OPTIMAL PRICING OF INTRADAY LIQUIDITY

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Abstract

This paper presents a general equilibrium model where intraday liquidity is needed because the timing of payments is uncertain. A necessary and sufficient condition for an equilibrium to be efficient is that the nominal intraday interest rate be zero, even when the overnight rate is strictly positive. Because a market for liquidity may not achieve efficiency, this creates a role for the central bank. I allow for the possibility of moral hazard and study policies commonly used by central banks to reduce their exposure to risk. I show that collateralized lending achieves the efficient allocation, while, for certain parameters, caps cannot prevent moral hazard.

1   Introduction

This paper studies a model in which intraday liquidity is needed because the timing of receipts is not well coordinated with the timing of obligations. There is a role for a central bank to provide this liquidity because a market can not always achieve the efficient allocation. The model allows for the possibility of moral hazard, and I study policies commonly used by central banks to reduce their exposure to risk. Collateralized lending is shown to achieve the efficient allocation. Caps, on the other hand, can not always prevent moral hazard.

Intraday liquidity is needed in this model because it is costly to make precise the time at which payments are received. If liquidity shocks are uninsurable, a necessary and sufficient condition for the equilibrium allocation to be efficient is for the intraday interest rate to be zero.[1] To obtain this result, I use a mechanism design approach. I consider the efficient allocation (i.e., the allocation achieved by a central planner) and I compare it to the equilibrium allocation. In equilibrium, the overnight interest rate is the inverse of the discount factor, while the interest rate in a market for intraday funds may not be equal to zero. This creates a role for a central bank that has the ability to create money that is willingly held: It can supply the needed liquidity at an interest rate of zero and achieve efficiency.

The reason for the difference between overnight and intraday interest rates is that money plays two different roles, which, in this model, require two different prices. Money is used as an asset (a store of value) and is also used to make transactions (medium of exchange). Because of the mismatch between receipts and uses of funds, it is optimal for money to be available at a zero interest rate when it is used as a medium of exchange. Yet, because it is an asset, it must have a positive return in order to be held in equilibrium. Because liquidity shocks are uninsurable, the market is unable to price liquidity efficiently. As I will show, achieving efficiency requires the stock of money to be increased temporarily. A central bank (or any special agent with the right incentives and power to temporarily increase the money supply) can do that, but the market cannot.

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[1]I assume a constant money supply, so the real and the nominal interest rate are the same. In the appendix, I show that the results generalize to constant growth of the money stock.

This might help explain the fact that many central banks, including the Federal Reserve and the European Central Bank, set the cost of intraday funds–funds that must be paid by the end of the day–at zero, or very close to zero, while the cost of overnight funds–funds that must be paid the next day–typically varies between 3 and 6 percent.

There is a concern among regulators that lending intraday funds at a very low interest rate could create moral hazard and lead to the central bank being exposed to excessive risk. This model allows me to study standard policies that central banks use to try to reduce their exposure to credit risk, and investigate their effectiveness. To do this, I introduce risk in the model and allow for the possibility of moral hazard. In that context I study three policies: 1) charging a small positive interest rate, 2) requiring collateral, and 3) imposing a quantitative limit or cap. I show that imposing collateral is the only policy that can always achieve the efficient allocation.

My approach to modelling payments is similar to that of Freeman (1996), Green (1997), or Kahn and Roberds (2001). I do not directly model banks, and some features of actual payment systems are missing. Instead, I model agents who face friction that are thought to be relevant to participants in the payment system. In this paper, the friction is the mismatch between receipts and uses of funds.

The introduction of an endogenous choice of risk might be the primary novelty of the paper.[2] Agents in the model have an incentive to invest in excessively risky technologies because they can strategically default on the funds they borrowed from the central bank. I show that agents will not choose this option if the central bank makes collateralized loans. On the other hand, I show that, even under the tightest cap consistent with the efficient allocation, agents will have an incentive to invest in excessively risky technologies for certain parameters values. The third option, charging a small but positive interest rate, also leads to an inefficient allocation.

Another interesting result is that a zero intraday interest rate is necessary to implement the efficient allocation in an environment where the overnight interest rate can be strictly

[2]Freeman (1999), Temzelides and Williamson (2001) consider environments with exogenous risk. Kahn and Roberds (2001) consider the case where a central bank may not enforce repayment of its loans.

positive. The basic friction is essentially the same as in Freeman (1996) (see also Green, 1997), but in his model both intraday and overnight interest rates must be zero to achieve the efficient allocation. This is an application of the Friedman rule. Zhou (2000), Kahn and Roberds (2001) show that the intraday interest rate should still be zero in a second-best equilibrium if inflation makes the overnight rate strictly positive. By considering a model where inflation has no welfare costs, this paper shows that the efficiency of a zero intraday interest rate is not a consequence of the Friedman rule, nor is it a way of offsetting a cost of inflation.[3]

The remainder of the paper is organized as follows. Section 2 provides a brief review of how payment systems function. Section 3 presents the model, defines an equilibrium, characterizes the efficient allocation, and shows that a zero intraday interest rate is a necessary condition for an equilibrium to be efficient. Section 4 shows that allowing the timing of payments to be endogenous does not guarantee the market intraday interest rate will be efficient. Section 5 introduces risk and analyzes policies designed to prevent moral hazard. Section 6 concludes.

2   A brief review of payment systems

This section describes how payments are typically made over large-value payment systems and the policies under which central banks lend to institutions which use these systems. I will be particularly interested in two systems: Fedwire, a service provided by the Federal Reserve, and TARGET, the system of the European Monetary Union. The material in this section is largely based on Zhou (2000), in which more details can be obtained.

To get a sense of the importance of large-value payment systems, one can consider the value of transfers done in the U.S. on the two main systems, CHIPS, which is privately owned and operated, and Fedwire. Zhou (2000) reports that in 1999, the *daily* value of transfers on the two systems was approximately 35 percent of the U.S. *annual* GDP for the same year.

When financial institutions (which, for simplicity, I will refer to as banks) need to transfer

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[3]It is also worth noting that the paper demonstrates that the main results in this literature, which typically studies overlapping generation models, survive in an infinite horizon framework.

funds to each other, they usually use a large-value payment system. On can think of a payment as consisting of two operations: When bank A needs to transfer funds to bank B, it first sends a payment message which is processed by the system and transmitted to bank B. The second operation is the settlement, at which time the funds are actually transferred. This may happen immediately or at a later time, depending on the system. Typically, the means of settlement on large-value payment systems is central bank funds (base money).

My focus will be on Real Time Gross Settlement (RTGS) systems, which have the property that payment messages are processed as they arrive and settlement occurs immediately. Both Fedwire and TARGET are RTGS systems. This is in contrast to net settlement systems where settlement only occurs at a pre-specified time, usually the end of the day, after all claims have been netted. CHIPS is an example of a net settlement system. In the previous example, if bank A transfers funds to bank B over Fedwire, the account of bank A with the Federal Reserve is immediately debited and the account of bank B immediately credited.

Central banks operating RTGS systems often extend intraday credit to banks. If the amount of reserves bank A has on its account is not enough to cover the payment made to bank B, the Federal Reserve will extend credit to bank A and process the payment. Two important questions are whether central banks should extend such credit, and if they do, at what price should funds be lent?[4]

By extending intraday credit, the central bank exposes itself to a certain amount of credit risk. This has raised some concerns among regulators, and different policies have been put in place in order to moderate that risk. Zhou (2000) lists three such policies: 1) imposing a small positive interest rate on intraday credit, 2) requiring collateral or intraday repos,[5] and 3) imposing an upper limit (also known as a cap) on the amount of intraday credit a bank may receive.

