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Takushi Kurozumi and Willem Van Zandweghe

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Takushi Kurozumi* Willem Van Zandweghe†

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Abstract

Previous literature shows that in the presence of staggered price setting, high trend inflation induces not only a large loss in steady-state output relative to its natural rate but also indeterminacy of equilibrium under the Taylor rule. This paper examines the implications of a “smoothed-off” kink in demand curves for the natural rate hypothesis and macroeconomic stability using a canonical model with staggered price setting. An empirically plausible calibration of the model demonstrates that the kink in demand curves mitigates the influence of high trend inflation on aggregate output through the average markup and (when relevant) the relative price distortion, thereby ensuring that the violation of the natural rate hypothesis is minor and preventing indeterminacy caused by high trend inflation.

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Keywords: Smoothed-off kink in demand curve, Trend inflation, Staggered price setting,

Natural rate hypothesis, Taylor principle

*Bank of Japan, 2-1-1 Nihonbashi Hongokuchō, Chuo-ku, Tokyo 103-8660, Japan. *E-mail address:* takushi.kurozumi@boj.or.jp

†Corresponding author. Research Department, Federal Reserve Bank of Kansas City, 1 Memorial Drive, Kansas City, MO 64198, USA. *E-mail address:* willem.vanzandweghe@kc.frb.org

1 Introduction

“[T]here is always a temporary trade-off between inflation and unemployment; there is no permanent trade-off.” Thus spoke Milton Friedman (1968, p. 11). Since then the natural rate hypothesis (NRH, henceforth)—in the long run output is at its natural rate regardless of trend inflation—has been widely accepted in macroeconomics. The Calvo (1983) model of staggered price setting, however, fails to satisfy this hypothesis, as McCallum (1998) forcefully criticized. Nevertheless, it has been a leading model of price adjustment for monetary policy analysis in the past decade and a half. One likely reason for this is that the introduction of price indexation makes the Calvo model meet the NRH, as shown by Ascari (2004). In fact, a considerable amount of research incorporates price indexation to trend inflation as in Yun (1996) or to past inflation as in Christiano et al. (2005). Yet the presence of price indexation raises another issue. The resulting model is not consistent with micro evidence that each period a fraction of prices is kept unchanged at a positive rate of trend inflation.¹ Because firms that do not reoptimize prices use price indexation, all prices change in every period.

Another likely reason why the Calvo model has thrived is that its violation of the NRH may be too small to induce grossly misleading implications for monetary policy. However, Ascari (2004), Levin and Yun (2007), and Yun (2005) examine the steady-state relationship between output and inflation in the model and show that the loss in steady-state output relative to its natural rate becomes larger as trend inflation rises. Higher trend inflation causes price-adjusting firms to choose a higher markup and erodes non-adjusting firms’ markups more severely. This then increases the average markup because the effect of adjusting firms’ higher markups dominates that of non-adjusting firms’ more severely eroding markups. Higher trend

¹Moreover, Cogley and Sbordone (2008) demonstrate that price indexation to past inflation is not empirically important once drift in trend inflation is taken into account.

inflation thus reduces aggregate output. In addition to this average markup distortion, there is another distortion associated with the dispersion of relative prices (i.e., the relative price distortion) when labor is homogeneous. Higher trend inflation widens the dispersion of relative prices of goods, because of adjusting firms' increasing relative prices and non-adjusting firms' eroding relative prices. It thus increases the dispersion of demand for goods, thereby reducing aggregate output.

The large violation of the NRH in the Calvo model has implications for monetary policy. Higher trend inflation reduces not only steady-state output but also the long-run inflation elasticity of output in the model. In their analysis of determinacy of equilibrium under the Taylor (1993) rule, Ascari and Ropele (2009), Kurozumi (2014), and Kurozumi and Van Zandweghe (2012) show that this elasticity plays a key role in the determinacy condition called the long-run version of the Taylor principle: in the long run the interest rate should be raised by more than the increase in inflation. Higher trend inflation reduces the elasticity substantially once the trend inflation rate exceeds a certain positive threshold. Because the long-run version of the Taylor principle is less likely to be satisfied with a lower value of the elasticity, it imposes a more severe upper bound on the output coefficient of the Taylor rule as trend inflation rises. Moreover, higher trend inflation gives rise to another condition for determinacy that imposes more severe lower bounds on the inflation and output coefficients of the Taylor rule. Therefore, indeterminacy of equilibrium under the Taylor rule is more likely with higher trend inflation.²

