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Asset Purchases in a Monetary Union with Default and Liquidity Risks*

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Abstract

Using a two-country monetary-union framework with financial frictions, we study sovereign default and liquidity risks and quantify the efficacy of asset purchases. Default risk increases with government indebtedness and shifts in the fiscal limit perceived by investors. Liquidity risks increase when the default probability affects credit market tightness. The framework indicates that shifts in fiscal limits, more than rising government debt, played a crucial role for Italy around 2012. While both default and liquidity risks can dampen economic and financial conditions, the model suggests that the magnifying effect from liquidity risks can be more consequential. In this context, asset purchases can stabilize economic conditions especially under scenarios of elevated financial stress.

JEL Classification: E58, E63, F45

Keywords: Monetary and fiscal policy interactions; Unconventional monetary policy; Regime-switching model

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1 Introduction

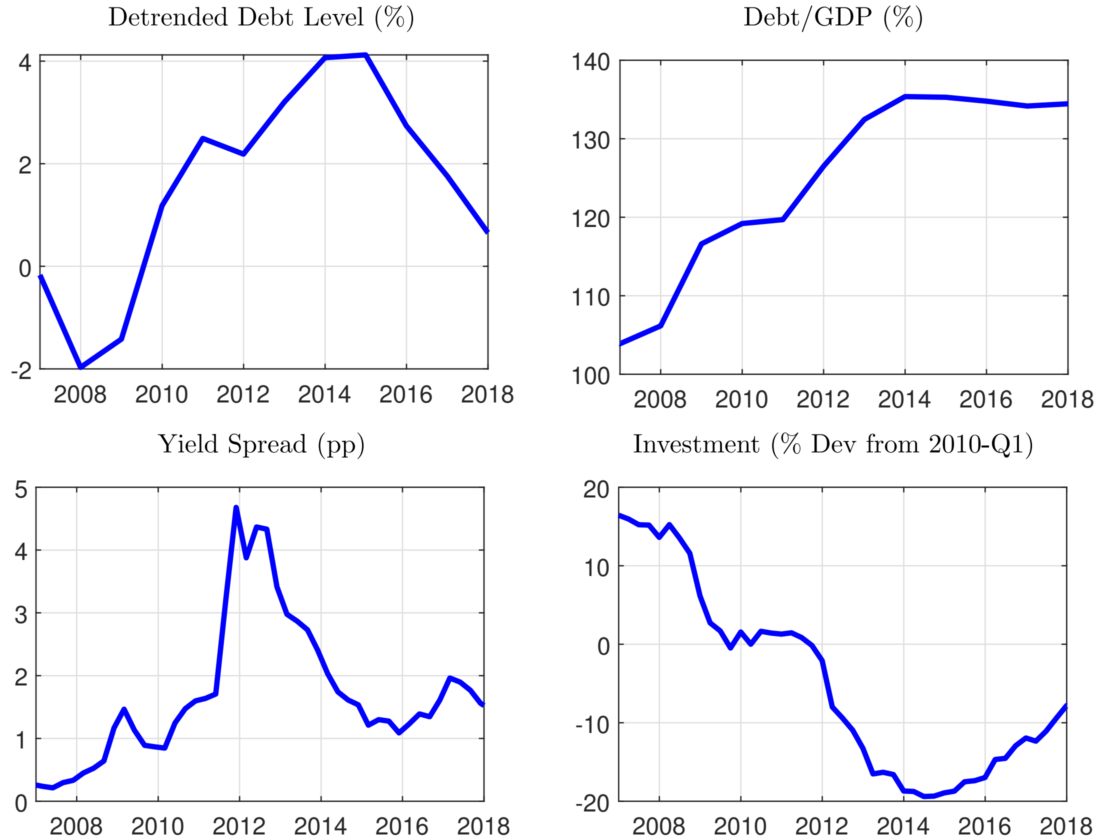
Since 2012, monetary policy of the European Union has expanded to include various asset-purchasing tools with specific objectives. In particular, the European Central Bank's (ECB) programs now differentiate liquidity risks from solvency risks when intervening in sovereign bond markets. For example, a policy program introduced at the height of the European sovereign debt crisis in 2012, the Outright Monetary Transactions (OMT), allows the ECB to purchase government bonds in countries that suffer from distressed interest rates driven by country-specific fundamentals. By contrast, in June 2022, the ECB introduced a new policy program, the Transmission Protection Instrument (TPI), which allows for asset purchases of securities issued in jurisdictions "experiencing deterioration in financing conditions not warranted by country-specific fundamentals" (ECB, 2022).[1] In other words, the ECB has implemented distinct policy tools to respond to different types of risks in the economy, reflecting distinctions between risks originating from fiscal fundamentals versus those arising from financial market conditions.

While these separate programs attempt to target different risks in the economy—namely default and liquidity risks—the risks can be intimately related. Rapid increases in government bond yields driven by fiscal fundamentals can lead to increased liquidity risks that further tighten financial markets. Likewise, a deterioration in financing conditions — not warranted by country-specific fundamentals — can turn into a solvency crisis if it sharply restricts economic activity. Thus, to effectively evaluate the design and efficacy of asset purchases in stabilizing the financial and fiscal sectors, it is important to assess the relative importance of the transmission of default and liquidity risks simultaneously.

In this paper, we quantify the importance of default and liquidity risks through the lens of a monetary union model. Following Bi (2012) and Bi and Traum (2014), we model sovereign default as an endogenous regime-switching process, where default is determined endogenously by fiscal limits – the government's willingness and capacity to service its debt – as well as the underlying fiscal fundamentals. In this framework, the default probability increases with the level of government indebtedness and has two main channels of elevating risk. First, the default probability can change when the fiscal limit perceived by investors shifts. For example, during the European debt crisis, the sharp deterioration in the fiscal outlook for the Greek government may have undermined investors' beliefs in Italy's fiscal space, shifting down Italian fiscal limits perceived by investors. As a consequence, a level of government debt that was previously sustainable may have been viewed as unsustainable by investors at the peak, leading to a rapid rise in Italian government bond yields. Second, the

[1] A key goal of the program is to ensure financial stability, as it was created to prevent financial market fragmentation and ensure a smooth transmission of monetary policy (Schnabel, 2023).

Figure 1: Debt, Yield Spreads, and Investment in Italy, 2008-2018



Notes: Variables plotted are the annual debt level relative to a linear trend from 1998-2008, the quarterly debt-GDP ratio, the quarterly average of yields of Italian debt relative to German debt, and HP-filtered quarterly real investment expressed as percent deviation from the 2010-Q1 value.

default probability affects the domestic financial sector and level of liquidity in the domestic economy. We quantify the significance of each type of risk and how effective asset purchases are in their presence.

Figure 1 shows key aspects of financial and macroeconomic data in Italy surrounding the European debt crisis that inform our model and analysis. First, there was a marked increase in the debt level, relative to an already increasing trend, that started in 2008 and proceeded for the next several years. This accumulation of debt was both in absolute terms but also relative to the size of Italy's economy: the debt-to-GDP ratio jumped from 105% to around 135%. During the height of the crisis in 2012, the yield spread of Italian government bonds relative to German bonds spiked from below 1% prior to the crisis to nearly 5%. Over the next four years the spread gradually fell before settling at a higher level. These dynamics suggest a bit of a puzzle, since the spread dynamics do not closely follow the debt dynamics, indicating spreads were driven by something beyond pure fundamentals such as investor beliefs about the fiscal limit in Italy. Finally, the spillovers from financial stress to the macroeconomy were substantial. Investment fell sharply by about 20% from 2012 to 2014, which was from a level already depressed due to the Global Financial Crisis. Our model seeks to capture these dynamics, understand the relative importance of default versus liquidity risks, and then ask to what extent asset purchases by the central bank are effective.

Our model extends the two-country monetary-union framework of Nakamura and Steinsson (2014), and incorporates financial intermediaries as in Gertler and Karadi (2011) and Sims and Wu (2021). We refer to each country as Home and Foreign. In both countries, banks collect deposits from households and use these funds, along with their own net worth, to lend to domestic firms as well as buy government bonds from both Home and Foreign countries. This intermediation has real economic consequences, as firms are required to have external financing to buy capital goods. In this environment, a higher risk of the Home government defaulting prompts the financial intermediary to demand a lower government bond price, depressing the financial intermediary's net worth and raising its leverage. The deteriorating conditions can prompt a sale of private bonds, as the financial intermediary must adjust its assets to satisfy its balance sheet constraint. This deterioration in turn depresses private lending, investment, and economic activity. This chain reaction can occur regardless of the source of default risk, namely fiscal fundamentals or investors' perception of the risks of government debt.

We consider an additional channel of higher default risk stemming from liquidity issues: in anticipation of a possible default and its associated fire sale of assets, a financial intermediary can tighten its liquidity conditions before default (Bocola, 2016). To capture this notion, we allow the overall tightness of the credit market to endogenously vary with the probability of a sovereign default, which we refer to as the liquidity risk channel. In this case, rising default risk raises liquidity risk and tightens financial conditions, further depressing asset prices and the financial intermediary's net worth.

To provide a quantitative evaluation of the importance of these risks, we explicitly account for the nonlinearity associated with default when solving the model. We use the perturbation solution method developed for endogenous regime switching models Benigno, Foerster, Otrok, and Rebucci (2024), as it is challenging to solve our large-scale model using global methods. We calibrate the Home country using Italian data and Foreign country using German data. We consider a baseline case that roughly captures the macroeconomic dynamics in Italy during the European debt crisis: the fiscal limit shifts lower, reflecting deterioration in market sentiment, and the level of government debt also increases. We explore the impact from default and liquidity risks as well as asset purchases targeting Home government debt.

We find the presence of liquidity risks is crucial for outcomes, while the direct impact of rising government default risk is modest. Although the possibility of default lowers asset prices, net worth of financial intermediaries, investment and output, the quantitative impact is modest when there is no liquidity risk channel. For instance, the higher default risk reduces investment by 3 percent in the absence of the liquidity risk channel, but lowers investment by 10 percent when the default risk also induces liquidity risks. In the latter case, tighter financial conditions directly amplify the decline in asset prices and net worth, which further depresses economic activity.

We introduce a credit intervention by the central bank, whereby the monetary authority conducts Home asset purchases in response to movements in the Home credit spread. Following an increase in Home default and liquidity risks, the spread between Home government bonds and the central bank's interest rate target rises, inducing the central bank to purchase Home sovereign debt. This action lessens the decline in the government bond price, which, in turn, dampens the negative effects reverberating throughout the financial market. With fewer constraints, private lending also contracts less, weakening the decline in economic activity. Overall, the model suggests that asset purchases can help stabilize the Home economy, with their impact more pronounced in scenarios of elevated financial stress.

Related Literature This paper is related to several strands of the literature. First, it is related to a theoretical literature utilizing New Keynesian models of a monetary union to study fiscal policy, see Erceg and Lindé (2013), Nakamura and Steinsson (2014), and Farhi and Werning (2017) among many others. Maćkowiak and Schmidt (2022) study the fiscal theory of the price level in a monetary union with heterogeneous fiscal policies, while Bianchi, Melosi, and Rogantini-Picco (2023) explore the interactions of monetary and fiscal policy following the issuance of euro bonds.

Secondly, the paper is closely related to a large literature that study the effects of asset purchases through the lens of DSGE models with segmented asset markets and financial frictions following Gertler and Karadi (2011), Carlstrom, Fuerst, and Paustian (2017), and Sims and Wu (2021).[2] For instance, Kirchner and Wijnbergen (2016) highlight the crowding-out mechanism through reduced private access to credit when leverage-constrained banks accumulate government debt. Krenz (2022) studies unconventional monetary policy in a monetary union and focuses on the welfare implications of union-wide versus country-specific asset purchase rules.

In addition, our paper contributes to the literature of the sovereign-bank diabolic loop.

[2]Kollmann, Enders, and Muller (2011), Kollmann (2013), Dedola, Karadi, and Lombardo (2013), and Wu, Xie, and Zhang (2023) explore the importance of financial connections and financial participants for the transmission of shocks across countries.

Some papers explore the interaction of sovereign default risk and liquidity risk in a closed economy.[3] For instance, Bocola (2016) estimates such a model with exogenous sovereign default, while Coimbra (2020) models financial frictions through an occasionally binding Value-at-Risk constraint. Other papers focus on the interactions from the perspective of banking regulations, for instance Abad (2019) and Fueki, Huertgen, and Walker (2024).

More closely related, Bianchi, Callegari, Hitaj, and Theodoridis (2024) introduce default risk and financial frictions into a two-country model of the Eurozone. They show that higher default risk raises borrowing costs, magnifying the unpleasant effects of adverse energy shocks, and argue countries would be better off deviating from fiscal rules and accommodating the adverse shock. In addition, Auray, Eyquem, and Ma (2018) introduce an asset purchase program into a perfect-foresight, two-country model without inter-region trades to study responses to a sovereign debt crisis.