Different policies have been used by central banks, and these policies may change over time. For example, from 1987 to 1999, Switzerland had a RTGS system (called SIC) under which

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[4]Note that net settlement systems implicitly allow banks to borrow at an interest rate of zero.

[5]An intraday repo is a sale of securities, combined with a promise to repurchase the securities later the same day.

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the Swiss National Bank (SNB) would not lend intraday funds. This changed in October 1999, however, and it now allows intraday, repos-backed, interest-free overdrafts. On the other hand, the Federal Reserve always allowed intraday overdrafts for transactions taking place on Fedwire. It used to lend at no cost and without limitations until 1986. At that time, a cap on the amount of allowable intraday overdrafts was imposed because of concerns about the Fed's exposure to credit risk. In 1994, motivated by the same concerns, the Federal Reserve also started charging a small interest rate on overdrafts. Under the TARGET system, fully collateralized overdrafts have always been allowed and can be obtained interest-free.

The analysis in this paper suggests that the interest rate charged for intraday overdrafts on Fedwire creates inefficiencies while imposing a cap may not be able to prevent moral hazard. In contrast, the design of TARGET, and SIC since 1999, appear to be best. In particular, collateralized lending is shown to prevent moral hazard.

3 The environment

There are three types of agents (consumers, producers, and retailers) each of mass 1, who live forever. There is one consumption good in each period t, $t = 0, 1, 2, \ldots$. In each even period, there is also a good x from which no utility can be directly derived, but which can be used to produce good c.

Each consumer has an endowment of money M at date zero. Also, at the beginning of every even date, each consumer is endowed with an amount ω_x of good x. Producers are endowed with a technology that produces one unit of good c for each unit of good x used as input. Retailers have access to a technology that stores good c from one period to the next. This technology returns one unit for each unit invested.[6]

Producers specialize in transforming good x into good c, and I assume they cannot sell these goods directly to the consumers. This is the job of the retailers. One can tell the following story: Each producer makes a slightly different type of good c, and the consumers

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[6]Note that good c can be stored between odd and even periods–from an afternoon to the next morning. However, no good will be stored in equilibrium. The appendix shows how the results generalize for any return γ of the storage technology such that $\gamma \leq \frac{1}{\beta}$.

want some of each of these goods (as in Schreft (1992)). The retailers' job is to get all these goods together in one location where it is convenient for consumers to buy them.

Consumers have preferences that can be represented by a utility function U, where $U = \sum_{t=0}^{\infty} \beta^t u(c_t)$, and u is strictly increasing and strictly concave, and $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. Producers and retailers maximize profits.

I want to think of each period as half a day. A new day begins with every even period. Even periods can be thought of as the morning and odd periods as the afternoon. The main friction in this model is related to the definition of a day. I assume that it is costly to make the timing of payments very precise, which, in the model, translates into some uncertainty as to when a payment is received.[7] Think of the payment as being mailed, and there is some probability that the mail is delayed.

Agents in this model can guarantee that a payment will be made on a particular day, but might not be able to guarantee that it will be made in a particular period. For example, I can guarantee payment on October 26th, but it might be costly to guarantee that the payment will be made the 26th before 10 am. To allow for endogenous timing of payments, I assume that a payment will arrive at a specified time with probability 1, if a cost, ε, is payed that day.

This friction is central to the results in this paper. It is meant to capture the uncertainty described by Zhou (2000): "In general, a bank has little control over the arrival of its customers' outgoing payment requests, whether they are urgent (time sensitive) requests, and the flow of its incoming funds transfers (which depend on other banks' timing decisions of payments initiation)."

Money is needed in this economy because agents are spatially separated and communication is limited, as in Townsend (1980, 1987). Assume that consumers are separated into two groups of equal size. Let C_1 denote the set of consumers in the first group and C_2 the set of consumers in the second group. Similarly, retailers and producers are also separated into two groups of equal size, denoted by R_1, R_2 and P_1, P_2, respectively. Agents of the same

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[7]This uncertainty is similar to the one faced by agents in Freeman (1996) who don't know if they will have to leave some market early or late.

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type but belonging to different groups never meet. For example, consumers in C_1 never meet consumers in C_2.

The timing of meetings and events goes as follows:

Even periods

1) *Agents in C_1 meet with agents in R_1 and P_1, while in a separate meeting agents in C_2 meet with agents in R_2 and P_2.*

In these meetings, consumers decide how much of their endowment they want to sell to retailers and producers. Producers want to buy goods they can transform and sell to the retailers. Retailers want to get cash to pay for the goods the producers make. Producers and retailers offer consumers claims for cash in the future.

2) *R_1 meets with P_1 and, separately, R_2 meets with P_2.*

Producers transform the good x they bought into good c and then sell these goods to retailers. Retailers choose how many goods to sell in this period and how much to store for the next period.

3) *Consumers in C_1 meet and, separately, so do consumers in C_2.*

Some (randomly chosen) consumers receive payments from the producers. Consumers can borrow or lend cash to and from each other, provided they belong to the same group, at some interest rate r on an intraday market. Because each group is large, the proportion of consumers that receive their payment early is the same in both C_1 and C_2 and so is the intraday interest rate.

4) *C_1 meets with R_2 and, separately, C_2 meets with R_1.*

Consumers meet with retailers they have not had contact with during the period. Since they don't have claims against these retailers, they must use cash to obtain goods. Consumers then consume.

Odd periods

1) *Consumers in C_1 meet and, separately, so do consumers in C_2.*

Consumers who didn't receive a payment from producers in the previous period receive it now. Consumers who borrowed on the intraday market repay their debts.

2) *C_1 meets with R_2 and, separately, C_2 meets with R_1.*

Consumers buy goods from retailers and consume.

3) C_1 *meets with* R_1 *and, separately,* C_2 *meets with* R_2.

After the goods market closes, consumers meet the retailer they lent cash to. The retailers are now able to repay their debts with cash that the consumers will be able to use next period.

<div align="center">TIME LINE</div>

Retailers need money to buy good c from producers. Also, because of the pattern of meetings between groups of consumers and retailers, consumers need money to purchase good c.

By symmetry, the problems of consumers in both C_1 and C_2 are the same. Thus it is not necessary to keep track of the group to which a particular consumer belongs. The same holds true for retailers and producers.

3.1   The consumers' problem

In exchange for their endowment and the cash they currently hold, consumers can buy claims on cash in the future. Producers offer them claims for \$1 in cash later today, and retailers offer them claims for \$1 the next day. The uncertainty about the timing of payments described above is modelled in the following way: The payment from the producer to which a consumer sold her endowment is received early (in the same period good was sold) with probability θ. With probability $1 - \theta$ the payment is received late (in the next period).

Consumers cannot insure themselves against this risk, in particular, they cannot write contracts that are contingent on receiving their payments early or late. Several reasons can be invoked to justify this assumption. For example, if early payment is private information, a contract cannot incorporate that information. A self-enforcing arrangement in which consumers reveal that information truthfully is possible if the same group of consumers meets every day. However, such a scheme will break down if consumers are sufficiently impatient or if, because of spatial separation, they meet only infrequently.