This paper examines implications of a “smoothed-off” kink in demand curves for the NRH

²In the Calvo model, Levin and Yun (2007) endogenize firms' probability of price changes along the lines of the literature, such as Ball et al. (1988), Romer (1990), Kiley (2000), and Devereux and Yetman (2002), and investigate its implications for the NRH. They show that the loss in steady-state output relative to its natural rate remains non-trivial under moderate trend inflation but that such a loss wanes and eventually disappears under much higher trend inflation because the probability of price changes approaches the one in the flexible-price economy. In this context, Kurozumi (2011) analyzes determinacy of equilibrium under the Taylor rule and shows that indeterminacy caused by higher trend inflation is less likely.

and macroeconomic stability in the Calvo model. This kink in demand curves has been analyzed by Kimball (1995), Dotsey and King (2005), and Levin et al. (2008), and generates strategic complementarity in price setting.³ Recent empirical literature emphasizes the importance of such complementarity for reconciling the Calvo model with micro evidence on the frequency of price changes.⁴ The strategic complementarity arising from the kinked demand curves gives the New Keynesian Phillips curve (NKPC, henceforth) a flat slope—i.e., a small elasticity of inflation with respect to real marginal cost—reported in the empirical literature, keeping the average frequency of price changes consistent with micro evidence.⁵

A calibration of the Calvo model that is consistent with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC shows that a smoothed-off kink in demand curves mitigates the influence of high trend inflation on aggregate output through the average markup (and the relative price distortion in the case of homogeneous labor), thereby ensuring that the violation of the NRH is minor and preventing equilibrium indeterminacy caused by high trend inflation. As noted above, in the absence of such a kink in demand curves, higher trend inflation increases the average markup (and the relative price distortion in the homogeneous-labor case), thereby reducing aggregate output. The kink causes the relative demand for a differentiated good to become *more* price-elastic for an *increase* in the relative price of the good. Higher trend inflation then decreases the average markup, because

³See also Levin et al. (2007) and Shirota (2007). In this strand of literature, a kink in demand curves arises from a formulation of households' preferences for differentiated consumption goods or final-good firms' production technology that combines differentiated intermediate goods. For a micro-foundation of concave or quasi-concave demand curves, see, e.g., Benabou (1988), Heidhues and Koszegi (2008), and Gourio and Rudanko (2014). Benabou develops a model of customer search, where a search cost gives rise to a reservation price above which a customer continues to search for a seller. Heidhues and Koszegi consider consumers' loss aversion, which increases the price responsiveness of demand at higher relative to lower market prices. Gourio and Rudanko construct a model of customer capital, in which firms have a long-term relationship with customers whose demand is unresponsive to a low price.

⁴For recent micro evidence on price changes, see, e.g., Bils and Klenow (2004), Kehoe and Midrigan (forthcoming), Klenow and Kryvtsov (2008), Klenow and Malin (2010), and Nakamura and Steinsson (2008).

⁵For empirical analysis of the NKPC, see Galí and Gertler (1999), Galí et al. (2001), Sbordone (2002), and Eichenbaum and Fisher (2007).

the kinked demand curves dampen the increase in price-adjusting firms' markups induced by higher trend inflation, so that the effect of non-adjusting firms' eroding markups dominates that of adjusting firms' increasing markups. Therefore, the kinked demand curves mitigate the distortion of aggregate output associated with the average markup. Moreover, they cause the relative demand for a differentiated good to become *less* price-elastic for a *decline* in the relative price of the good. Consequently, as higher trend inflation increases the dispersion of relative prices, the associated increase in the dispersion of relative demand is subdued and thus the kinked demand curves mitigate the relative price distortion. This dampens the decline in output associated with the relative price distortion when labor is homogeneous. Because of these effects, the violation of the NRH is minor in the presence of the kink in demand curves. In addition, the mitigating effects of the kinked demand curves reverse a decline in the long-run inflation elasticity of output caused by higher trend inflation and thus makes the long-run version of the Taylor principle much more likely to be met than in the absence of the kink. It also makes irrelevant the other determinacy condition that induces lower bounds on the inflation and output coefficients of the Taylor rule. Therefore, determinacy of equilibrium under the Taylor rule is much more likely in the presence of the kink in demand curves.⁶