2 Model

Since we are interested in the effects of asset purchases in a monetary union when there is default risk, our model has a number of necessary features. It requires two countries that are linked by trade and a degree of financial flows, financial intermediaries that are subject to a financial friction, governments that issue debt, one country where the government may default on this risk as economic conditions change, and a monetary authority that sets a union-level interest rate but can also purchase country-level debt.

Our model builds on Bi and Traum (2023) to incorporate endogenous default risk of the government. It extends the two-country monetary-union framework of Nakamura and Steinsson (2014) and incorporates financial intermediaries as in Gertler and Karadi (2011) and Sims and Wu (2021). Importantly, we model sovereign default as an endogenous regime switching process following Bi (2012) and Bi and Traum (2014). The default probability increases with respect to the state of government indebtedness, and default is driven by the underlying fiscal and macro fundamentals as well as liquidity conditions.

We refer to each region as Home and Foreign. Both regions have the same economic structure. The financial market is segmented as households cannot hold government and private bonds directly; instead, financial intermediaries collect deposits from households and lend to the domestic private sector and governments in both regions. The two economies produce differentiated tradable goods, and trade is friction-less. Thus, Home and Foreign households pay the same nominal prices for the differentiated goods produced in each region. Both governments also set their own taxes and public expenditures and issue bonds, while a

[3] Bianchi and Mondragon (2022) study the interaction of default and liquidity risks by focusing on rollover debt crises and monetary independence.

central bank conducts monetary policy at the union level. Since both regions are symmetric, we focus on a description of the Home economy. We will consider a unit of time to be a quarter.

2.1 Home Households The representative household maximizes the expected intertemporal utility given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \Theta_t \left[\frac{c_t^{1-\sigma_c}}{1-\sigma_c} - \chi \frac{l_t^{1+\sigma_l}}{1+\sigma_l} \right] \right\}, \tag{2.1}$$

where c_t is composite consumption and l_t is the number of hours worked. Θ_t represents an endogenous discount factor, which in the absence of complete international markets ensures stationarity (Schmitt-Grohe and Uribe, 2003).[4]

The composite consumption c_t aggregates Home and Foreign consumption sub-baskets, $c_{H,t}$ and $c_{F,t}$, in Armington form:

$$c_t = \left[\alpha_H^{\frac{1}{\phi^c}} \left(c_{H,t}\right)^{\frac{\phi^c-1}{\phi^c}} + (1-\alpha_H)^{\frac{1}{\phi^c}} \left(c_{F,t}\right)^{\frac{\phi^c-1}{\phi^c}} \right]^{\frac{\phi^c}{\phi^c-1}}, \tag{2.2}$$

where $\phi^c > 0$ denotes the elasticity of substitution between Home and Foreign goods, and α_H is the household's relative preference for Home goods. The sub-basket $c_{H,t}$ aggregates Home differentiated consumption varieties $c_{H,t}(j)$, and $c_{F,t}$ aggregates Foreign differentiated consumption varieties $c_{F,t}(j)$, with the elasticity of substitution across goods of $\theta^c > 1$.

The household's period budget constraint in real terms is:

$$d_t + b_t^i + c_t\left(1+\tau^c\right) = \frac{R_{t-1}^d d_{t-1}}{\pi_t} + \frac{R_{t-1}^d b_{t-1}^i}{\pi_t} + w_t l_t + \Pi_t^f + div_t - x - t_t + T_t^{cb}, \tag{2.3}$$

where $w_t \equiv W_t/P_t$. Households make one-period savings deposits at financial intermediaries. We denote d_t as the amount of savings deposits, and R_{t-1}^d as the nominal interest rate on deposits between $t-1$ and t, which is known with certainty in $t-1$. Households pay consumption taxes τ^c to their government. In addition, households receive profits from ownership of firms Π_t^f as well as equity from financial intermediaries div_t. Each period, households make a fixed real equity transfusion to newly born financial intermediaries, which we denote by x, and pay a lump-sum tax t_t. T_t^{cb} denotes a transfer from the monetary authority as it conducts asset purchases.

Across countries, households can only trade nominal one-period bonds b_t^i. This bond trades at the same rate as the Home country's deposit rate, which also equals the interest

[4] Appendix A lists details of the model as well as all the equilibrium conditions.

rate set by the common central bank.[5] For reference below, we define $\Lambda_{t,t+1}$ as household's real stochastic discount factor.

2.2 Home Government The Home government finances its consumption and public investment by collecting distortionary tax revenue, by receiving lump-sum taxes, as well as by floating government bonds. Its budget constraint in real terms is given by

$$\rho_{H,t} g + (1-\Delta_t)(1+\kappa^b Q_t^b)\frac{b_{t-1}}{\pi_t} = Q_t^b b_t + t_t + \tau^i p_t^w y_t + \tau^c c_t \tag{2.4}$$

where g is a constant level of government consumption. Public bonds are defined as a perpetuity with coupons that decay exponentially, as in Woodford (2001). A bond issued at date t pays $(\kappa^b)^{k-1}$ at date $t+k$ with the coupon decay factor κ^b capturing the average maturity of the bond portfolio.

Importantly, the bond contract is not enforceable. At each period, the government may default on its government bonds by taking a haircut if the debt-GDP ratio is higher than a stochastic threshold $\mathcal{B}_t^*$,

$$\Delta_t = \begin{cases} 0, & \text{if } s_{t-1} < \mathcal{B}_t^* \\ \delta_b, & \text{otherwise} \end{cases} \tag{2.5}$$

where $s_{t-1} = \frac{Q_{t-1}^b b_{t-1}}{4y_{t-1}}$ denotes the debt to annual GDP ratio. $\mathcal{B}_t^*$ can be interpreted as the fiscal limit that captures the government's capacity to service debt. Bi (2012) shows that fiscal limits can arise endogenously from dynamic Laffer curves and depend on the underlying macroeconomic shocks. Therefore, the likelihood of the government defaulting on its debt depends on the fiscal limit as well as its outstanding debt liability: it increases with a higher level of outstanding debt and a lower level of fiscal limit.

Following Bi and Traum (2014), we specify the conditional probability of a government's default at time t as a logistic function of existing debt-GDP ratio s_{t-1} with parameters η_0^{FL}, η_s^{FL}, and η_ϵ^{FL} dictating its shape:

$$P(s_{t-1} \geq \mathcal{B}_t^*) = \frac{\exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_t^P)]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_t^P)]}. \tag{2.6}$$

The shock ϵ_t^P plays a key role in our analysis. It is a shock to the fiscal limit distribution, which affects the default probability directly without a movement in the debt-GDP ratio.

[5] Since Home deposits and the traded bond have the same interest rate, the setup is similar to allowing Foreign households to directly hold deposits at the Home financial intermediary.

The shock reflects investors' perception of government's willingness and capability to service its debt that may or may not be driven by underlying macroeconomic conditions. In absence of the shock, the default probability increases only when the government debt-GDP ratio rises. With a positive ϵ_t^P, the fiscal limit distribution shifts down, raising default probability even if the debt-GDP ratio remains unchanged. During the European debt crisis, the sharp deterioration in the fiscal outlook of Greek government may have undermined investors' belief in Italian government fiscal space, effectively shifting down the distribution of Italian fiscal limits. Therefore, a debt level that would have been sustainable during normal times may become unsustainable during the crisis, leading to a rapid rise in interest rates. In section 4, we explore a scenario with such a shift in fiscal limits and highlight the transmission mechanism through default versus liquidity risks. We assume that ϵ_t^P follows an AR(1) process.

Finally, in addition, we assume that the lump-sum tax follows a fiscal rule, ensuring debt is at least partially financed over time:

$$\frac{t_t - t}{t} = \phi^T \frac{Q_{t-1}^b b_{t-1} - Q^b b}{Q^b b}. \tag{2.7}$$

2.3 Home Production The model includes three different types of production firms, similar to Sims and Wu (2021). A representative wholesale firm produces output using labor as well as its own capital. These firms additionally face a loan-in-advance constraint, thus issuing long-term bonds to finance a portion of their capital purchases. Retail firms repackage wholesale output for resale and are subject to price stickiness. In addition, a representative investment producer generates new capital using Home and Foreign retail goods.

Investment Producers Competitive investment producing firms use Home and Foreign goods, in the same Armington form as consumption, to obtain a composite investment I_t. In turn, composite investment is used to produce new capital I_t^w and is sold to wholesale firms at price P_t^k or in real terms, $p_t^k \equiv P_t^k / P_t$. Investment producers make optimal decisions on I_t to maximize its present value of expected future profits. The production function is $I_t^w = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t$, where $S(.)$ denotes an investment adjustment cost.

Wholesale Firms A representative wholesale firm produces output according to,

$$y_t^w = A_t l_t^{1-\alpha} K_{t-1}^{\alpha}. \tag{2.8}$$

K_{t-1} is private capital that is owned by wholesale firms and evolves according to a standard law of motion,

$$K_t = I_t^w + (1-\delta)K_{t-1}. \tag{2.9}$$

Similar to Sims and Wu (2021), we assume that the wholesale firm must issue perpetual bonds to finance a fraction η^I of new physical capital, I_t. The loan-in-advance constraint is

$$Q_t^f\left(f_t - \kappa^f \frac{f_{t-1}}{\pi_t}\right) \geq \eta^I p_t^k I_t^w, \tag{2.10}$$

where f_t denotes the amount of private bonds. The firm chooses labor, investment, and bond issuance to maximize the present value of their profits,

$$\max \quad \sum_{t=0}^{\infty} E_0 \left[\Theta_t \Lambda_{t,t+1}\left(p_t^w y_t^w(1-\tau^i) - w_t l_t - p_t^k I_t^w - \frac{f_{t-1}}{\pi_t} + Q_t^f\left(f_t - \kappa^f \frac{f_{t-1}}{\pi_t}\right)\right)\right], \tag{2.11}$$

subject to equations (2.8), (2.9), and (2.10).

Retail Firms We assume producer currency pricing for pricing exports. The law of one price holds and implies

$$\rho_{H,t} = rer_t \rho^*_{H,t} \quad \text{and} \quad \rho^*_{F,t} = \frac{\rho_{F,t}}{rer_t}, \tag{2.12}$$

where $rer_t = P_t^*/P_t$ denotes the real exchange rate, $\rho^*_{H,t} = P^*_{H,t}/P^*_t$ and $\rho^*_{F,t} = P^*_{F,t}/P^*_t$.[6]

The retail firm h repackages wholesale output, $y_t(h) = y_t^w(h)$, and sells it for the price $P_t(h)$. The firm faces a Rotemberg price adjustment cost, where the real cost is denoted by: $\frac{\psi}{2}\left(\frac{P_t(h)}{P_{t-1}(h)}\frac{1}{\pi} - 1\right)^2 y_t$. The firm h sets its price while taking into account demand for its product, which comes from multiple sources: Home and Foreign private consumers and investment firms and the Home regional government. It chooses labor and its price to optimize real profits. Total demand for good h is given by

$$y_t(h) = p_t(h)^{-\theta^c}\left(c_{H,t} + c^*_{H,t} + i_{H,t} + i^*_{H,t} + g\right) \tag{2.14}$$

[6] In general, the law of one price implies that for a nominal exchange rate ϵ_t,

$$P_{F,t} = \epsilon_t P^*_{F,t}, \quad P^*_{HF,t} = \frac{P_{H,t}}{\epsilon_t}. \tag{2.13}$$

In the above expressions, we have used the fact that the nominal exchange rate in our currency union is one.

where $p_t(h) = P_t(h)/P_t^H$. After imposing equilibrium, optimal price setting implies

$$\frac{p_t^w}{\rho_t^H} = \frac{\theta^c - 1}{\theta^c} + \frac{\psi}{\theta^c}\left(\frac{\pi_t^H}{\pi^H} - 1\right)\frac{\pi_t^H}{\pi^H} - \frac{\psi}{\theta^c} E_t \beta(c_t)\Lambda_{t,t+1}\left(\frac{\pi_{t+1}^H}{\pi^H} - 1\right)\frac{\pi_{t+1}^H}{\pi^H}\frac{y_{t+1}}{y_t}. \tag{2.15}$$

2.4 Home Financial Intermediaries

In the Home region, financial intermediaries are structured as in Gertler and Karadi (2011) and Sims and Wu (2021). There is a continuum on the unit interval. Each period, a fraction of financial intermediaries stochastically exit and are replaced by the same number of new intermediaries with startup funds from households. Financial intermediaries accumulate net worth until they exit, whereby they return their net worth to households.