An indirect way to insure against the liquidity risk is for producers to pay a cost ε, which guarantees that their payments arrive early. In this section, I assume $\varepsilon = \infty$. In the next

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section, I consider $\varepsilon \in (0, \infty)$. I assume that an appropriate law of large numbers holds so that the proportion of consumers receiving their payment early is exactly θ. The payment from the retailer is received at the end of the day (the end of the following odd period) and may not be used until the next day (the beginning of the next even period).

Let D and D' denote the quantity of claims a consumer buys from a producer and a retailer, respectively, and let q and q' denote their price. Let p_x denote the price of good x, M denote the quantity of cash that consumers start the day with and M_s the quantity of cash they decide to keep for later. Consumers cannot save more cash that they hold which implies $M_s \le M$. The budget constraint for the household can be written as

$$qD + q'D' + M_s \le p_x\omega_x + M. \tag{1}$$

Let p and p' denote the price of the consumption good in even and odd periods, respectively. Let c and c' denote the amount consumed in even and odd periods, respectively, by those who receive their payment early, and let B denote the amount borrowed by these consumers on the intraday market. The constraints for these consumers are

$$pc \quad \le \quad B + D + M_s, \tag{2}$$
$$p'c' + (1 + r)B \quad \le \quad 0. \tag{3}$$

It is immediately obvious that consumers who receive their payments early will choose to lend funds on the intraday market, so $B < 0$. Combine the two equations above to get

$$(1 + r)pc + p'c' \le (1 + r)\left[D + M_s\right]. \tag{4}$$

I use an upper bar to distinguish consumers who receive their payment late. For them the constraints are

$$p\bar{c} \quad \le \quad \bar{B} + M_s, \tag{5}$$
$$p'\bar{c}' + (1 + r)\bar{B} \quad \le \quad D, \tag{6}$$

and they can be combined to yield

$$(1 + r)p\bar{c} + p'\bar{c}' \le D + (1 + r)M_s. \tag{7}$$

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Call $(1 + r)[D + M_s]$ and $D + (1 + r)M_s$ the interim wealth of consumers who receive their payments early, and late, respectively. Consumers will not be able to consume the same amount unless their interim wealth is exactly the same. This is the case if the interest rate in the intraday market is zero, or $r = 0$. A strictly positive intraday interest rate introduces risk in the interim wealth and thus in the levels of consumption. This is disliked by risk-averse consumers.

Consumers who receive their payment early can either save these funds or lend them on the intraday market. There is no alternative use for these funds. In particular, they cannot be invested in any productive technology or on some other market. This assumption could be problematic if one observed that, in practice, banks always borrowed as much as possible from the central bank. This might suggest that there were some profitable investment opportunities in which banks could invest. Evidence from the U.S. suggests that there is no such opportunity. Banks' intraday borrowing is limited by a cap but very few banks even come close to borrowing as much as their cap allows.

The fact that intraday liquidity does not have any alternative use is important, especially if the central bank is unable to monitor the use of the funds it lends. Suppose, for example, that intraday liquidity could be invested to yield a positive return. In order for the central bank to be able to insure consumers against the risk of receiving their payment late, it must make sure that these funds won't be invested instead. If it cannot, consumers who received their payments early and do not need liquidity will nonetheless want to borrow in order to take advantage of the investment opportunity.

Now I can write the consumers' problem:

$$v\left(\frac{M}{p_x}\right) = \max \quad \theta\left[u(c) + \beta u(c')\right] + (1 - \theta)\left[u(\bar{c}) + \beta u(\bar{c}')\right] + \beta^2 v\left(\frac{M''}{p_x''}\right),$$

subject to

$$qD + q'D' + M_s \leq p_x\omega_x + M, \tag{8}$$

$$(1+r)pc + p'c' \leq (1+r)[D + M_s], \tag{9}$$

$$(1+r)p\bar{c} + p'\bar{c}' \leq D + (1+r)M_s, \tag{10}$$

$$M_s \leq M, \tag{11}$$

$$M'' \leq D', \tag{12}$$

where M'' is the amount of cash the consumer holds at the beginning of the next day, and p_x'' is the price of good x at that time.

The first order conditions and the envelope condition for this problem yield:

$$\frac{u'(c)}{\beta u'(c')} = \frac{u'(\bar{c})}{\beta u'(\bar{c}')} = (1+r)\frac{p}{p'}, \tag{13}$$

$$\frac{1}{p_x}v'\left(\frac{M}{p_x}, \theta\right) = \frac{1}{p}[\theta u'(c) + (1-\theta)u'(\bar{c})], \tag{14}$$

$$\frac{1}{p_x''}\beta^2 v'\left(\frac{M''}{p_x''}, \theta\right) = q'\frac{1}{p_x}v'\left(\frac{M}{p_x}, \theta\right), \tag{15}$$

$$(1-q)v'\left(\frac{M}{p_x}, \theta\right) = \frac{r}{1+r}\frac{1-\theta}{p}u'(\bar{c}). \tag{16}$$

I assume there is no money growth, so that $M = M''$ and $p_x = p_x''$. Thus, the third equation implies that $q' = \beta^2$.

The price consumers pay to get \$1 tomorrow is q', so the "overnight interest rate" in this economy is $\frac{1}{\beta^2}$, which is exactly what we would expect since there is no money growth.[8] There are two related notions of intraday interest rate: Producers sell claims to \$1 later in the day at price q, and consumers can borrow and lend on an intraday market for funds at a rate of interest r. Equation (16) provides a link between these two notions and shows, in particular, that $q = 1$ if and only if $r = 0$. In what follows, I mean r when I talk about the intraday interest rate, unless I specify otherwise.

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[8]The discount factor β is squared because a day is composed of two periods.

3.2 The producers' problem

Producers buy good x from consumers at price p_x, sell claims to \$1 later in the day at price q, and sell good c to retailers at price p_c in order to maximize their profits. Let $\hat{c}$ denote the amount of good c that a producer sells to retailers. By symmetry, all producers end up buying ω_x from consumers, so that $\hat{c} \leq \omega_x$. The producers' problem can thus be written:

$$\max \quad q p_c \hat{c} - p_x \omega_x,$$

subject to

$$\hat{c} \leq \omega_x. \tag{17}$$

The condition for an interior solution is that $q p_c = p_x$. Since one unit of good x can be transformed into one unit of good c, the price at which good c is sold must be equal to the cost of acquiring good x, which is the price of a unit of x times the interest rate on the funds borrowed from the consumers, $\frac{1}{q}$.

The producers cannot guarantee that the payments they make arrive early, unless they pay a cost ε, which in this section is assumed to be infinite. They cannot choose which payments arrive early or late. One way to think about this is that cash is sent by mail to each consumer. All the envelopes leave at the same time, but some get delayed along the way. Nobody has access to the delayed cash.

3.3 The retailers' problem

Retailers sell claim for cash tomorrow at price q' in order to obtain the cash they need to purchase good c from producers. They then decide how much of these goods to sell immediately to consumers at price p and how much to save until next period to sell at price p' in order to maximize their profits. Let $\hat{M}$ denote the amount of cash retailers get from consumers, and $\hat{s}$ denote the amount of good c saved until next period.