The desirable properties of the smoothed-off kink in demand curves in terms of preventing both large violations of the NRH and equilibrium indeterminacy caused by high trend inflation are not shared by firm-specific labor, which is another source of strategic complementarity in price setting analyzed in existing literature.⁷ As is the case with homogeneous labor, high

⁶The implications of the smoothed-off kink in demand curves for the NRH and equilibrium determinacy obtained in the Calvo model would apply to the Taylor (1980) model of staggered price setting. See King and Wolman (1999) for a welfare analysis of trend inflation in the Taylor model. Hornstein and Wolman (2005) and Kiley (2007) show that in the Taylor model higher trend inflation is more likely to induce indeterminacy of equilibrium under the Taylor rule.

⁷The smoothed-off kink in demand curves and firm-specific labor have been regarded as isomorphic in their implications for log-linear dynamics in existing literature, but indeed differ at a non-zero rate of trend inflation in the absence of price indexation.

trend inflation induces not only a large loss in steady-state output relative to its natural rate but also indeterminacy of equilibrium under the Taylor rule in the Calvo model with firm-specific labor. In this model, introducing a smoothed-off kink in demand curves once again mitigates the influence of high trend inflation on aggregate output through the average markup. Therefore, the kinked demand curves ensure that the violation of the NRH is minor and prevent equilibrium indeterminacy caused by high trend inflation, regardless of whether labor is homogeneous or firm-specific.

To account for the U.S. economy's shift from the Great Inflation era to the Great Moderation era, the previous literature, including Clarida et al. (2000) and Lubik and Schorfheide (2004), has stressed the key role played by the Federal Reserve's switch from a passive to an active policy response to inflation. In addition, Coibion and Gorodnichenko (2011) argue for the importance of a decline in trend inflation to explain the shift, using the Calvo model with firm-specific labor but no kink in demand curves. This model, however, induces a large loss in steady-state output relative to its natural rate during the Great Inflation era. The Calvo model with a smoothed-off kink in demand curves yields a minor violation of the NRH and supports the view of the previous literature.

The remainder of the paper proceeds as follows. Section 2 presents the Calvo model with a smoothed-off kink in demand curves. In this model, Section 3 examines implications of the kink for the NRH, while Section 4 analyzes those for determinacy of equilibrium under the Taylor rule. In Section 5, these implications are compared with those obtained in the model with firm-specific labor. Section 6 concludes.

2 The Calvo model with a smoothed-off kink in demand curves

The model economy is populated by a representative household, a representative final-good firm, a continuum of intermediate-good firms, and a monetary authority. Key features of the model are that each period a fraction of intermediate-good firms keeps prices of their differentiated products unchanged, while the remaining fraction reoptimizes its prices in the face of the final-good firm's demand curves in which a smoothed-off kink is introduced. The behavior of each economic agent is described in turn.

2.1 Household

The representative household consumes C_t final goods, supplies N_t homogeneous labor, and purchases B_t one-period riskless bonds to maximize the utility function $E_0 \sum_{t=0}^{\infty} \beta^t [\log C_t - N_t^{1+\sigma_n}/(1+\sigma_n)]$ subject to the budget constraint $P_t C_t + B_t = P_t W_t N_t + i_{t-1} B_{t-1} + T_t$, where E_t denotes the expectation operator conditional on information available in period t , $\beta \in (0, 1)$ is the subjective discount factor, $\sigma_n \geq 0$ is the inverse of the elasticity of labor supply, P_t is the price of final goods, W_t is the real wage, i_t is the gross interest rate on the bonds, and T_t consists of lump-sum public transfers and firm profits.

Combining first-order conditions for utility maximization with respect to consumption, labor supply, and bond holdings yields

$$W_t = C_t N_t^{\sigma_n}, \quad (1)$$

$$1 = E_t \left(\frac{\beta C_t}{C_{t+1}} \frac{i_t}{\pi_{t+1}} \right), \quad (2)$$

where $\pi_t = P_t/P_{t-1}$ denotes gross inflation.