The intermediary j purchases Home government bonds $b_t^{H,j}$, as well as private bonds, $f_t^{H,j}$. In addition, we allow the intermediary to purchase Foreign government bonds, $b_t^{F,j}$.[7] These purchases are financed by deposits from domestic households d_t^j and the firm's net worth n_t^j. The balance sheet condition in real terms is given by

$$Q_t^b b_t^{H,j} + Q_t^f f_t^j + Q_t^{b,*} b_t^{F,j} = d_t^j + n_t^j. \tag{2.16}$$

If intermediary j survives, then its net worth evolves as,

$$\begin{aligned} n_t^j = \left(R_t^b - R_{t-1}^d\right)\frac{Q_{t-1}^b b_{t-1}^{H,j}}{\pi_t} + \left(R_t^f - R_{t-1}^d\right)\frac{Q_{t-1}^f f_{t-1}^j}{\pi_t} \\ + \left(R_t^{b,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{b,*} b_{t-1}^{F,j}}{\pi_t} + \frac{R_{t-1}^d n_{t-1}}{\pi_t} \end{aligned} \tag{2.17}$$

where $R_t^b - R_{t-1}^d$, $R_t^f - R_{t-1}^d$, and $R_t^{b,*} - R_{t-1}^d$ are, respectively, the excess returns from holding Home government, Home private as well as Foreign government bonds related to the cost of funding through deposits. The realized returns on holding these bonds are

$$R_t^b = (1 - \Delta_t)\frac{1 + \kappa^b Q_t^b}{Q_{t-1}^b}, \quad R_t^f = \frac{1 + \kappa^f Q_t^f}{Q_{t-1}^f}, \quad R_t^{b,*} = \frac{1 + \kappa^{b,*} Q_t^{b,*}}{Q_{t-1}^{b,*}}. \tag{2.18}$$

The return on Home government bonds reflects the possibility that the government could default, in which case there is a haircut on payments.

Each period, a fraction $1 - \sigma$ of financial intermediaries exit and return their net worth to domestic households. The objective of the intermediary j is to maximize its expected

[7] Cross-region exchange of multiple assets could induce multiple unit-roots into the model. Below, we assume home bias in preferences for assets, which ensures there are no unit root dynamics from cross-region financial intermediary asset holdings.

terminal net worth according to,

$$\max V_t^j = (1-\sigma) E_t \beta(c_t) \Lambda_{t,t+1} n_{t+1}^j + \sigma \beta E_t \beta(c_t) \Lambda_{t,t+1} V_{t+1}^j, \tag{2.19}$$

where it discounts future net worth by the stochastic discount factor of households.

If it can make positive excess returns from investing in bonds, an intermediary would want to expand its assets indefinitely by collecting deposits from households. To put a limit on its ability to do so, we assume that financial intermediaries face a costly enforcement problem as in Gertler and Karadi (2011): at the end of a period, an intermediary can divert a fraction of its assets and transfer them to household owners, in which case depositors can recover the remaining assets and force the intermediary into bankruptcy. Thus, an incentive constraint must be satisfied for depositors to be willing to lend in the first place. The constraint is given by,

$$V_t^j \geq \eta_t^v (Q_t^f f_t^j + \theta^b m_t^{b,j}). \tag{2.20}$$

The above condition implies that the value of continuing as an intermediary V_t^j should be larger or equal to the funds that the intermediary j can divert. Should it choose to enter bankruptcy, the incentive constraint implies the intermediary can keep a fraction η_t^v of its private bonds. This variable captures the tightness of the overall credit market: the higher η_t^v is, the more funds financial intermediaries can divert, making depositors less willing to lend funds.

In addition, it can also keep a fraction $\eta_t^v \theta^b$ of government bonds with $0 \leq \theta^b \leq 1$, implying it is easier for the intermediary to divert private bonds than public bonds. Following Krenz (2022), we assume that government bonds are assembled in terms of CES composite portfolios of Home and Foreign bonds,

$$m_t^{b,j} = \left[\gamma_b^{\frac{1}{\sigma_b}} \left(Q_t^b b_t^{H,j} \right)^{\frac{\sigma_b - 1}{\sigma_b}} + (1-\gamma_b)^{\frac{1}{\sigma_b}} \left(Q_t^{b,*} b_t^{F,j} \right)^{\frac{\sigma_b - 1}{\sigma_b}} \right]^{\frac{\sigma_b}{\sigma_b - 1}}. \tag{2.21}$$

The parameter $\sigma_b < 0$ denotes the interest rate elasticity of asset demand, and the parameter γ_b denotes the home bias in asset holdings. The assumption of imperfect substitutability between Home and Foreign assets in the incentive constraint can be motivated by the owners' preference for different asset types, different attitudes towards risks across regional assets, and differential convenience benefits due to institutional differences across countries (see Alpanda and Kabaca, 2020 and Krenz, 2022).

The financial intermediary solves its optimisation problem and has the following first-order conditions:

$$E_t\beta(c_t)\Lambda_{t,t+1}\Omega_{t+1}\frac{R^f_{t+1}-R^d_t}{\pi_{t+1}} = \frac{\lambda^v_t}{1+\lambda^v_t}\eta^v_t \tag{2.22}$$

$$E_t\beta(c_t)\Lambda_{t,t+1}\Omega_{t+1}\left(\frac{R^b_{t+1}-R^d_t}{\pi_{t+1}}\frac{Q^b_t b^H_t}{m^b_t}+\frac{R^{b,*}_{t+1}-R^d_t}{\pi_{t+1}}\frac{Q^{b,*}_t b^F_t}{m^b_t}\right) = \frac{\lambda^v_t}{1+\lambda^v_t}\eta^v_t\theta^b \tag{2.23}$$

$$E_t\beta(c_t)\Lambda_{t,t+1}\frac{\Omega_{t+1}}{\pi_{t+1}}R^d_t = \frac{\phi_t}{1+\lambda^v_t}\eta^v_t \tag{2.24}$$

where λ^v_t is the multiplier on the incentive constraint. Without the incentive constraint, the expected returns on the two assets equal the cost of funds. In our analysis, the incentive constraint binds, meaning there are excess returns of long-term public and private bonds over the deposit rate. If $\theta^b < 1$, then the excess returns of government bonds will be lower than those of private bonds. In addition,

$$\Omega_t = 1-\sigma+\sigma\eta^v_t\phi_t \tag{2.25}$$

$$\phi_t = \frac{Q^f_t f_t + \theta^b m^b_t}{n_t} \tag{2.26}$$

where ϕ_t is an endogenous leverage ratio, and all financial intermediaries make the same optimal decisions.

We model liquidity risk in the financial sector as fluctuations in η^v_t. Importantly, the liquidity risk can depend on the default probability, a mechanism we call the *liquidity risk channel*.

$$\frac{\eta^v_t}{\bar{\eta}^v} = 1+\phi^\eta\left(\exp\left(\phi^\eta_s\left(\frac{s_{t-1}+\epsilon^P_t}{\bar{s}}-1\right)\right)-1\right) \tag{2.27}$$

If $\phi^\eta > 0$, the liquidity risk channel is present and η^v_t increases with government debt-GDP ratio as well as shift in the fiscal limits. Therefore, liquidity risk rises with the probability of sovereign default. This channel captures the idea that a deterioration in fiscal fundamentals or investors' perception on fiscal limits can directly tighten the financial intermediary's incentive constraint. If $\phi^\eta = 0$, the liquidity risk channel is not present and the default probability does not have a direct impact on liquidity conditions. This setup allows us to discuss default risks and liquidity risks, as well as their interactions. Bocola (2016) shows that sovereign default risks can cause the incentive constraint to tighten and force a fire sale of a financial intermediary's assets. Thus, anticipation of a possible default and the associated fire sale can tighten the financial intermediary's liquidity conditions today. The liquidity risk channel captures this spillover from fiscal risks to liquidity conditions.

Finally, we assume that newly entering intermediaries receive start-up funds from households, denoted by x in real terms. The aggregate net worth in the financial intermediary sector evolves as,

$$
\begin{aligned}
n_t &= \sigma\left[\left(R_t^b - R_{t-1}^d\right)\frac{Q_{t-1}^b b_{t-1}^H}{\pi_t} + \left(R_t^f - R_{t-1}^d\right)\frac{Q_{t-1}^f f_{t-1}}{\pi_t} + \left(R_t^{b,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{b,*} b_{t-1}^F}{\pi_t}\right] \\
&\quad + \sigma R_{t-1}^d \frac{n_{t-1}}{\pi_t} + (1-\sigma)x. \qquad (2.28)
\end{aligned}
$$

2.5 Policy Interventions The central bank sets a common monetary policy for the two regions by following a Taylor-rule for the economy-wide nominal interest rate:

$$
\ln\frac{R_t^d}{R^d} = \phi_\pi \ln\frac{\pi_t^{ag}}{\pi^{ag}} + \phi_y \ln\frac{y_t^{ag}}{y^{ag}} \qquad (2.29)
$$

The money authority responds to variation in the weighted average of consumer price inflation, $\ln\frac{\pi_t^{ag}}{\pi^{ag}} = 0.5\ln\frac{\pi_t}{\pi} + 0.5\ln\frac{\pi_t^*}{\pi^*}$, and the weighted average of output in each region, $\ln\frac{Y_t^{ag}}{Y^{ag}} = 0.5\ln\frac{Y_t}{Y} + 0.5\ln\frac{Y_t^*}{Y^*}$.

We model the unconventional policy of liquidity injections directly on the issuance of new public bonds. In equilibrium, the total public bonds satisfy

$$
b_t = b_t^{cb} + b_t^H + b_t^{H,*} rer_t. \qquad (2.30)
$$

The central bank's holdings of public bonds are denoted as $Q_t^b b_t^{cb} = re_t$. Following Sims and Wu (2021), we assume that the operating surplus is returned to households via a lump-sum transfer:

$$
T_t^{cb} = R_t^b Q_{t-1}^b \frac{b_{t-1}^{cb}}{\pi_t} - Q_t^b b_t^{cb}. \qquad (2.31)
$$

In steady state, the central bank does not hold any public bonds with $b_t^{cb} = 0$. During a crisis, the central bank can raise b_t^{cb} to inject liquidity into the financial market. We model this as

$$
re_t = re + \phi_{cb}(R_t^{spread} - R^{spread}), \qquad (2.32)
$$

where $R_t^{spread} \equiv E_t R_{t+1}^b - R_t^d$ measures the spread between expected returns on government bonds and deposits. When $\phi_{cb} > 0$, the central bank injects credit when a crisis can imply a sharp increase in the spread on government bonds. Gertler and Karadi (2011) consider a similar rule for a closed-economy model without default risk.

2.6 Goods Market Clearing and Additional Equilibrium Conditions Home goods market clearing implies $y_t = c_{H,t} + c^*_{H,t} + i_{H,t} + i^*_{H,t} + g$. Adjustment of international relative prices is summarized by the condition linking the real exchange rate to relative inflation (since the nominal exchange rate is fixed): $rer_t/rer_{t-1} = \pi^*_t/\pi_t$. The equation for net foreign asset accumulation can be written as:

$$b^i_t = \frac{R^d_{t-1} b^i_{t-1}}{\pi_t} + \rho^H_t (c^*_{H,t} + i^*_{H,t}) - \rho^{F,*}_t rer_t (c_{F,t} + i_{F,t}). \tag{2.33}$$

3 Calibration and Solution Method

In this section, we discuss how we calibrate the parameters of the model and solve it.

3.1 Calibration We apply a symmetric calibration on the deep parameters that are common across countries. To have a steady-state price markup of 10 percent, we let $\theta^c = 11$. The price adjustment cost parameter, ψ, is calibrated to replicate firms adjusting prices 25% of the time in a Calvo-type Phillips curve in the absence of strategic price complementary.[8] The Frisch elasticity is set to 1, and we also assume logarithmic preferences ($\sigma_c = 1$). The investment adjustment cost $S\left(\frac{I_t}{I_{t-1}}\right)$ takes the functional form of $\frac{\omega_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, and the investment adjustment cost parameter ω_I is set to 2.

For the endogenous discount factor, we calibrate ω_β to 0.01, as in Devereux and Sutherland (2009). The small value ensures the influence of the endogenous discount factor is muted in the dynamics. We assume the steady-state real interest rate is 2 percent annualized, which implies $\beta(c) = 0.995$. Given our calibration of ω_β, we set β_c to ensure the calibration for $\beta(c)$. The elasticity of substitution between goods is set to $\phi^c = 1.3$ and the degree of home bias α_H to 0.7, in the range of estimates of Albonico, Calès, Cardani, Croitorov, Ferroni, Giovannini, Hohberger, Pataracchia, Pericoli, Raciborski, and Rat (2017). For monetary policy, the Taylor rule parameters are set to standard values with $\phi_\pi = 1.5$ and $\phi_y = 0.15$.

The financial sector as well as the government sector play a key role in our results. For those two sectors, we calibrate the Home country using Italian data and the Foreign counterpart using German data, as summarized in Table 1. The steady-state value of debt-to-GDP ratio is set to 105 percent for Home country and 60 percent for Foreign country, roughly in line with the average government debt level in Italy and Germany between 2000 and 2008. The rest of the fiscal variables are calibrated following Bianchi, Melosi, and Rogantini-Picco (2023).[9] The government expenditures-to-GDP ratio is calibrated to 19

[8] The mapping implies $\psi = \theta^c/[(1-\xi_p)/\xi_p(1-\beta\xi_p)]$, where $\xi_p = 0.75$ is the Calvo adjustment parameter.
[9] See the European Commission, DG Taxation and Customs Union, Taxes in Europe database and IBFD data.

percent, the consumption tax is set to 0.22, and the income tax is calibrated to 0.2 for Home country to match Italian fiscal data. The average duration of government debt, $1 - 1/\kappa^b$, is set to 7 years. In the Foreign country, government expenditures are set to 20 percent of GDP, the consumption tax to 0.19, and the income tax to 0.25 to be consistent with German fiscal data. The average duration of government debt, $1 - 1/\kappa^b$, is set to 6 years. We calibrate the response of lump-sum taxes to debt, ϕ^T, as 3.