Retailers must sell enough goods to satisfy the claims they sell, which I can write as

$$D' \leq p\hat{c} + (p' - p)\,\hat{s}. \tag{18}$$

This expression makes it clear that unless $p = p'$, retailers will choose either $\hat{s} = \infty$ or $\hat{s} = -\infty$, neither of which can be true in equilibrium. The retailers' problem can thus be written as

$$\max \quad q'D' - p_c \hat{c},$$

subject to

$$D' \leq p\hat{c}. \tag{19}$$

The condition for an interior solution is that $q'p = p_c$. This means that the price at which retailers sell a unit of good c, which is p, must be equal to the cost of obtaining that unit which is p_c times the cost of obtaining the cash, $\frac{1}{q'}$.

3.4 The central bank

The central bank (CB) can print money at no cost. Hence, it can modify the money supply by agreeing to lend at a given interest rate while the intraday market is open. Only intraday loans are considered, so funds that are lent in the morning must be repaid in the afternoon.

3.5 Equilibrium

I only consider symmetric stationary equilibria, by which I mean equilibria where consumers start each day with the same amount of cash so that agents face the same problem at the beginning of every day.

I define an equilibrium without CB intervention (the CB sets the interest rate at which it lends to infinity). Below I describe how the definition changes if the CB lends.

Definition 3.1. *Given ω_x, M, θ, a stationary equilibrium is a set of prices $\{q, q', p, p', p_x, p_x'', p_c, r\}$ and an allocation $\left\{D, D', c, c', \bar{c}, \bar{c}', \hat{c}, \hat{s}, \hat{M}, M_s, M''\right\}$ such that:*
1) Given prices, the allocation solves the problem of the consumers, the retailers, and the producers.

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2) Markets clear:

$$\theta c + (1 - \theta)\,\bar{c} = \hat{c} - \hat{s}, \tag{20}$$

$$\theta c' + (1 - \theta)\,\bar{c}' = \hat{s}, \tag{21}$$

$$M = \hat{M} + M_s, \tag{22}$$

$$\theta\,(D + M_s - pc) = (1 - \theta)\,(p\bar{c} - M_s), \tag{23}$$

$$\theta p' c' = (1 - \theta)\,(D - p'\bar{c}'). \tag{24}$$

Equations (20) and (21) correspond to the goods market in even and odd periods, respectively. Equation (22) says that the cash set aside plus the cash lent to retailers has to be equal to the total cash available. Finally, equations (23) and (24) correspond to the intraday market for funds in even and odd periods, respectively.

I call an equilibrium with a private market for intraday funds a *market* equilibrium. Instead of letting a private market for intraday funds operate, I can assume that a CB lends funds at an interest rate r^{CB}. This interest rate is a policy tool, not a variable determined by the market. The definition of an equilibrium in this case would be the same as above, with r^{CB} replacing r and without equations (23) and (24). Note that if $r^{CB} > r$ then nobody borrows from the CB and the situation is just as if the CB did not exist. More interesting are the cases where $r^{CB} \leq r$. I call such equilibria *equilibria with cental bank.*

In equilibrium, $p = p'$ and must clear the retail market for good c, which implies $p = \frac{M}{\omega_x}$. From the consumers' problem, $D' = M''$ and from the producers' problem, $D = \hat{M}$ so that, $p_c = \frac{\hat{M}}{\omega_x}$. This, in turn, implies that $\frac{p_c}{p} = \frac{\hat{M}}{M} = \beta^2$ and $M_s = M - \hat{M} = M\,(1 - \beta^2)$.

The interest rate r clears the intraday market for liquidity. If $r > 0$, it is determined as follows. First, one can rewrite equations (23) and (24) to get

$$\omega_x\left[\theta + (1 - \theta)\left(1 - \beta^2\right)\right] = \theta c + (1 - \theta)\,\bar{c}, \tag{25}$$

$$\omega_x \beta^2\,(1 - \theta) = \theta c' + (1 - \theta)\,\bar{c}'. \tag{26}$$

In order to get a closed form expression for the intraday interest rate, assume that u is CRRA:

$$u(c) = \frac{c^{1-\sigma}}{1 - \sigma}, \quad \sigma > 0. \tag{27}$$

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Taking that into account, one of the first order conditions from the consumers' problem becomes

$$\frac{c'}{c} = \frac{\bar{c}'}{\bar{c}} = [\beta(1+r)]^{\frac{1}{\sigma}}.$$

(28)

It follows that

$$[\theta c + (1-\theta)\,\bar{c}]\,[\beta(1+r)]^{\frac{1}{\sigma}} = \theta c' + (1-\theta)\,\bar{c}'.$$

(29)

Using (25) and (26), I can write

$$\frac{(1-\theta)\,\beta^2}{\theta + (1-\theta)\,(1-\beta^2)} = [\beta(1+r)]^{\frac{1}{\sigma}}.$$

(30)

From that I can conclude that $r > 0$ if

$$(1-\theta)\left[\beta^2\left(\beta^{-\frac{1}{\sigma}} + 1\right) - 1\right] > \theta.$$

(31)

If this inequality doesn't hold, there is excess liquidity on the market and $r = 0$. If $\sigma < 0.5$, the term in brackets in the above equation is negative and the inequality will not hold. When σ is small, consumption is more substitutable between the morning and the afternoon, which reduces the demand for intraday funds. The term in brackets is also negative if $\sigma > 0.5$ and β is small enough. This is somewhat surprising since one might have thought that if consumers were very impatient, they would want funds early and the price of these funds would have to be positive for the market to clear. Instead, consumers would set aside a lot of cash in this case. Recall that, $M_s = M\,(1-\beta^2)$, which increases as β becomes small. On the other hand, if β is sufficiently close to 1, the term in brackets will be positive. In this case the inequality will hold if θ is sufficiently small; the intuition is that since θ is a measure of how many consumers receive their payment early, the smaller it is, the less cash there is available in the market, and thus the higher the interest rate must be for the market to clear. Figure 1 shows the pairs (θ, β) for which the intraday interest rate is strictly positive.

FIGURE 1

If $r > 0$ (and thus $q < 1$), cash seems to be dominated by claims on producers, and one might wonder why consumers would choose to hold $M_s > 0$ (obviously, if $1 + r = q = 0$, they

are indifferent between cash and these claims). The reason is that claims on producers are risk, while cash is not. The more cash consumers hold, the less they will have to borrow if their payment arrives late. Once this is taken into account, cash is not dominated by claims on producers.

3.6   The efficient allocation

The efficient allocation is the allocation obtained by a planner who tries to maximize the consumers' utility, given an endowment ω_x of good x, the same technology as the producers in order to transform good x into good c, and the same technology as the retailers to store good c from one period to the next. Since I am interested in symmetric stationary allocations, I can solve the problem for one day, and define the efficient allocation to be the allocation that gives consumers that daily allocation every day. The daily problem is

$$\max \quad u(c) + \beta u(c'),$$

subject to

$$c + s \quad \leq \quad \omega_x, \tag{32}$$

$$c' \quad \leq \quad s. \tag{33}$$

Thus, it is easily seen that the efficient allocation is completely determined by

$$c + c' \quad = \quad \omega_x, \tag{34}$$

$$\frac{u'(c)}{u'(c')} \quad = \quad \beta. \tag{35}$$

Proposition 3.2. *The market allocation will be equal to the efficient allocation if and only if $r = 0$.*

Proof. If $r = 0$, the first order conditions from the consumers' problem yield equation (35). Also, in that case all consumers have the same interim wealth so that $c = \bar{c}$ and $c' = \bar{c}'$, which means that equations (20) and (21) can be combined to yield (34). If $r > 0$, consumers' don't have the same interim wealth, which implies that they cannot all consume the same allocation. $\qquad\qquad\square$

This result does not depend on the utility function being CRRA. The role of that assumption is to characterize the equilibrium intraday interest rate. As noted above, it will typically not be the case that $r = 0$ if θ is sufficiently small.