2.2 Final-good firm

As in Kimball (1995), the representative final-good firm produces Y_t homogeneous goods under perfect competition by choosing a combination of intermediate inputs $\{Y_t(f)\}$ so as to maximize profit $P_t Y_t - \int_0^1 P_t(f) Y_t(f) df$ subject to the production technology

$$\int_0^1 F \left(\frac{Y_t(f)}{Y_t} \right) df = 1, \quad (3)$$

where $P_t(f)$ is the price of intermediate good $f \in [0, 1]$. Following Dotsey and King (2005) and Levin et al. (2008), the production technology is assumed to be of the form

$$F \left(\frac{Y_t(f)}{Y_t} \right) = \frac{\gamma}{(1 + \epsilon)(\gamma - 1)} \left[(1 + \epsilon) \frac{Y_t(f)}{Y_t} - \epsilon \right]^{\frac{\gamma-1}{\gamma}} + 1 - \frac{\gamma}{(1 + \epsilon)(\gamma - 1)},$$

where $\gamma = \theta(1 + \epsilon)$. The parameter $\epsilon \leq 0$ governs the curvature of the demand curve for each intermediate good. In the special case of $\epsilon = 0$, the production technology (3) is reduced to the CES one $Y_t = [\int_0^1 (Y_t(f))^{\theta-1} df]^{\theta/(\theta-1)}$, where the parameter $\theta > 1$ represents the price elasticity of demand for each intermediate good.

The first-order conditions for profit maximization yield the final-good firm's relative demand curve for intermediate good f ,

$$\frac{Y_t(f)}{Y_t} = \frac{1}{1 + \epsilon} \left[\left(\frac{P_t(f)}{P_t d_{1t}} \right)^{-\gamma} + \epsilon \right], \quad (4)$$

where d_{1t} is the Lagrange multiplier on the production technology (3) in profit maximization, given by

$$d_{1t} = \left[\int_0^1 \left(\frac{P_t(f)}{P_t} \right)^{1-\gamma} df \right]^{\frac{1}{1-\gamma}}. \quad (5)$$

Fig. 1 illustrates the demand curve (4) with various values of the curvature parameter ϵ . As can be seen in this figure, the price elasticity of demand for each intermediate good f , given by $\eta_t = \gamma - \theta\epsilon(Y_t(f)/Y_t)^{-1}$, varies inversely with the relative demand for the good. Therefore,

in the presence of the kink in demand curves, the relative demand for an intermediate good becomes *more* price-elastic for an *increase* in the relative price of the good, while it become *less* price-elastic for a *decline* in the relative price of the good.

The final-good firm's zero-profit condition implies that its product's price P_t satisfies

$$1 = \frac{1}{1 + \epsilon} d_{1t} + \frac{\epsilon}{1 + \epsilon} d_{2t}, \quad (6)$$

where

$$d_{2t} = \int_0^1 \frac{P_t(f)}{P_t} df. \quad (7)$$

Note that in the special case of $\epsilon = 0$, where the production technology (3) becomes the CES one, Eqs. (4)–(6) can be reduced to $Y_t(f) = Y_t(P_t(f)/P_t)^{-\theta}$, $P_t = [\int_0^1 (P_t(f))^{1-\theta} df]^{1/(1-\theta)}$, and $d_{1t} = 1$, respectively.

The final-good market clearing condition is given by

$$Y_t = C_t. \quad (8)$$

2.3 Intermediate-good firms

Each intermediate-good firm f produces one kind of differentiated good $Y_t(f)$ under monopolistic competition. Firm f 's production function is linear in its labor input

$$Y_t(f) = N_t(f). \quad (9)$$

The labor market clearing condition is given by

$$N_t = \int_0^1 N_t(f) df. \quad (10)$$

Given the real wage W_t , the first-order condition for minimization of production cost shows that real marginal cost is identical among all intermediate-good firms, given by $mc_t = W_t$.