Turning to the financial sector, we target an excess return over the deposit rate, $R^f - R^d$, of 3 percent annualized in both Home and Foreign to match the spread between non-financial lending rate and deposit rate. Following Sims and Wu (2021), we use the total credit to private non-financial sector to calibrate the outstanding private debt to annualized GDP, $\frac{Q^f f}{4y}$, which is set to 1.1 for Home and 1.2 for Foreign. With investment accounting for 17 percent of GDP in Home country, the fraction of investment the wholesale firms must finance by issuing debt, η^I, is 0.65. For Foreign country, investment accounts for 16 percent of GDP and 75 percent of investment is financed through corporate debt. We calibrate the private debt maturity, $1 - 1/\kappa^f$, to 10 years in both countries. The interest rate elasticity of asset demand, σ_b, is calibrated to -2 for both countries in the baseline case, inline with Poutineau and Vermandel (2015). In Section 4, we explore alternative calibrations on σ_b to discuss cross-country asset holdings by the financial sector during a debt crisis. We also calibrate the parameter governing home bias in asset holding, γ_b, to 0.7 for Home country and 0.8 for Foreign, which respectively match the shares of government debt held by domestic financial sectors in Italy and Germany. The rest of the financial variables are calibrated using similar metrics from the existing literature (Sims and Wu, 2021; Gertler and Karadi, 2011). Financial intermediaries have a survival probability, σ, of 0.95. The value of startup funds to new financial intermediaries, x, is chosen to be consistent with a leverage ratio of 4.

Finally, we calibrate the fiscal limit distribution to match the Italian data during the European debt crisis as shown in Figure 1. Prior to the crisis, Italian government debt was 105 percent of GDP while its long-term yield spread against German counterpart was around 0.2 percentage points. During the crisis, Italian debt rose to 120 percent of GDP in 2012 when its yield spread peaked close to 5 percentage points. After the crisis, the yield spread declined and reached 1 percentage point in 2015 when Italian debt remained elevated at 135 percent of GDP. We calibrate the logistic function as well as a shift in the fiscal limit distribution to match the Italian data prior to, during, and after the debt crisis. In addition, we assume that the haircut parameter, δ_b, to be 0.1, meaning when it defaults, the Home government takes a haircut of 40 percent at annual frequency.

Under this assumption of haircut, we set the parameters governing the logistic function,

Table 1: Calibration

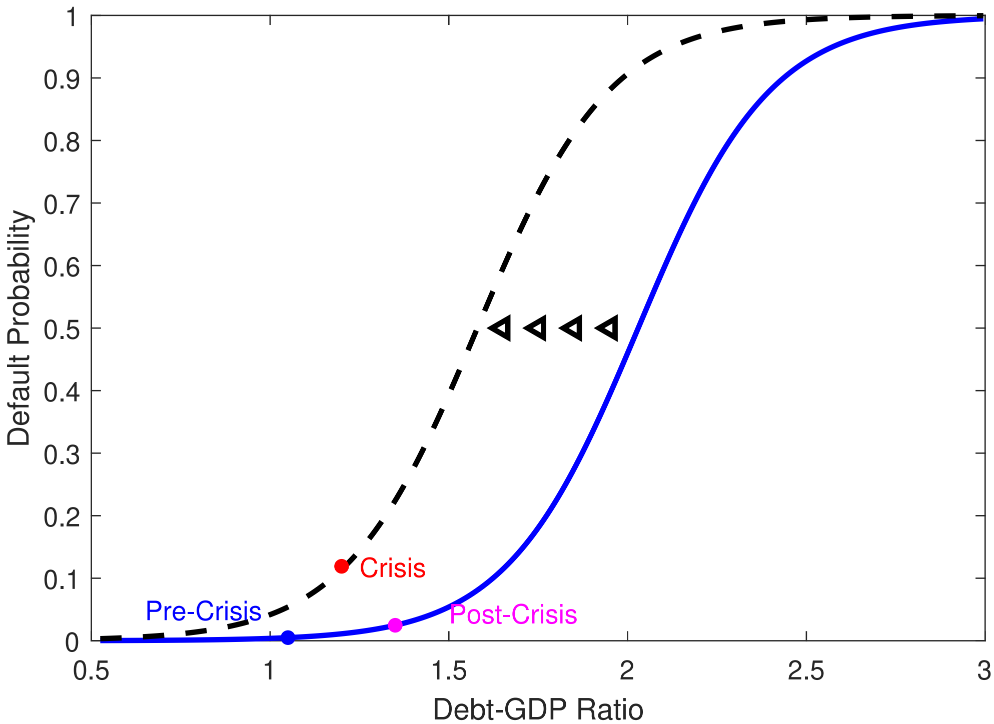
Parameter	Value	Description
		Home Country
κ^f	$1-40^{-1}$	Coupon decay parameter for private bonds
κ^b	$1-28^{-1}$	Coupon decay parameter for government bond
η^I	0.65	Fraction of investment from debt
ϕ	4	Leverage ratio
η^v	0.59	Recoverability parameter
$\frac{Q^f f}{4y}$	1.1	Private bonds as share of GDP
$\frac{Q^b b}{4y}$	1.05	Government bonds as share of GDP
τ^c	0.22	Consumption tax rate
τ^i	0.2	Income tax rate
$\frac{g^c}{y}$	0.19	Government consumption as share of GDP
		Foreign Country
$\kappa^{f,*}$	$1-40^{-1}$	Coupon decay parameter for private bonds
$\kappa^{b,*}$	$1-24^{-1}$	Coupon decay parameter for government bond
$\eta^{I,*}$	0.75	Fraction of investment from debt
ϕ^*	4	Leverage ratio
$\eta^{v,*}$	0.59	Recoverability parameter
$\frac{Q^{f,*} f^*}{4y^*}$	1.2	Private bonds as share of GDP
$\frac{Q^{b,*} b^*}{4y^*}$	1.05	Government bonds as share of GDP
$\tau^{c,*}$	0.19	Consumption tax rate
$\tau^{i,*}$	0.25	Income tax rate
$\frac{g^{c,*}}{y^*}$	0.2	Government consumption as share of GDP

equation (2.6), and the probability of default to match the Italian data. Specifically, We calibrate η_0^{FL} and η_s^{FL} by targeting the debt-GDP ratio and the yield spread prior to and after the crisis: when debt-GDP was 1.05 the probability of default was 0.5%, and when debt-GDP was 1.35 the probability of default was 2.5%.[10] As shown in Figure 2, the solid blue line shows the fiscal limit distribution in the case without a shock to the distribution, $\epsilon_t^P = 0$, with the blue dot representing the data point prior to the crisis and the purple dot after the crisis. In addition, we calibrate the shock to fiscal limit distribution to match the data during the crisis with $(\hat{s}_c, \hat{p}_c) = (1.2, 0.12)$. The black dashed line shows the shifted fiscal limit distribution during the crisis with the red dot representing the data point in 2012.

3.2 Solving the Model Given the evolution of the haircut (2.5) and the default probability (2.6), the model is an endogenous regime-switching model. We introduce a regime

[10]A property of the logistic function is that for any given two points on the distribution, $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$, the parameters η_0^{FL} and η_s^{FL} can be uniquely determined by $\eta_s^{FL} = \frac{1}{\tilde{s}-\hat{s}} \log\left(\frac{\tilde{p}}{\hat{p}}\frac{1-\hat{p}}{1-\tilde{p}}\right)$, and $\eta_1^{FL} = \log\frac{\tilde{p}}{1-\tilde{p}} - \eta_s^{FL}\tilde{s}$.

Figure 2: Fiscal Limit Distribution



variable def_t, and note that there are two regimes: one in which there is no default and one with default, denoted $def_t = 0$ and $def_t = 1$, respectively. The haircut can then be written as

$$\Delta_t = \begin{cases} 0, & \text{if } def_t = 0 \\ \delta_b, & \text{if } def_t = 1 \end{cases} \tag{3.1}$$

The transition matrix is time varying depending on the state $s_t = \frac{Q_t^b b_t}{4y_t}$ and the underlying macroeconomic shocks, and has elements $\mathbb{P}_{ij,t} = \Pr\left(def_{t+1} = j | def_t; s_t\right)$. Using our assumption that the transition follows a logistic function, we have

$$\mathbb{P}_t = \begin{bmatrix} \mathbb{P}_{00,t} & \mathbb{P}_{01,t} \\ \mathbb{P}_{10,t} & \mathbb{P}_{11,t} \end{bmatrix} = \begin{bmatrix} 1 - pdef_t & pdef_t \\ 1 - pdef_t & pdef_t \end{bmatrix}$$

where

$$pdef_t = \frac{\exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_t^P)]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_t^P)]}.$$

In order to solve the model, we use the perturbation approach for solving endogenous regime-switching models in Benigno, Foerster, Otrok, and Rebucci (2024), which generates a set of approximated decision rules conditional on each regime. Since Benigno, Foerster, Otrok, and Rebucci (2024) show that first-order approximations to the decision rules are insufficient for capturing behavior induced by endogenous probabilities, we approximate our model to the second order. This second-order approximation captures important features of our model, such as households and firms internalizing the fact that default becomes more likely as the debt-to-output ratio increases. Appendix B provides additional details on the solution procedure.

4 Results

In this Section, we investigate how default risk affects the economy, the importance of various channels, and the effectiveness of asset purchases. Our main exercises are motivated by the dynamics in Italy during the 2012 European debt crisis, as shown in Figure 1. The key features of this episode were a rapid accumulation of debt – which we model as an unexpected, exogenous increase in the debt level – and a downward shift in the fiscal limit as highlighted in figure 2, reflecting deterioration in market sentiment.[11] Both of these factors serve to raise the default risk in the Home country, and by extension increase liquidity risk as well.

We start by illustrating the effects of the crisis in our baseline model that includes both default and liquidity risks without any asset purchases. Then we decompose our baseline results by investigating two key model features. Since liquidity risks are a compounding factor for default risk, we show that liquidity risks are a key feature in the model that amplifies the effects of default risk on both financial markets and the macroeconomy. Then we consider a case where a crisis is driven only by a shift in the fiscal limit and not a contemporaneous increase in the debt level – these results indicate that shifts in the fiscal limit, rather than debt, are key for understanding the 2012 crisis. Finally, we address the effect of asset purchases for limiting the financial and macroeconomic repercussions of a debt crisis.

4.1 Baseline In our first set of results, the fiscal limit shifts lower and at the same time, the level of Home government debt increases, in line with the Italian data. Both factors raise the default as well as the liquidity risks, and Figure 3 shows the responses related to the Home financial intermediary. In this baseline case, we assume that the central bank does not conduct asset purchases (i.e., $\phi_{cb} = 0$ in equation 2.32) to highlight the transmission mechanism.

A higher government bond supply tightens financial conditions. At $t = 0$, government

[11] Abstracting from the drivers of this accumulation, such as changes in government spending or taxes, enables us to focus on the economic effects of higher debt rather than the drivers of it.

debt increases from its steady state by 6 percent, in line with the Italian data in figure 1. The higher bond supply depresses the price of government bonds and also induces a fire sale of private bonds as the financial intermediary needs to meet its balance sheet constraint. Asset prices decline for both private and government bonds, lowering the financial intermediary's net worth. With a larger amount of government debt to absorb as well as a lower net worth, the financial intermediary faces a higher leverage ratio and thus demands a higher excess return on government bonds, which is the spread between the expected return on government bonds and the risk-free deposit rate paid to households.

This transmission mechanism is strengthened by two factors: higher default risks as well as higher liquidity risks. As shown in the bottom right panel, the default probability increases to close to 7 percent with the higher level of government debt as well as a shift in fiscal limit. A higher likelihood of receiving a haircut prompts the financial intermediary to demand a lower government bond price, further tightening financial conditions. Importantly, with $\phi_\eta > 0$, the liquidity risk channel further amplifies this channel. The higher default probability transmits to a higher η^v, and, subsequently, a more binding incentive constraint further tightens the credit market.