Corollary 3.3. *A central bank can implement the efficient allocation by setting $r^{CB} = 0$.*

If it is the case that the market allocation is not efficient, there is a role for the CB to lend funds at a zero interest rate on the intraday market. As mentioned in section 2, Switzerland had a private market for intraday funds between 1987 and 1999. The interest rate on this market was typically positive. So it should not be a surprise, in the face of proposition 3.2, that the Swiss National Bank decided in 1999 to resume lending intraday funds at a zero interest rate.

As noted already, the result of proposition 3.2 is similar to what has been obtained in the literature. Lacker (1997), however, obtains very different results. In his model, the intraday rate should be at least equal to the overnight rate. This is explained by an important difference in the modelling assumptions. He assumes there is no uncertainty about the arrival of intraday funds; the uncertainty is about overnight funds. This means that intraday and overnight funds are close substitutes. In contrast, there is no uncertainty about overnight funds in my model, while the uncertainty is about the timing of intraday funds. This uncertainty implies that overnight funds are not good substitutes for intraday liquidity. Knowing that I will receive some payment later is not helpful if I need these funds now.

Introducing overnight uncertainty as in Lacker (1997) would not change the results of the model. First, in contrast to his paper, the overnight rate can never be smaller than the intraday rate in the present model. If that were the case, consumers would borrow at the cheaper overnight rate and nobody would borrow intraday. Second, lending at zero interest rate overnight has consequences that are dramatically different from lending at zero interest rate intraday. This is because an overnight loan can be rolled over indefinitely, while an intraday loan cannot. Thus, a CB might decide not to lend at zero interest overnight even if by doing so it might be able to eliminate the cost of some uncertainty.

Indeed, the fact that CB lending at zero interest rate intraday is optimal depends on the

assumption that there is no alternative use for these funds. While this assumption can be defended for intraday lending, it cannot be for overnight lending. Because an overnight loan can be rolled over, borrowed funds could be used to invest in some productive activity. Hence, eliminating the cost of the overnight uncertainty might conflict with other policy goals.

It is worth noting that in this model the Friedman rule is not a necessary condition for the equilibrium allocation to be optimal. This feature is needed to show that the optimality of intraday lending at a zero interest rate is not a consequence of the Friedman rule. In contrast, in Freeman (1996) and Green (1997) all interest rates must be zero for the equilibrium allocation to be efficient. It also allows me to show that the optimality of intraday lending at a zero interest rate is not due to the introduction of some other friction. In Zhou (2000) and Kahn and Roberds (2001) inflation introduces such a friction.

Since the Friedman rule has been shown to be necessary for optimality in a wide class of models (see Chari, Christiano, and Kehoe, 1996 and Correia and Teles, 1996) it is interesting to understand why it is not in this paper. In most models, money is a dominated asset if the Friedman rule does not hold. Thus, if money is to be valued, some constraint (for example, a cash-in-advance constraint) must force agents to hold money. This creates an inefficiency that can only be eliminated if the Friedman rule holds, and money is not a dominated asset.

In this paper, in contrast, money is not a dominated asset even when the Friedman rule does not hold. Consumers have the choice between holding money or claims on producers for their intraday needs. If $r = 0$, claims on producers have a price of 1 so consumers are indifferent between these claims and money. If $r > 1$, claims on producers sell for less than 1 (they have a positive rate of return), and consumers are exactly compensated for the risk they take by holding them. Consumers hold claims on retailers for their overnight needs. From the first order conditions of the consumers' problem, claims on retailers must have a real return of $\frac{1}{\beta^2}$. Hence consumers' decisions are not distorted by inflation.

Retailers are able to pay a positive real return on their claims because they sell goods to consumers at a higher price than they buy these goods from producers. This wedge between the "wholesale" price of good c and its "retail" price is not distortionary because it only affects the level of the prices consumers care about, but not their relative values. Hence retailers are

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able to "absorb" the potentially distorting inflation tax and in its place introduce a wedge that does not distort consumers decisions.

Green (1997) points out that special private agents can achieve the same allocation as the CB. Call such an agent a clearinghouse. Assume that a clearinghouse is allowed to issue pieces of papers called private notes that must later be redeemed for cash. Retailers will accept these pieces of papers if it is known they will be redeemed. By issuing enough such notes, the clearinghouse can supply enough liquidity to achieve the efficient allocation. Moreover, if issuing private notes is costless, competitive clearinghouses will supply liquidity at a market interest rate of zero. As in Green (1997), what is important in order to achieve efficiency is the ability to increase the total supply of liquidity. Whether this is done by a CB or a private clearinghouse doesn't matter.

Implicit in this model is the fact that the CB is not paying interest on reserve. It is sometimes argued that if the CB paid interest on reserves, there would be no need for it to supply intraday liquidity. One way to think about this is to assume the CB pays an interest rate equal to $\frac{1}{\beta}$ on cash held by consumers. In that case, consumers are indifferent between holding claims on retailers and cash because these assets have the same return. In particular, they can increase M_s to the point where they never have to borrow intraday funds. However, there will still be some consumers who receive their payments earlier than others. This risk remains, and the efficient allocation cannot be achieved only by a policy of paying interest rate on reserves.

4 Endogenous timing

In this section I show the result I obtained above is robust to letting the timing of payment be endogenous. I do that by letting $\varepsilon \in (0, \infty)$. Each day, if a producers pays ε, the payment made by this producer arrives early with probability 1. As in section 3, if ε is not payed, the payment will arrive early with probability θ. Note that because of perfect competition, it is irrelevant whether producers or consumers have to pay the cost.

I show that a low but positive ε cannot drive the intraday interest rate all the way to zero.

Low values of ε will lower the intraday interest rate, but it will remain strictly positive. In other words, figure 2 is the same for all values $\varepsilon > 0$.

The intuition is as follows: If enough other consumers obtain their liquidity early, then the interest rate on the intraday market will be zero, and there is no need for me to pay the cost to get my payments early as well. In equilibrium, consumers will pay the cost to receive their payment early only if there is a cost of receiving a late payment. This cost is the interest paid on intraday funds and it must be strictly positive.

Proposition 4.1. *Having all producers incurring the cost ε with probability 1 is not an equilibrium.*

If some producers pay ε, then $r > 0$.

Proof. First, I show that not all producers will pay the cost. To establish a contradiction, I assume they all do and consider the incentives of a single producer who is thinking of deviating. Since all other consumers will receive their payment early, there will be an excess amount of liquidity on the intraday market and the interest rate will be zero. A producer thus has incentives not to pay the fixed cost and attract more consumers by sharing the amount saved. The consumers have nothing to lose from the fact that their payments will arrive late because they know that they can borrow for free. This remains true no matter how small ε is, as long as it is strictly positive.

Now I show that if some producers pay ε, then $r > 0$. In equilibrium, consumers must be indifferent between producers that pay ε and those that don't. Suppose that $r = 0$, then as discussed above, there is no cost to choosing a producer who didn't pay ε. So all consumers strictly prefer producers that didn't pay ε. This contradicts the assumption that some producers pay ε. $\qquad\square$

Another way of expressing this result is that if the intraday interest rate is strictly positive for $\varepsilon = \infty$, it is also for $\varepsilon \in (0, \infty)$. Note, however, the intraday interest rate will vary with ε. If technological progress or better communications allows banks to reduce the cost of coordinating payments, we should expect the interest rate on private intraday markets to decrease.