Combining this equation with Eqs. (1), (4), (8), (9), and (10) yields

$$mc_t = Y_t^{1+\sigma_n} \left(\frac{s_t + \epsilon}{1 + \epsilon} \right)^{\sigma_n}, \quad (11)$$

where

$$s_t = \int_0^1 \left(\frac{P_t(f)}{P_t d_{1t}} \right)^{-\gamma} df. \quad (12)$$

In the face of the final-good firm's demand curve (4) and the marginal cost (11), intermediate-good firms set prices of their products on a staggered basis as in Calvo (1983). Each period a fraction $\alpha \in (0, 1)$ of firms keeps previous-period prices unchanged, while the remaining fraction $1 - \alpha$ of firms sets the price $P_t(f)$ to maximize the profit function

$$E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \left(\frac{P_t(f)}{P_{t+j}} - mc_{t+j} \right) \frac{1}{1 + \epsilon} \left[\left(\frac{P_t(f)}{P_{t+j} d_{1t+j}} \right)^{-\gamma} + \epsilon \right] Y_{t+j},$$

where $q_{t,t+j} = \beta^j C_t / C_{t+j}$ is the stochastic discount factor between period t and period $t + j$.

For this profit function to be well-defined, the following assumption is imposed.

Assumption 1 *The three inequalities $\alpha\beta\pi^{\gamma-1} < 1$, $\alpha\beta\pi^\gamma < 1$, and $\alpha\beta\pi^{-1} < 1$ hold, where π denotes gross trend inflation.*

Using the final-good market clearing condition (8), the first-order condition for Calvo staggered price setting leads to

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \prod_{k=1}^j \pi_{t+k}^\gamma \left[\left(p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} - \frac{\gamma}{\gamma-1} mc_{t+j} \right) d_{1t+j}^{-\gamma} - \frac{\epsilon}{\gamma-1} \left(p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} \right)^{1+\gamma} \right] = 0, \quad (13)$$

where p_t^* is the relative price set by firms that reoptimize prices in period t .

Moreover, under the Calvo staggered price setting, the price dispersion equations (5), (7),

and (12) can be reduced to, respectively,

$$(d_{1t})^{1-\gamma} = (1-\alpha)(p_t^*)^{1-\gamma} + \alpha \left(\frac{d_{1t-1}}{\pi_t} \right)^{1-\gamma}, \quad (14)$$

$$d_{2t} = (1-\alpha)p_t^* + \alpha \left(\frac{d_{2t-1}}{\pi_t} \right), \quad (15)$$

$$(d_{1t})^{-\gamma} s_t = (1-\alpha)(p_t^*)^{-\gamma} + \alpha \left(\frac{d_{1t-1}}{\pi_t} \right)^{-\gamma} s_{t-1}. \quad (16)$$

2.4 Monetary authority

The monetary authority follows a policy rule as in Taylor (1993). This rule adjusts the interest rate i_t in response to deviations of inflation and output from their steady-state values,

$$\log i_t = \log i + \phi_\pi(\log \pi_t - \log \pi) + \phi_y(\log Y_t - \log Y), \quad (17)$$

where i and Y are steady-state values of the interest rate and output and $\phi_\pi, \phi_y \geq 0$ are the policy responses to inflation and output.⁸

2.5 Log-linearized equilibrium conditions

The log-linearized model is presented for the subsequent analysis of equilibrium determinacy. Under Assumption 1, log-linearizing equilibrium conditions (2), (6), (8), (11), (13)–(16), and (17) and rearranging the resulting equations leads to

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \left(\hat{i}_t - E_t \hat{\pi}_{t+1} \right), \quad (18)$$

$$\begin{aligned} \hat{\pi}_t = & \beta E_t \hat{\pi}_{t+1} + \frac{(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^\gamma)}{\alpha\pi^{\gamma-1}[1-\tilde{\epsilon}\gamma/(\gamma-1-\tilde{\epsilon})]} \hat{m}c_t - \frac{1}{\alpha\pi^{\gamma-1}} \left(\hat{d}_{1t} - \alpha\beta\pi^{\gamma-1} E_t \hat{d}_{1t+1} \right) + \hat{d}_{1t-1} - \alpha\beta\pi^{\gamma-1} \hat{d}_{1t} \\ & - \frac{\gamma(1-\alpha\pi^{\gamma-1})[\alpha\beta\pi^{\gamma-1}(\pi-1)(\gamma-1) + \tilde{\epsilon}(1-\alpha\beta\pi^\gamma)]}{\alpha\pi^{\gamma-1}[\gamma-1-\tilde{\epsilon}(\gamma+1)]} \hat{d}_{1t} + \xi_t + \psi_t, \end{aligned} \quad (19)$$