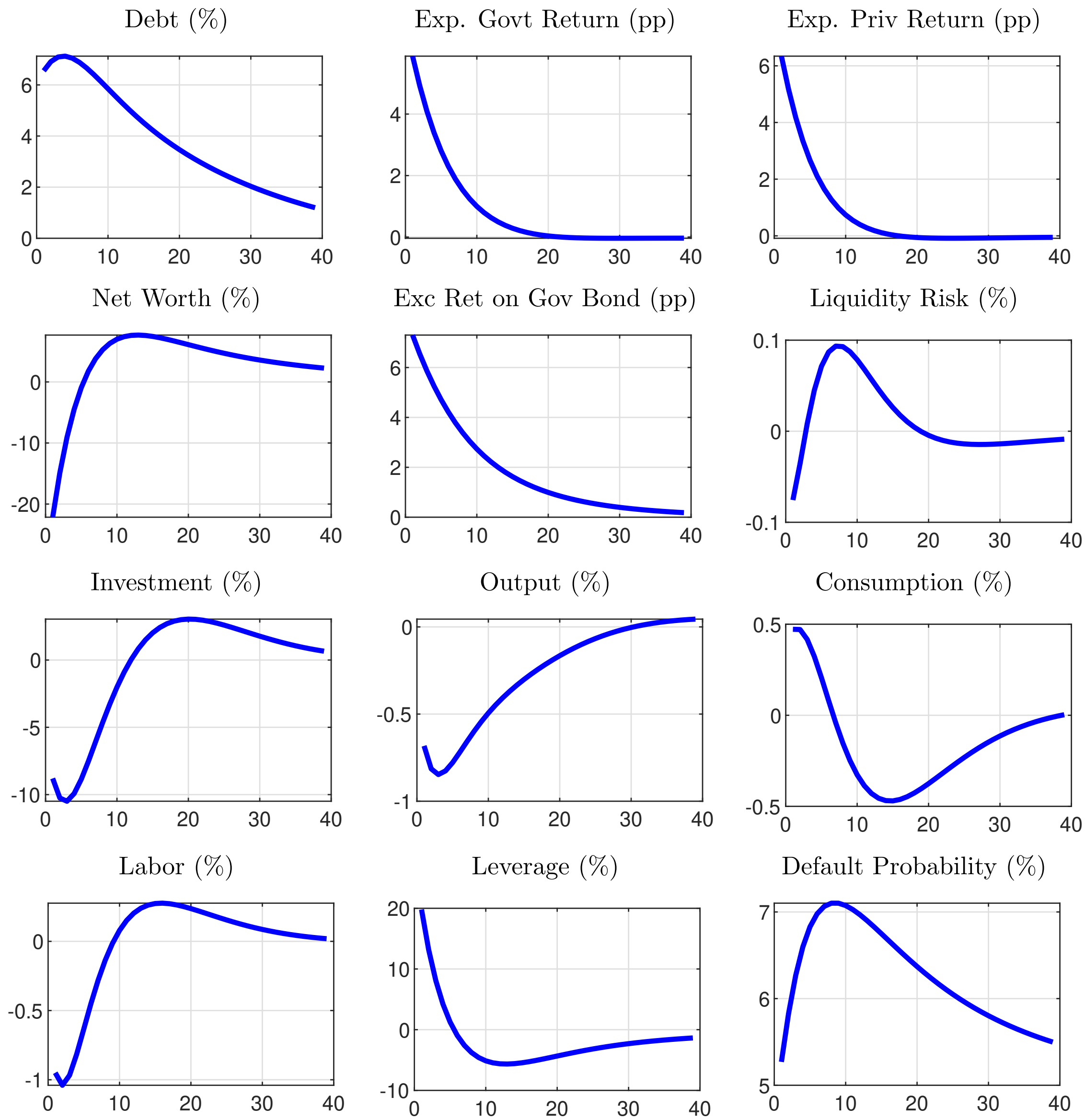
Turning to the impact beyond the financial sector, figure 4 compares the macroeconomic responses between the Home and Foreign economies. The solid blue lines show that in the Home economy, the higher government bond supply crowds out private investment through tighter financial conditions. Labor supply also declines and the real wage falls. Lower investment, as well as a lower labor supply, leads to a sustained decline in output. Consumption increases initially, but quickly declines as income falls. Tighter financial conditions also increase the relative price of Home goods ($\rho_{H,t}$), as firms face higher financing costs. With home bias in goods, this implies higher overall inflation.

In contrast, the Foreign economy, shown in dashed blue lines, sees an increase in investment. This is because the tightening financial conditions in the Home economy prompts an increase in foreign asset holdings. The capital outflow raises foreign investment. Supported by higher investment, Foreign output increases over the medium term. Its consumption also is higher than in the Home country, while stronger demand raises Foreign inflation.

4.2 Default risks vs. liquidity risks In the baseline case, both default and liquidity risks contribute to the deterioration in Home financial conditions and macroeconomic performance. We distinguish the impact from each channel in this section, as highlighted in figure 5.

The direct impact from Home sovereign default risk is modest. To see this, the dashed black lines show the case with only default risk and the liquidity risk channel is turned off

Figure 3: Response to a Default Risk Crisis, Home Country

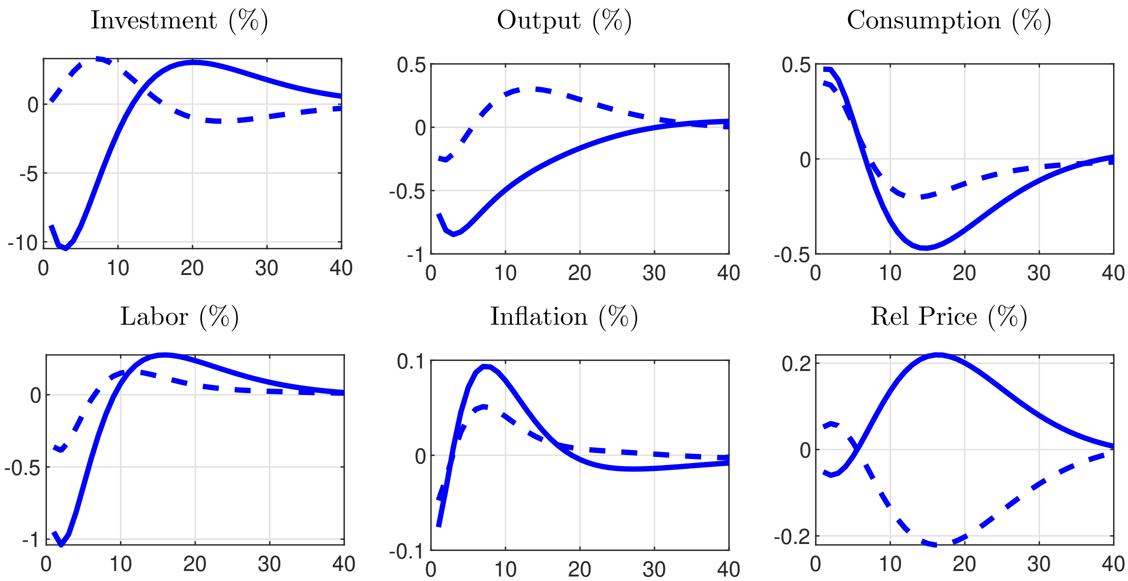


Notes: All variables are plotted as deviations from their stochastic steady states. Expected spread and deposit rate are shown in annualized rates, while default probability is in level.

(with $\phi_\eta = 0$). The possibility of receiving a haircut in the future prompts the financial intermediary to demand a higher expected return on government bonds, tightening financial conditions. Net worth decreases, while leverage rises. The tighter financial conditions weigh on the private bond market and depress investment.

On the other hand, when interacted with the liquidity risk channel, default risks imply a more pronounced deterioration in financing conditions and the economic outlook. To see this, the gap between the solid and the dashed lines reflects the amplification from the

Figure 4: Response to a Default Risk Crisis, Home vs. Foreign Country



Notes: Blue solid lines are responses for the Home economy and blue dashed lines are for the Foreign economy. All variables are plotted as deviations from their stochastic steady states. Expected spread and deposit rate are shown in annualized rates, while default probability is in level.

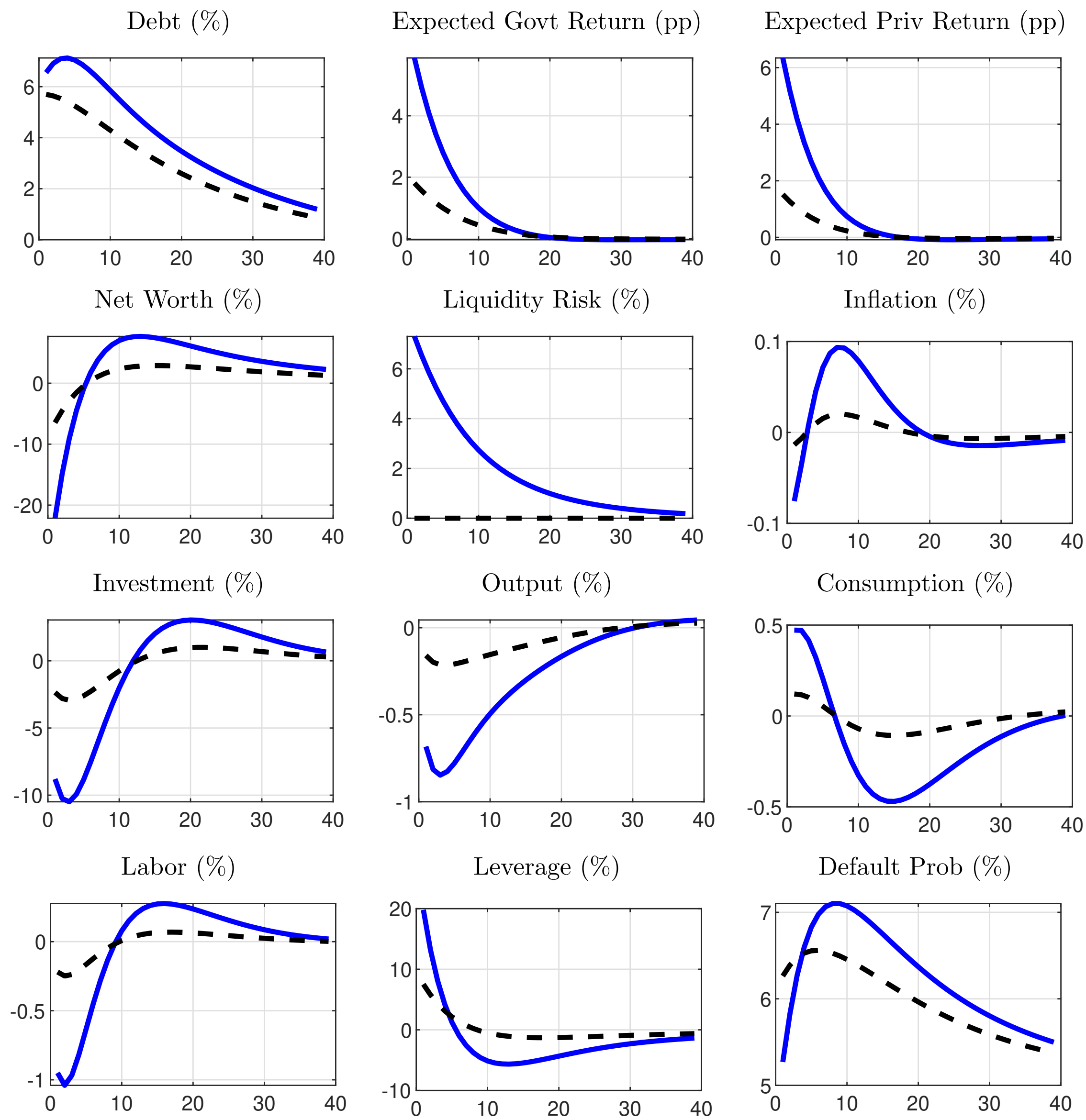
liquidity risk channel. Higher government debt not only raises the likelihood of the financial intermediary receiving a haircut, but also tightens the incentive constraint since $\phi_\eta > 0$. A more binding constraint amplifies the decline in asset prices and net worth, as well as the rise in leverage. The quantitative impact from default risk is much more significant in this case. For instance, the default risk reduces investment by 3 percent when the liquidity risk channel is turned off, but lowers investment by 10 percent when the default risks also trigger liquidity risks.

4.3 Shifts in fiscal limit

In the section, we highlight the impact from the shift in fiscal limit in figure 6 by turning off the shift in fiscal limits. The dashed black lines show this alternative case, while the solid blue lines show the baseline case with responses to both a rise in government debt as well as a shift in fiscal limit. The comparison shows that the shift in fiscal limit plays a more important role in explaining the quantitative impact, while both higher government debt and a deterioration in fiscal limit tighten financial conditions and depress private investment. For instance, the initial increase in government debt contributes to a decline of 3 percent in investment out of the overall contraction of more than 10 percent.