This result is similar to the delay-of-payment effect described, for example, by Angelini (1998, 2000), Kahn and Roberds (1999), or McAndrews and Rajan (2000). These authors show that, in the face of a positive interest rate, payments may be delayed because of a network externality. Here, however, I do not assume but instead show that the interest rate in a private market for intraday funds will be positive even if the timing of payments is endogenous.

It is sometimes argued that CBs who lend intraday funds should lend them at a positive interest rate because such an interest rate would prevail in a private market. According to proposition 4.1, we should indeed expect private market for intraday funds to have a positive interest rate, but by proposition 3.2 this is precisely what should be avoided. Even with endogenous timing of payments, the market equilibrium cannot achieve the efficient allocation. This strengthens the case for a CB to lend intraday funds at a zero interest rate.

5   Risk and moral hazard

One concern associated with a CB lending intraday liquidity is that this might lead to excessive risk-taking by banks and thus, potentially, high losses to the CB. In practice, several policies have been adopted to reduce this risk. These policies are: 1) imposing a cap on the amount of borrowing allowed, 2) taking collateral, and 3) charging a strictly positive intraday interest rate.

By introducing a risky technology which promises a high return with a small probability, I can extend the model to study the effectiveness of the policies listed above. If consumers can default on their debts when projects fail, they might have an incentive to invest in the risky technology and borrow as much as they can. If the investment is successful, this will allow them to consume more, while if the project fails, they default and bear little cost. I will show that, if the CB lends funds without restrictions, moral hazard can occur.

Assume producers can invest in two different technologies. The safe technology is the same as before and transforms one unit of good x into one unit of good c. The risky technology will transform one unit of good x into $2 - \delta$ units of good c with probability $\frac{1}{2}$ and returns

nothing with probability $\frac{1}{2}$, $\delta \in [0, 1)$. If δ is zero, the risky technology is a mean-preserving spread of the safe technology, while if δ is positive it has a lower expected return. In an environment where consumers are risk averse, using the risky technology cannot improve welfare for everyone. I define *moral hazard* to mean the risky technology is being used.

Consumers will sell their endowment to producers who choose the risky technology rather than the safe one if this yields higher expect utility. Although producers can be allowed to invest in a diversified set of risky projets, they will never choose to do this since it can yield no more than investing in the safe technology.

This section is concerned with how to prevent moral hazard if it occurs. As shown below, it can occur under the following assumptions:

Assumptions.

A1. There are no direct costs associated with the failure of a producer (it just goes out of business and is replaced by a new producer).

A2. There are no direct costs to consumers when they default on their debt.

A3. The CB cannot remember which consumers have defaulted in the past.

A4. The CB does not lend more than a consumer can possibly repay.

Assumptions A1 to A3 guarantee that the CB cannot punish defaulting agents enough to induce them not to invest in the risky technology. These assumptions can be weakened by assuming a greater punishment in case of default. This would make moral hazard less likely. For example, instead of A3, it could be assumed that consumers who default on their debts are excluded from the intraday market. Excluded consumers can choose to set M_s high enough so they are able to consume the amount they desire in the morning even if their payment arrives late. This puts a lower bound on the cost of being excluded from the market. In this case, moral hazard will occur only if consumers are sufficiently impatient.

Note that even if the CB has the ability to inflict heavy punishment, it might hesitate to do so if it is difficult to distinguish agents who invest in excessively risky technology from others. This would happen, for example, if with a small positive probability the safe technology has a return of zero.

Because of A3, the CB cannot obtain funds from consumers after they have been paid by

the retailers. Thus, if assumption A4 is relaxed, there can be no equilibrium as all consumers want to borrow an infinite amount. In odd periods, the CB can obtain funds from consumers equal to the amount they receive from the producers. This is equivalent to assuming that the CB takes as collateral the claims consumers have on producers, and never lends more than what it can hope to get back if the project is a success. If the project in which the producer has invested fails, the claims are worthless and the consumer will default on the loan. If the project is a success, the collateral is worth at least as much as the debt owed, and thus, the debt is repayed.

For simplicity, I will again assume the timing of payments is exogenous. Thus, if a producer invests in the risky technology, and the project succeeds, the payment is made early with probability θ and late otherwise. Until the beginning of the odd period, it is not possible to distinguish between a late payment and a failed project that will lead to no payment at all. This is the reason moral hazard can occur. All the results in this section remain valid if the timing of payment is endogenous as in section 4.

I have already shown in section 3 that the efficient allocation cannot be achieved if the intraday interest rate is strictly positive. In the rest of the section I prove the following proposition:

Proposition 5.1. *Under assumptions A1 to A4,*

1) moral hazard occurs if:

    a) The CB lends freely at zero intraday interest rate.

    b) θ is close to 1 and the tightest cap consistent with the efficient allocation is imposed.

Hence, unrestricted lending and caps cannot achieve the efficient allocation.

2) Moral hazard does not occur if the CB makes collateralized loans at zero intraday interest rate. This achieves the efficient allocation.

5.1 Free lending at zero intraday interest rate

If the CB lends freely, all consumers choose to borrow as much as they can. Investing in the risky technology allows them to receive more, if the project succeeds, than they would with

the safe technology. It also allows them to borrow more intraday funds and, if the project doesn't succeed, they can default on the debt. It follows that all producers will invest in risky technologies. All consumers end up consuming exactly the same amount, because they have access to the same amount of funds regardless of when, or if, their payment is received. The cost for the economy is $\frac{1}{2}\delta$, which is the expected loss from using the risky, rather than the safe, technology. Notice that every consumer in the economy is made worse off when the risky technology is used, yet every single consumer has an incentive to invest in that technology.

Since moral hazard will occur if the CB lends freely, I can now turn to policies that are designed to reduce the CB's exposure to risk. First I consider caps:

5.2 Caps

Assume the CB limits the amount consumers are allowed to borrow. I will show, for certain parameters, this policy cannot achieve the efficient allocation. I consider the tightest cap that could implement this allocation, and show that moral hazard will occur: If all producers invest in the short term technology, a deviating producer can make consumers better off.

Let a superscript $*$ denote equilibrium variables when only the safe technology is available, and the CB lends at $r^{CB} = 0$. This equilibrium allocation is efficient. Assume that $\theta = 0$, so all payments arrive late. Then, the smallest cap that can implement the efficient allocation is $Cap = pc^* - M_s^*$. If the cap is any tighter, consumers are not able to borrow enough in the even period to buy c^*. Consumption has to satisfy the interim budget constraint:

$$p\left(c^* + c^{*\prime}\right) = D^* + M_s^*. \tag{36}$$

Now I construct a feasible allocation that yields higher utility to a deviating consumer. Let c and c' denote consumption in an even and odd period, respectively, if the payment is received early. Let $\bar{c}$ denote consumption in an even period if the payment is not received early. Finally, $\bar{c}'$ and $\hat{c}'$ denote consumption in an odd period, if the payment is received late

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or if the project fails, respectively. This allocation must satisfy the following constraints:

$$D + M_s \leq D^* + M_s^*, \tag{37}$$

$$p(c + c') \leq (2 - \delta) D + M_s, \tag{38}$$

$$p(\bar{c} + \vec{c}') \leq (2 - \delta) D + M_s, \tag{39}$$

$$p(\bar{c} + \hat{c}') \leq Cap + M_s. \tag{40}$$

The first constraint says that the alternative allocation must not cost more than the efficient allocation. The other three constraints are the interim wealth constraints of the consumer if the payment arrives early, late, or not at all, respectively.

