume that $p(\theta) = 1-\theta$, $y(\theta) = \theta$.

1. Consider $f(k) = k(1-k)$ defined on $[0, 1/2]$ so that $f'(k) > 0$. For this functional form, $\varepsilon(k)$ decreases with k. In this case, equilibrium risk-shifting is strictly increasing in r_j and is given by:

$$\theta^*(r_j) = \frac{1}{4}\left(1 + \sqrt{1+8r_j}\right).$$

2. Consider $f(k) = \sqrt{k_0 + k}$ with $k_0 > 0$ an $k \geq 0$. In this case, the elasticity of investment is increasing in k, i.e., $\varepsilon'(k) > 0$. The equilibrium risk-shifting is given by:

$$\theta^*(r_j) = \frac{1}{2} + \frac{\sqrt{2}\left(1 - 24k_0r_j^2 + \sqrt{1-12k_0r_j^2}\right)}{12\left(1 - 6k_0r_j^2 + \sqrt{1-12k_0r_j^2}\right)^{1/2}}.$$

The above expression is decreasing in r_j.

3. Finally, let $f(k) = k^{\delta}$ with $\delta \in (0, 1)$. In this case, $\varepsilon(k) = \delta$ for all k. The optimal risk-taking is given by $\theta^*(r_j) = (2 - \delta)^{-1}$, which is independent of the loan rate.