The results highlight the importance of investor sentiment on financial market as well as macro economy. During the European debt crisis, the sharp deterioration in the fiscal outlook of Greek government may have undermined investors' belief in Italian government

Figure 5: Response to a Default Risk Crisis, with and without Liquidity Risk

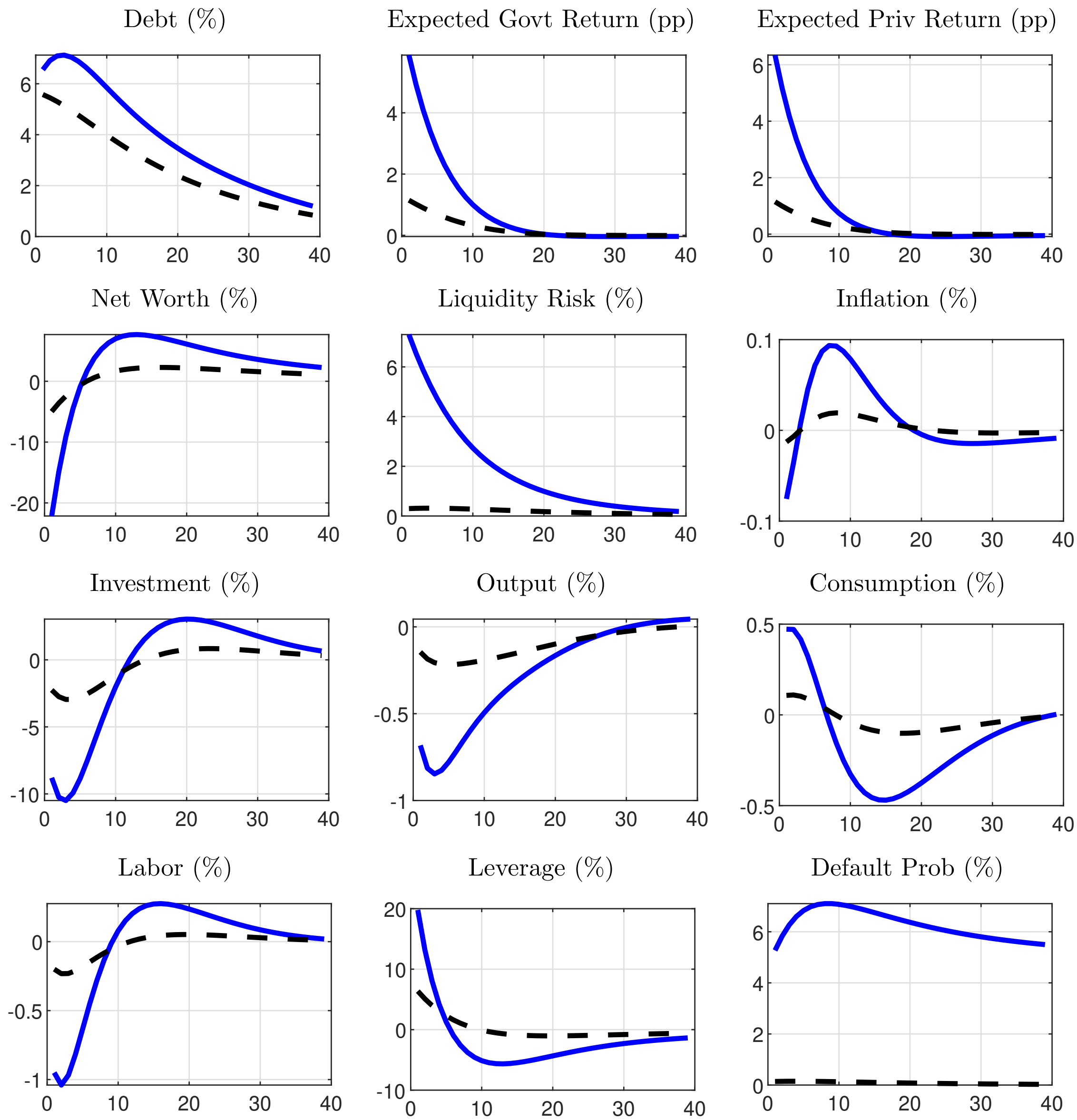


Notes: Solid blue lines show the response in the baseline case with both default and liquidity risk channels. Dashed black lines show the response in the alternative case with default risks but abstracting from the liquidity risk channel.

fiscal space, effectively shifting down the distribution of Italian fiscal limits. Therefore, a debt level that would have been sustainable during normal times may become unsustainable during the crisis.

4.4 Asset purchases We now consider the impact of credit interventions by the central bank, by allowing the central bank to conduct asset purchases in response to movements in the Home credit spread. To understand the effect of asset purchases, we compare the

Figure 6: Response to a Fiscal Limit Shift

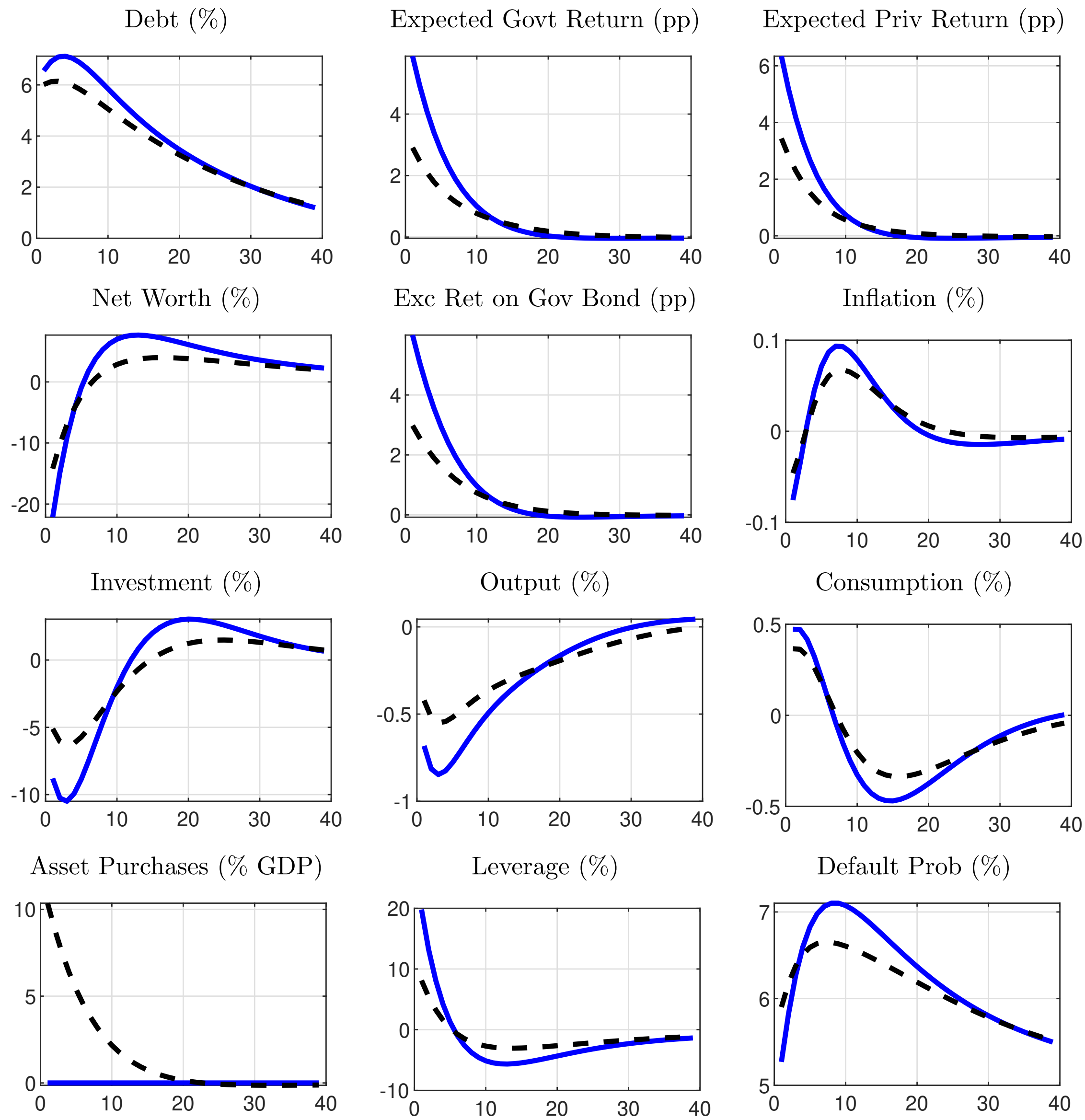


Notes: Solid blue lines show the response in the baseline case with a shift in fiscal limits. Dashed black lines show the response in the alternative case with an initial increase in Home government debt but without a shift in fiscal limits.

responses from the baseline case with those from a case in which the central bank conducts asset purchases as dictated by equation (2.32). Figure 7 displays the responses in both cases for the Home economy following a shift in its fiscal limit as well as an increase in its government debt.

The central bank's credit policy, as modeled in this framework, can help stabilize the Home economy. To see this, figure 7 shows the responses with the endogenous asset purchases

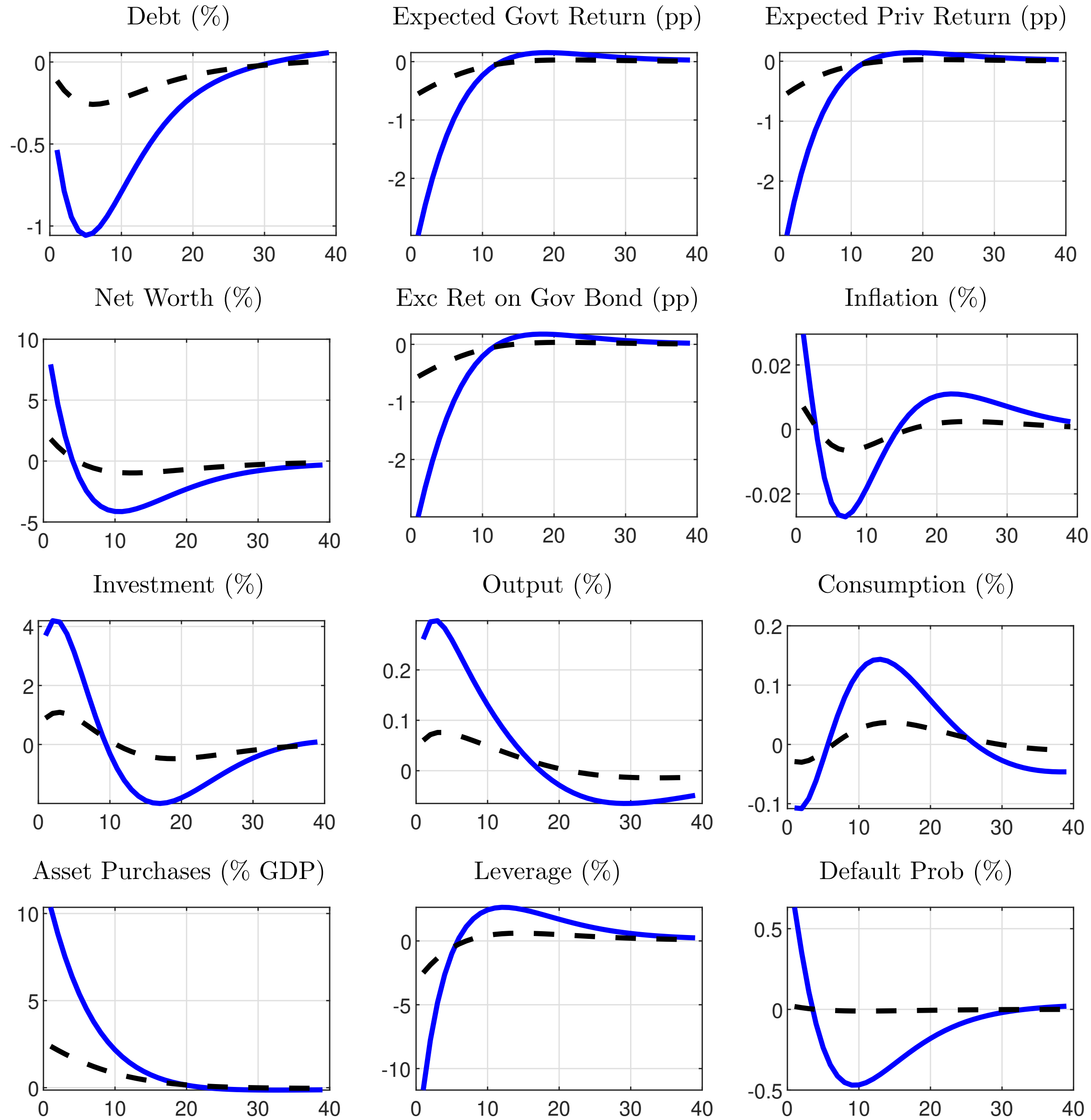
Figure 7: Response to a Default Risk Crisis, Effects of Asset Purchases



Notes: Home responses following an increase in Home government debt. Solid blue lines show the responses in the baseline case, and dashed black lines show the case with endogenous asset purchases.

(dashed black lines), as well as the responses from the baseline case (solid blue lines). The central bank's asset purchases significantly moderate the rise in government bond yields and improve the financial conditions relative to the baseline case. With asset purchases, there is a more subdued decline in net worth and a less sharp rise in leverage. Macroeconomic conditions are also improved, as the declines in investment and output are more muted. Endogenous asset purchases imply lower funding costs for firms, which dampens the increase in Home relative prices and Home inflation.