Let $M_s = D^* + M_s^* - Cap$. Since $\delta < 1$, $Cap + M_s = D^* + M_s^* < (2 - \delta) D + M_s$ so deviating yields at least as much consumption in every state. It follows that it is not an equilibrium for all producers to invest in the safe technology. If liquidity needs are high enough, caps cannot prevent moral hazard.

5.3 Collateralized lending at zero intraday interest rate

Finally, I consider the policy of making collateralized loans. Assume that the CB asks for the claims consumers have on retailers as collateral. To see that this policy will yield the efficient allocation, two things must be noted. First, $D' > D$, so that there are enough claims on retailers to use as collateral. Second, these claims are risk-free: The retailers always end up with all the cash available in the economy, regardless of which technologies the producers have invested in. Consequently, there can be no strategic default from the consumers under this policy. The CB can always recover the funds it lends, and the consumer never finds it beneficial to default. Thus there will be no moral hazard. This, in turn, means that this policy implements the efficient allocation in this model.

For the same reasons a CB can prevent moral hazard by asking for the right kind of collateral, moral hazard will not occur on a market for intraday funds. Consumers will ask for collateral from each other. This suggests another interpretation of the results obtained in section 5.1 and 5.3: Moral hazard is likely to occur if the CB finds it difficult to evaluate

the quality of the collateral against which it lends. One can think of the CB in section 5.1 as lending against claims on producers. However, it accepts these claims at face value, rather than at the value at which they would be traded on a market. In other words, the CB does not realize how risky these claims are. In section 5.3, the CB values the collateral against which it lends correctly.[9]

The Federal Reserve lends intraday funds at a small interest rate for transactions made over Fedwire. A cap is also imposed on the amount of intraday credit allowed. Proposition 3.2 suggests that charging a positive interest rate creates inefficiencies, while proposition 5.1 indicates that the cap may not be effective in preventing moral hazard, which could lead to excessive exposure to credit risk.

Under the TARGET system, intraday loans are interest-free and collateralized. This design is the best according to the results presented in this paper. The Swiss system, SIC, used to function without the SNB lending intraday funds. Proposition 4.1 indicates that this should lead to inefficiencies. Interestingly, since October 1999, the SNB allows intraday, repos-backed interest-free overdrafts making SIC more similar to TARGET.

6   Conclusion

This paper makes several contributions. It shows that the large differences observed between intraday and overnight interest rates can be optimal in an environment where precise timing of payments is costly. It also shows that allowing for endogenous timing of payments cannot lower intraday interest rate all the way to zero. This provides even stronger support for CBs to lend intraday funds since they can achieve efficiency.

Maybe most importantly, this paper studies policies that CBs have adopted to manage their exposure to risk. It shows that collateralized lending prevents moral hazard and achieves the efficient allocation if the interest rate is zero, while for certain parameters, caps will not prevent moral hazard, which leads to an inefficient outcome.

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[9]It is not unusual to assume that private agents are better at evaluating assets than is the CB. In particular, this is an argument often used to justify providing liquidity through open market operations rather than a discount window.

This study is conducted using a general equilibrium model in which intraday liquidity shortage can occur because it is costly to make the timing of payments precise. In the model, the overnight interest rate is the inverse of the discount factor. Having an intraday interest rate of zero is the only way to insure consumers against the risk of receiving their payments late.

A private market for intraday funds might not clear if the interest rate is zero; this creates a role for the CB. In equilibrium, consumers must be indifferent between receiving payments early or late, the cost of receiving payments late–the intraday interest rate–must be equal to the cost of receiving payments early, which is strictly positive. The CB allows agents to economize on this cost by lending at zero interest rate, thus improving welfare.

By introducing a risky technology in the model, I can study policies intended to protect the CB from excessive credit risk. The incentive for investing in a risky technology arises because, if the project succeeds, consumers can obtain more goods than they would have with the safe technology, while if it does not they can default on their debts. Investing in the risky technology is individually rational, but makes everyone worse off.

If loans are collateralized with a safe asset, the incentive to default disappears and moral hazard can be prevented. On the other hand, caps will not achieve the efficient allocation if the liquidity needs are too high. This is because the tightness of the cap is limited by the fact that consumers must be able to borrow enough to achieve the efficient allocation if they invest in the safe technology. Hence, when liquidity needs are high either moral hazard occurs or the cap is too tight.

Appendix

I have assumed so far that the stock of money is constant and that the return of the storage technology is 1. Because of these two assumptions, prices are constant and nominal and real interest rates are the same. In this appendix, I allow for the return of the storage technology to take any value $\gamma \leq \frac{1}{\beta}$, and I allow the stock of money to grow at constant rate $\pi \geq \beta$. I show that $r = 0$ is still a necessary and sufficient condition for the equilibrium to be efficient. I also show that this cannot be achieved by a constant rate of growth of the money stock. In order to achieve efficiency, a CB must have the ability to temporarily change the stock of money.

If the return of the storage technology is $\gamma \leq \frac{1}{\beta}$, the condition that retailers must sell enough goods to satisfy the claims they sell is now

$$D' \leq p\left(\hat{c} - \hat{s}\right) + p'\gamma\hat{s} = p\hat{c} + \left(p'\gamma - p\right)\hat{s}. \tag{A.1}$$

Thus in equilibrium, $p = \gamma p'$. From the consumer's problem, the ratio of marginal utility of consumption in the morning and the afternoon is still

$$\frac{u'\left(c\right)}{u'\left(c'\right)} = \frac{u'\left(\bar{c}\right)}{u'\left(\bar{c}'\right)} = \beta(1+r)\frac{p}{p'}. \tag{A.2}$$

The efficient allocation solves

$$\max \quad u(c) + \beta u(c'),$$

subject to

$$c + s \leq \omega_x, \tag{A.3}$$

$$c' \leq \gamma s. \tag{A.4}$$

and is completely characterized by

$$c + c' = \omega_x, \tag{A.5}$$

$$\frac{u'(c)}{u'(c')} = \gamma\beta. \tag{A.6}$$

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It is thus immediate that $r = 0$ is a necessary and sufficient condition for the equilibrium allocation to be efficient. In this case, the real rate of interest on intraday loans is $(1+r)\frac{p}{p'} = \gamma$, and since $\frac{p}{p'} = \gamma$, the nominal rate is $r = 0$.

Now I want to consider the case where the stock of money grows at a constant rate $\pi \geq \beta$. Note that the rate of growth of the money stock doesn't affect the ratio $\frac{p}{p'}$ which is pinned down by the return of the storage technology. Thus, $r = 0$ is still a necessary and sufficient condition for the equilibrium allocation to be efficient. The interesting question is whether there is a value of $\pi \geq \beta$ that will yield $r = 0$ without requiring the CB to lend on the intraday market. I show that this is not generally the case.

I assume that money is injected or removed from the economy every period (twice a day) before borrowing and lending takes place on the intraday market. This means that the interim wealth constraints for the consumers who receive their payments early can now be written as

$$pc \leq B + D + M_s + (\pi - 1)M, \tag{A.7}$$

$$p'c' + (1 + r)B \leq (\pi - 1)\pi M, \tag{A.8}$$

and be combined to get

$$(1 + r)pc + p'c' \leq (1 + r)\left[D + M_s(\pi - 1)M\right] + (\pi - 1)\pi M. \tag{A.9}$$

Similarly, for consumers who receive their payments late

$$p\bar{c} \leq \bar{B} + M_s + (\pi - 1)M, \tag{A.10}$$

$$p'\bar{c}' + (1 + r)\bar{B} \leq D + (\pi - 1)\pi M, \tag{A.11}$$

and they can be combined to yield

$$(1 + r)p\bar{c} + p'\bar{c}' \leq (1 + r)\left[M_s + (\pi - 1)M\right] + D + (\pi - 1)\pi M. \tag{A.12}$$

Equations (23) and (24) now become, respectively,

$$\omega_x \left[\frac{1}{\pi} - (1 - \theta)\beta^2\right] = \theta c + (1 - \theta)\bar{c}, \tag{A.13}$$

$$\omega_x \left[(1 - \theta)\beta^2 + 1 - \frac{1}{\pi}\right] = \theta c' + (1 - \theta)\bar{c}'. \tag{A.14}$$

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Assuming that the utility function is CRRA, the intraday interest rate is determined by

$$\frac{(1-\theta)\,\beta^2 + 1 - \frac{1}{\pi}}{\frac{1}{\pi} - (1-\theta)\,\beta^2} = [\beta(1+r)]^{\frac{1}{\sigma}}, \tag{A.15}$$

so that $r > 0$ if and only if

$$\left[(1-\theta)\beta^2 + 1 - \frac{1}{\pi}\right]\beta^{-\frac{1}{\sigma}} > \frac{1}{\pi} - (1-\theta)\beta^2. \tag{A.16}$$

For any value of $\pi > \beta$ this inequality holds if $\beta = 1$ and, by continuity, it holds for high enough values of β. For these values, intraday lending at a zero interest rate by a CB is required to achieve the efficient allocation.

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Consumers sell endowment

Late payments arrive
Intra-day market is open

Early payments arrive
Intra-day market is open

Retailers repay consumers

Producers make good c
Retailers buy good c
and choose storage

Consumers buy good c and consume

Consumers buy good c and consume
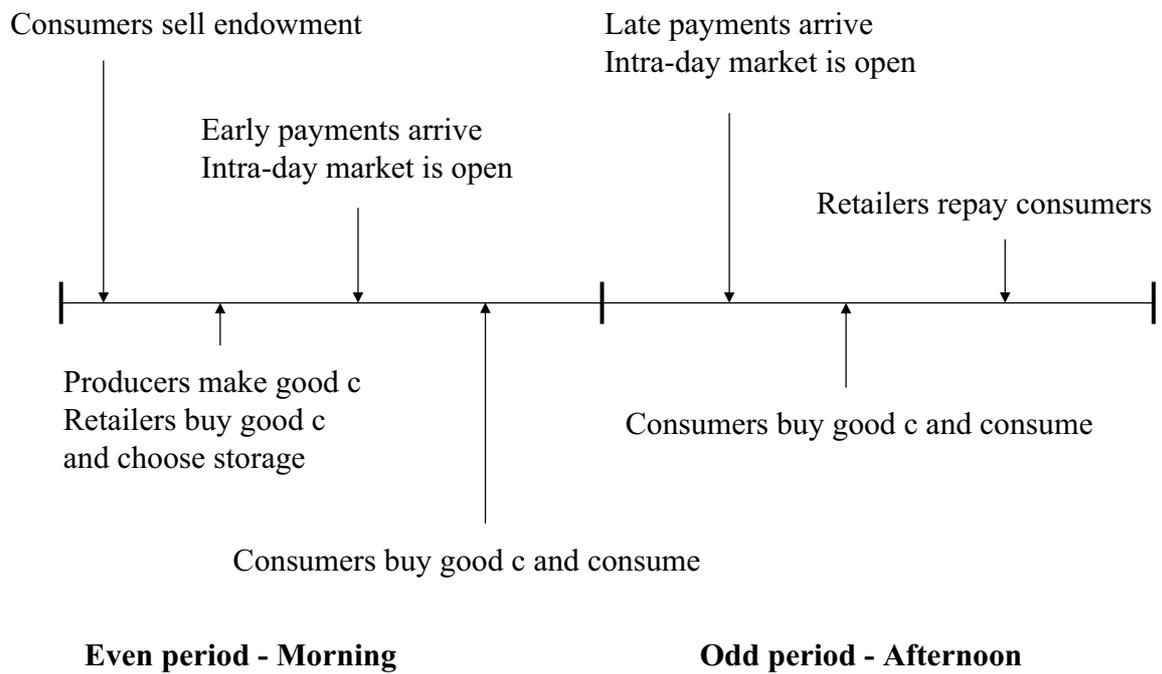
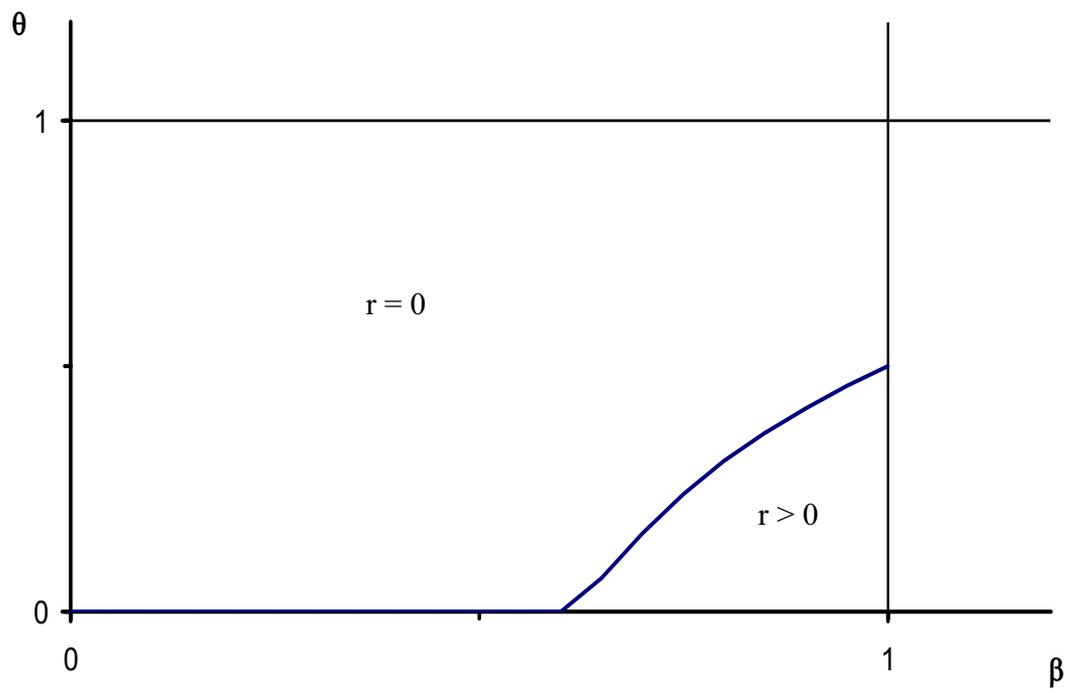**Even period - Morning**

Odd period - Afternoon

Fig. 1. Model timing.

Fig. 2. Intra-day interest rate ($\sigma = 1$)