Figure 8: Effectiveness of Asset Purchases, with and without a Fiscal Limit Shift



Notes: This chart shows the response differences between the case with and without asset purchases. Solid blue lines show the baseline case with an increase in Home government debt as well as a shift in fiscal limit. Dashed black lines show the alternative case with an increase in Home government debt while fiscal limit remains unchanged.

The impact of credit policy depends on financial conditions. In figure 8, we consider two cases: 1) the baseline case with an initiate increase in Home government debt as well as a shift in fiscal limits; and 2) the alternative case with only the initial increase in debt but without a shift in fiscal limits. In each case, we plot the impulse response differences as a results of asset purchases. The solid blue lines show the stabilization impact associated with asset purchases in the baseline case, that is, they capture the differences between the two simulations in figure 7. On the other hand, the dashed black lines highlight the impact of credit policy in the case without a shift in fiscal limit. While asset purchases lower the government bond yield and stabilize investment, the magnitude is far smaller than the baseline case. It is consistent with figure 6, as the shift in fiscal limits perceived by investors plays a more important role in the deterioration in financial conditions.

5 Conclusion

In recent years, the European Union has grappled with multiple crises that have affected sovereign default risks and liquidity risks of Union members. In response, the European Central Bank has introduced various credit policies to stabilize the economy when subject to rising default and liquidity risks. In this paper, we quantify the efficacy of asset purchases in a two-country monetary-union framework subject to both default and liquidity risks.

Following a notable increase in the probability of sovereign default from a rise in government debt, we find that both default and liquidity risks dampen economic and financial conditions. However, the quantitative effects depend crucially on the presence of the liquidity risk channel. While the possibility of default lowers asset prices, net worth of financial intermediaries, and economic activity, the quantitative impact is moderate when there is no liquidity risk channel.

These quantitative effects have important implications for the effectiveness of asset purchases. The model suggests that credit policies can help stabilize the economy in the presence of default or liquidity risks, as in either case the policy aims to offset the increase in excess returns on government bonds. Lowering this return helps alleviate pressures in the financial market, which eases credit access for the private sector and lessens the declines in overall economic activity. At the same time, we find the majority of the intervention is in response to heightened liquidity risk, suggesting the policy is particularly effective when this channel is present.

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A Equilibrium Conditions

The Foreign economy is identical to the Home economy except that Home government debt may default while Foreign government debt doesn't. In this section, we present the full set of equilibrium conditions for the baseline model.

Household The endogenous discount factor evolves according to $\Theta_{t+1} = \Theta_t \beta(\tilde{c}_t)$ with $\Theta_0 = 1$, therefore

$$\beta(c_t) = \beta_c (1 + c_t)^{-\omega_\beta} \tag{A.1}$$

$$\beta(c_t^*) = \beta_c^* (1 + c_t^*)^{-\omega_\beta^*} \tag{A.2}$$

Definition of marginal utility of consumption:

$$U_{c,t}(1 + \tau^c) = c_t^{-\sigma_c} \tag{A.3}$$

$$U_{c,t}^*(1 + \tau^{c,*}) = (c_t^*)^{-\sigma_c^*} \tag{A.4}$$

Household's real stochastic discount factor (net of the endogenous discount factor):

$$\Lambda_{t,t+1} = U_{c,t+1} / U_{c,t} \tag{A.5}$$

$$\Lambda_{t,t+1}^* = U_{c,t+1}^* / U_{c,t}^* \tag{A.6}$$

Household Euler equation:

$$\frac{1}{R_t^d} = E_t \beta(c_t) \Lambda_{t,t+1} \frac{1}{\pi_{t+1}} \tag{A.7}$$

$$\frac{1}{R_t^d} = E_t \beta(c_t^*) \Lambda_{t,t+1}^* \frac{1}{\pi_{t+1}^*} \tag{A.8}$$

First order condition for labor:

$$\chi L_t^{\sigma_L} = U_{c,t} w_t \tag{A.9}$$

$$\chi^* L_t^{*,\sigma_L} = U_{c,t}^* w_t^* \tag{A.10}$$

Consumption allocations:

$$c_{H,t} = \alpha_H \rho_{H,t}^{-\phi} c_t \tag{A.11}$$

$$c_{F,t} = (1 - \alpha_H)(\rho_{F,t}^* rer_t)^{-\phi} c_t \tag{A.12}$$

$$c^*_{F,t} = \alpha_F(\rho^*_{F,t})^{-\phi^*} c^*_t \tag{A.13}$$

$$c^*_{H,t} = (1-\alpha_F)(\rho_{H,t}/rer_t)^{-\phi^*} c^*_t \tag{A.14}$$

$$1 = \alpha_H\left(\rho_{H,t}\right)^{1-\phi} + (1-\alpha_H)\left(\rho^*_{F,t} rer_t\right)^{1-\phi} \tag{A.15}$$

$$1 = \alpha_F\left(\rho^*_{F,t}\right)^{1-\phi^*} + (1-\alpha_F)\left(\rho_{H,t}/rer_t\right)^{1-\phi^*} \tag{A.16}$$

Government Government budget constraints:

$$\rho_{H,t} g_t + (1-\Delta_t)(1+\kappa^b Q^b_t)\frac{b_{t-1}}{\pi_t} = Q^b_t b_t + t_t + tax_t \tag{A.17}$$

$$\rho^*_{F,t} g^* + (1+\kappa^{b,*} Q^{b,*}_t)\frac{b^*_{t-1}}{\pi^*_t} = Q^{b,*}_t b^*_t + t_t + tax^*_t \tag{A.18}$$

Governments use transfers in response to changes to government debt level:

$$\frac{t_t - t}{t} = \phi_t \frac{Q^b_{t-1} b_{t-1} - Q^b b}{Q^b b} \tag{A.19}$$

$$\frac{t^*_t - t^*}{t^*} = \phi^* \frac{Q^{b,*}_{t-1} b^*_{t-1} - Q^{b,*} b^*}{Q^{b,*} b^*} \tag{A.20}$$

Default rule:

$$\Delta_t = \begin{cases} 0, & \text{if } s_{t-1} < \mathcal{B}^*_t \\ \delta_b, & \text{otherwise} \end{cases}$$

The conditional probability of a government default tomorrow,

$$P(s_{t-1} \geq \mathcal{B}^*_t) = \frac{\exp[\eta^{FL}_0 + \eta^{FL}_s(s_{t-1} + \epsilon^P_t)]}{1 + \exp[\eta^{FL}_0 + \eta^{FL}_s(s_{t-1} + \epsilon^P_t)]}. \tag{A.21}$$

with $s_{t-1} = \frac{Q^b_{t-1} b_{t-1}}{4 y_{t-1}}$. Definition of tax revenue:

$$tax_t = \tau^i p^w_t y_t + \tau^c c_t \tag{A.22}$$

$$tax^*_t = \tau^{i,*} p^w_t y^*_t + \tau^{c,*} c^*_t \tag{A.23}$$

Firms Private Investment allocations:

$$i_{H,t} = \alpha_H \rho^{-\phi}_{H,t} I_t \tag{A.24}$$

$$i_{F,t} = (1-\alpha_H)(\rho^*_{F,t} rer_t)^{-\phi} I_t \tag{A.25}$$

$$i^*_{F,t} = \alpha_F(\rho^*_{F,t})^{-\phi^*} I^*_t \tag{A.26}$$

$$i^*_{H,t} = (1-\alpha_F)(\rho_{H,t}/rer_t)^{-\phi^*} I^*_t \tag{A.27}$$

Investment producer's production function:

$$I_t^w = u_t^I \left(1 - \frac{\omega_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \tag{A.28}$$

$$I_t^{w*} = \left(1 - \frac{\omega_I}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2\right) I_t^* \tag{A.29}$$

The optimization problem for investment producer at Home is given by

$$\max \quad \sum_{t=0}^{\infty} E_0 \left[\Theta_t \Lambda_{t,t+1} \left(p_t^k u_t^I \left(1 - \frac{\omega_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t - I_t\right)\right].$$

Thus, investment producer's first-order condition:

$$\begin{aligned} 1 = p_t^k u_t^I &\left(1 - \frac{\omega^I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \omega^I \left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \\ &E_t u_{t+1}^I \beta(c_t) \Lambda_{t,t+1} p_{t+1}^k \omega^I \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \end{aligned} \tag{A.30}$$

$$\begin{aligned} 1 = p_t^{k*} &\left(1 - \frac{\omega^I}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2 - \omega^I \left(\frac{I_t^*}{I_{t-1}^*} - 1\right)\frac{I_t^*}{I_{t-1}^*}\right) + \\ &E_t \beta(c_t^*) \Lambda_{t,t+1}^* p_{t+1}^{k*} \omega^I \left(\frac{I_{t+1}^*}{I_t^*} - 1\right)\left(\frac{I_{t+1}^*}{I_t^*}\right)^2 \end{aligned} \tag{A.31}$$

Wholesale production function (after substituting for retail production function):

$$y_t = A_t l_t^{1-\alpha} K_{t-1}^{\alpha} \tag{A.32}$$

$$y_t^* = A^* l_t^{*,1-\alpha^*} K_{t-1}^{*,\alpha^*} \tag{A.33}$$

Law of motion for private capital:

$$K_t = I_t^w + (1-\delta) K_{t-1} \tag{A.34}$$

$$K_t^* = I_t^* + (1-\delta^*) K_{t-1}^* \tag{A.35}$$

Wholesale firm's first-order condition for labor:

$$w_t = (1-\alpha)\frac{p_t^w y_t^w}{L_t}(1-\tau^i) \tag{A.36}$$

$$w_t^* = (1-\alpha^*)\frac{p_t^{w,*} y_t^{w,*}}{L_t^*}(1-\tau^{i,*}) \tag{A.37}$$

Wholesale firm's first-order condition for capital:

$$\zeta_t^1 = E_t\beta(c_t)\Lambda_{t,t+1}\left(\frac{p_{t+1}^w \alpha y_{t+1}}{K_t}(1-\tau^i) + (1-\delta)\zeta_{t+1}^1\right) \tag{A.38}$$

$$\zeta_t^{1,*} = E_t\beta(c_t^*)\Lambda_{t,t+1}^*\left(\frac{p_{t+1}^{w,*} \alpha^* y_{t+1}^*}{K_t^*}(1-\tau^{i,*}) + (1-\delta^*)\zeta_{t+1}^{1,*}\right) \tag{A.39}$$

The loan-in-advance constraints for private capital:

$$Q_t^f\left(f_t - \kappa^f\frac{f_{t-1}}{\pi_t}\right) = \eta^I p_t^k I_t^w \tag{A.40}$$

$$Q_t^{f,*}\left(f_t^* - \kappa^{f,*}\frac{f_{t-1}^*}{\pi_t^*}\right) = \eta^{I,*} p_t^{k,*} I_t^{w,*} \tag{A.41}$$

Wholesale firm's first-order conditions for I_t^w:

$$\zeta_t^1 = (1+\eta^I\zeta_t^2) \tag{A.42}$$

$$\zeta_t^{1,*} = (1+\eta^{I,*}\zeta_t^{2,*}) \tag{A.43}$$

where ζ_t^1, ζ_t^2, $\zeta_t^{1,*}$, and $\zeta_t^{2,*}$ are the Lagrangian multipliers. Wholesale firm's first-order conditions for corporate bond:

$$Q_t^f(1+\zeta_t^2) = E_t\beta(c_t)\Lambda_{t,t+1}\frac{1}{\pi_{t+1}}\left(1+\kappa^f Q_{t+1}^f(1+\zeta_{t+1}^2)\right) \tag{A.44}$$

$$Q_t^{f,*}(1+\zeta_t^{2,*}) = E_t\beta(c_t^*)\Lambda_{t,t+1}^*\frac{1}{\pi_{t+1}^*}\left(1+\kappa^{f,*} Q_{t+1}^{f,*}(1+\zeta_{t+1}^{2,*})\right) \tag{A.45}$$

Retail producers choose labor and price to optimize real profits, given by:

$$\sum_{k=0}^{\infty} E_0\left[\Theta_{t+k}\Lambda_{t,t+k}\left(p_{t+k}(h)y_{t+k}(h) - \frac{p_{t+k}^w}{\rho_{t+k}^H}y_{t+k}(h) - \frac{\psi}{2}\left(\frac{P_{t+k}(h)}{P_{t+k-1}(h)}\frac{1}{\pi^H} - 1\right)^2 y_{t+k}\right)\right]$$

where $p_t(h) = P_t(h)/P_t^H$. Phillips equations from the optimization problem of retail producers:

$$\frac{p_t^w}{\rho_t^H} = \frac{\theta^c - 1}{\theta^c} + \frac{\psi}{\theta^c}\left(\frac{\pi_t^H}{\pi^H} - 1\right)\frac{\pi_t^H}{\pi^H} - \frac{\psi}{\theta^c} E_t \beta(c_t)\Lambda_{t,t+1}\left(\frac{\pi_{t+1}^H}{\pi^H} - 1\right)\frac{\pi_{t+1}^H}{\pi^H}\frac{y_{t+1}}{y_t} \tag{A.46}$$

$$\frac{p_t^{w,*}}{\rho_t^{F,*}} = \frac{\theta^{c*} - 1}{\theta^{c*}} + \frac{\psi^*}{\theta^{c*}}\left(\frac{\pi_t^{F,*}}{\pi^{F,*}} - 1\right)\frac{\pi_t^{F,*}}{\pi^{F,*}} - \frac{\psi^*}{\theta^c} E_t \beta(c_t^*)\Lambda_{t,t+1}^*\left(\frac{\pi_{t+1}^{F,*}}{\pi^{F,*}} - 1\right)\frac{\pi_{t+1}^{F,*}}{\pi^{F,*}}\frac{y_{t+1}^*}{y_t^*} \tag{A.47}$$

Financial Intermediary The financial intermediary's balance sheet conditions:

$$Q_t^b b_t^H + Q_t^f f_t + Q_t^{b,*} b_t^F = d_t + n_t \tag{A.48}$$

$$Q_t^{b,*} b_t^{F,*} + Q_t^{f,*} f_t^* + Q_t^b b_t^{H,*} = d_t^* + n_t^* \tag{A.49}$$

Evolutions of net worth:

$$\begin{aligned} n_t &= \sigma\left[\left(R_t^b - R_{t-1}^d\right)\frac{Q_{t-1}^b b_{t-1}^H}{\pi_t} + \left(R_t^f - R_{t-1}^d\right)\frac{Q_{t-1}^f f_{t-1}}{\pi_t} + \left(R_t^{b,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{b,*} b_{t-1}^F}{\pi_t}\right] \\ &\quad + \sigma R_{t-1}^d \frac{n_{t-1}}{\pi_t} + (1-\sigma)x \end{aligned} \tag{A.50}$$

$$\begin{aligned} n_t^* &= \sigma^*\left[\left(R_t^{b,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{b,*} b_{t-1}^{F,*}}{\pi_t^*} + \left(R_t^{f,*} - R_{t-1}^d\right)\frac{Q_{t-1}^{f,*} f_{t-1}^*}{\pi_t^*} + \left(R_t^b - R_{t-1}^d\right)\frac{Q_{t-1}^b b_{t-1}^{H,*}}{\pi_t^*}\right] \\ &\quad + \sigma^* R_{t-1}^d \frac{n_{t-1}^*}{\pi_t^*} + (1-\sigma^*)x^* \end{aligned} \tag{A.51}$$

Definitions of rates of return:

$$R_t^b = (1-\Delta_t)\frac{1+\kappa^b Q_t^b}{Q_{t-1}^b}, \quad R_t^f = \frac{1+\kappa^f Q_t^f}{Q_{t-1}^f} \tag{A.52}$$

$$R_t^{b,*} = \frac{1+\kappa^{b,*} Q_t^{b,*}}{Q_{t-1}^{b,*}}, \quad R_t^{f,*} = \frac{1+\kappa^{f,*} Q_t^{f,*}}{Q_{t-1}^{f,*}} \tag{A.53}$$

Adjusted leverage:

$$\phi_t = \frac{Q_t^f f_t + \theta^b m_t^b}{n_t} \tag{A.54}$$

$$\phi_t^* = \frac{Q_t^{f,*} f_t^* + \theta^{b,*} m_t^{b,*}}{n_t^*} \tag{A.55}$$

Definition of Ω:

$$\Omega_t = 1 - \sigma + \sigma \eta^v \phi_t \tag{A.56}$$

$$\Omega_t^* = 1 - \sigma^* + \sigma^* \eta^{v,*} \phi_t^* \tag{A.57}$$

Portfolio manager allocations in the government bond market:

$$\frac{Q_t^b b_t^H}{m_t^b} = \gamma_b \left(\frac{E_t R_{t+1}^b}{E_t R_{t+1}^m} \right)^{-\sigma_b} \tag{A.58}$$

$$\frac{Q_t^{b,*} b_t^F}{m_t^b} = (1 - \gamma_b) \left(\frac{E_t R_{t+1}^{b,*}}{E_t R_{t+1}^m} \right)^{-\sigma_b} \tag{A.59}$$

$$\frac{Q_t^{b,*} b_t^{F,*}}{m_t^{b,*}} = \gamma_{b,*} \left(\frac{E_t R_{t+1}^{b,*}}{E_t R_{t+1}^{m,*}} \right)^{-\sigma_{b,*}} \tag{A.60}$$

$$\frac{Q_t^b b_t^{H,*}}{m_t^{b,*}} = (1 - \gamma_{b,*}) \left(\frac{E_t R_{t+1}^b}{E_t R_{t+1}^{m,*}} \right)^{-\sigma_{b,*}} \tag{A.61}$$

$$R_t^m = \left[\gamma_b \left(R_t^b \right)^{1-\sigma_b} + (1 - \gamma_b) \left(R_t^{b,*} \right)^{1-\sigma_b} \right]^{\frac{1}{1-\sigma_b}} \tag{A.62}$$

$$R_t^{m,*} = \left[\gamma_{b,*} \left(R_t^{b,*} \right)^{1-\sigma_{b,*}} + (1 - \gamma_{b,*}) \left(R_t^b \right)^{1-\sigma_{b,*}} \right]^{\frac{1}{1-\sigma_{b,*}}} \tag{A.63}$$

First-order conditions for portfolios:

$$E_t \beta(c_t) \Lambda_{t,t+1} \Omega_{t+1} \frac{R_{t+1}^f - R_t^d}{\pi_{t+1}} = \frac{\lambda_t^v}{1 + \lambda_t^v} \eta_t^v \tag{A.64}$$

$$E_t \beta(c_t) \Lambda_{t,t+1} \Omega_{t+1} \left(\frac{R_{t+1}^b - R_t^d}{\pi_{t+1}} \frac{Q_t^b b_t^H}{m_t^b} + \frac{R_{t+1}^{b,*} - R_t^d}{\pi_{t+1}} \frac{Q_t^{b,*} b_t^F}{m_t^b} \right) = \frac{\lambda_t^v}{1 + \lambda_t^v} \eta_t^v \theta^b \tag{A.65}$$

$$E_t \beta(c_t^*) \Lambda_{t,t+1}^* \Omega_{t+1}^* \frac{R_{t+1}^{f,*} - R_t^d}{\pi_{t+1}} = \frac{\lambda_t^{v,*}}{1 + \lambda_t^{v,*}} \eta_t^{v,*} \tag{A.66}$$

$$E_t \beta(c_t^*) \Lambda_{t,t+1}^* \Omega_{t+1}^* \left(\frac{R_{t+1}^{b,*} - R_t^d}{\pi_{t+1}^*} \frac{Q_t^{b,*} b_t^{F,*}}{m_t^{b,*}} + \frac{R_{t+1}^b - R_t^d}{\pi_{t+1}^*} \frac{Q_t^b b_t^{H,*}}{m_t^{b,*}} \right) = \frac{\lambda_t^{v,*}}{1 + \lambda_t^{v,*}} \eta_t^{v,*} \theta^{b,*} \tag{A.67}$$

Evolution of adjusted leverage:

$$\frac{\phi_t}{1 + \lambda_t^v} \eta^v = E_t \beta(c_t) \Lambda_{t,t+1} \frac{\Omega_{t+1}}{\pi_{t+1}} R_t^d \tag{A.68}$$

$$\frac{\phi_t^*}{1 + \lambda_t^{v,*}} \eta^{v,*} = E_t \beta(c_t^*) \Lambda_{t,t+1}^* \frac{\Omega_{t+1}^*}{\pi_{t+1}^*} R_t^d \tag{A.69}$$

The Rest Goods' market clearing:

$$y_t = c_t^H + c_t^{H,*} + g^+ i_t^H + i_t^{H,*} \tag{A.70}$$

$$y_t^* = c_t^{F,*} + c_t^F + g^* + i_t^{F,*} + i_t^F \tag{A.71}$$

Monetary policy:

$$\ln \frac{R_t^d}{R^d} = \phi_\pi \ln \frac{\pi_t^{ag}}{\pi^{ag}} + \phi_y \ln \frac{y_t^{ag}}{y^{ag}} \tag{A.72}$$

Asset purchase policy:

$$re_t = re + \phi_{cb}(R_t^{spread} - R^{spread}) \tag{A.73}$$

with

$$re_t = Q_t^b b_t^{cb} \tag{A.74}$$

$$R_t^{spread} = E_t R_{t+1}^b - R_t^d \tag{A.75}$$

Market clearing in asset markets:

$$b_t = b_t^{cb} + b_t^H + b_t^{H,*} rer_t \tag{A.76}$$

$$b_t^* = b_t^{F,*} + \frac{b_t^F}{rer_t} \tag{A.77}$$

$$0 = b_t^i + rer_t b_t^{i,*} \tag{A.78}$$

Net foreign assets evolution:

$$b_t^i + Q_t^{b,*} b_t^F - Q_t^b b_t^{H,*} rer_t = \frac{R_{t-1}^d b_{t-1}^i}{\pi_t} + \frac{R_t^{b,*} Q_{t-1}^{b,*} b_{t-1}^F}{\pi_t}$$
$$-R_t^l Q_{t-1}^b \frac{b_{t-1}^{H,*} rer_{t-1}}{\pi_t} + \rho_t^H (c_{H,t}^* + i_{H,t}^*) - \rho_t^{F,*} rer_t (c_{F,t} + i_{F,t}) \tag{A.79}$$

Relative consumer price adjustment:

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^*}{\pi_t} \tag{A.80}$$

B Computational Details

This Appendix provides details of the solution method. As noted in section 3.2, the evolution of the haircut (2.5) and the default probability (2.6) implies that the model is an endogenous regime-switching model. There is a regime variable, and $def_t = 0$ indicates no default while $def_t = 1$ indicates default. The haircut can then be written as

$$\Delta_t = \begin{cases} 0, & \text{if } def_t = 0 \\ \delta_b, & \text{if } def_t = 1 \end{cases} \tag{B.1}$$

The transition matrix is time varying depending on the state $s_t = \frac{Q_t^b b_t}{4y_t}$ and the underlying macroeconomic shocks, and has elements $\mathbb{P}_{ij,t} = \Pr\left(def_{t+1} = j | def_{t-1}; s_t\right)$. Using our assumption that the transition follows a logistic function, we have

$$\mathbb{P}_t = \begin{bmatrix} \mathbb{P}_{00,t} & \mathbb{P}_{01,t} \\ \mathbb{P}_{10,t} & \mathbb{P}_{11,t} \end{bmatrix} = \begin{bmatrix} 1 - pdef_t & pdef_t \\ 1 - pdef_t & pdef_t \end{bmatrix} \tag{B.2}$$

where

$$pdef_t = \frac{\exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_t^P)]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL}(s_{t-1} + \epsilon_t^P)]}. \tag{B.3}$$

We use the perturbation approach for solving endogenous regime-switching models in Benigno, Foerster, Otrok, and Rebucci (2024), which generates a set of approximated decision rules conditional on each regime. More specifically, the equilibrium conditions in Appendix A can be written as

$$E_t f\left(y_{t+1}, y_t, x_t, x_{t-1}, \chi\varepsilon_{t+1}, \varepsilon_t, \theta_{t+1}, \theta_t\right) = 0 \tag{B.4}$$

where y_t denotes the non-predetermined variables, x_t the predetermined variables, ε_t the shocks, and θ_t the regime switching parameters. In our case, we follow Foerster, Rubio-Ramírez, Waggoner, and Zha (2016) and use the partition principle to write

$$\theta_t \equiv \Delta\left(def_t\right) = \bar{\Delta} - \chi\hat{\Delta}\left(def_t\right) \tag{B.5}$$

where $\bar{\Delta}$ denotes the ergodic mean of $\Delta\left(def_t\right)$ using the steady state matrix $\mathbb{P}_{ss}$, where

$$pdef_{ss} = \frac{\exp[\eta_0^{FL} + \eta_s^{FL} s_{ss}]}{1 + \exp[\eta_0^{FL} + \eta_s^{FL} s_{ss}]}. \tag{B.6}$$

Benigno, Foerster, Otrok, and Rebucci (2024) show that an iterative procedure can be used to solve for the steady state of the equilibrium conditions – this procedure is needed because the ergodic mean $\bar{\Delta}$ depends on the steady state debt-to-GDP ratio, which is itself a function of $\bar{\Delta}$.

After finding the steady state, we use perturbation to find second-order approximations to the regime-dependent decision rules

$$y_t = g\left(x_{t-1}, \varepsilon_t, \chi; def_t\right) \tag{B.7}$$

$$x_t = h\left(x_{t-1}, \varepsilon_t, \chi; def_t\right) \tag{B.8}$$

We then use these approximated decision rules for simulations. One relevant feature of the approximated decision rules is that second-order approximations are necessary to capture behavior induced by endogenous probabilities. Benigno, Foerster, Otrok, and Rebucci (2024) show that first-order approximations are identical to an exogenous regime-switching model with transition probabilities $\mathbb{P}_{ss}$. Intuitively, the second-order approximation is necessary to capture important features of our model, such as households and firms internalizing the fact that default becomes more likely as the debt-to-output ratio increases.