$$\hat{m}c_t = (1+\sigma_n)\hat{Y}_t + \frac{\sigma_n s}{s+\epsilon} \hat{s}_t, \quad (20)$$

⁸Subsection 5.3 analyzes implications for determinacy of equilibrium under more general specifications of the Taylor rule, which have been estimated for the Federal Reserve by Coibion and Gorodnichenko (2011).

$$\hat{s}_t = \frac{\alpha\gamma\pi^{\gamma-1}(\pi-1)}{1-\alpha\pi^{\gamma-1}} \left(\hat{\pi}_t + \hat{d}_{1t} - \hat{d}_{1t-1} \right) + \alpha\pi^\gamma \hat{s}_{t-1}, \quad (21)$$

$$\hat{d}_{1t} = -\frac{\tilde{\epsilon}\alpha\pi^{-1}(\pi^\gamma-1)(1-\alpha\beta\pi^{-1})}{(1-\alpha\pi^{-1})[1-\alpha\beta\pi^{\tilde{\theta}-1} + \tilde{\epsilon}(1-\alpha\beta\pi^{-1})]} \hat{\pi}_t + \frac{\alpha\pi^{-1}[1-\alpha\beta\pi^{\gamma-1} + \tilde{\epsilon}\pi^\gamma(1-\alpha\beta\pi^{-1})]}{1-\alpha\beta\pi^{\gamma-1} + \tilde{\epsilon}(1-\alpha\beta\pi^{-1})} \hat{d}_{1t-1}, \quad (22)$$

$$\xi_t = \alpha\beta\pi^\gamma E_t \xi_{t+1} + \frac{\beta(\pi-1)(1-\alpha\pi^{\gamma-1})}{1-\tilde{\epsilon}\gamma/(\gamma-1-\tilde{\epsilon})} \left[\gamma E_t \hat{\pi}_{t+1} + (1-\alpha\beta\pi^\gamma) \left(E_t \hat{m}c_{t+1} + \gamma E_t \hat{d}_{1,t+1} \right) \right], \quad (23)$$

$$\psi_t = \alpha\beta\pi^{-1} E_t \psi_{t+1} + \frac{\tilde{\epsilon}\beta(\pi^\gamma-1)(1-\alpha\pi^{\gamma-1})}{\pi^\gamma [\gamma-1-\tilde{\epsilon}(\gamma+1)]} E_t \hat{\pi}_{t+1}, \quad (24)$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t, \quad (25)$$

where hatted variables denote log-deviations from steady-state values, ξ_t and ψ_t are auxiliary variables, $\tilde{\epsilon} = \epsilon(1-\alpha\beta\pi^{\gamma-1})/(1-\alpha\beta\pi^{-1})[(1-\alpha\pi^{\gamma-1})/(1-\alpha)]^{-\gamma/(\gamma-1)}$, and $s = (1-\alpha)/(1-\alpha\pi^\gamma)[(1-\alpha)/(1-\alpha\pi^{\gamma-1})]^{-\gamma/(\gamma-1)}$.

The strategic complementarity arising from the kinked demand curves reduces the slope of the NKPC (19) by $1/[1-\tilde{\epsilon}\gamma/(\gamma-1-\tilde{\epsilon})]$. It thus allows to reconcile the model with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC.

At a zero rate of trend inflation (i.e., $\pi = 1$), Eqs. (21)–(24) imply that $\hat{s}_t = 0$, $\hat{d}_{1t} = 0$, $\xi_t = 0$, and $\psi_t = 0$, and hence Eqs. (19) and (20) can be reduced to

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha[1-\epsilon\theta/(\theta-1)]} \hat{m}c_t \quad (26)$$

and $\hat{m}c_t = (1+\sigma_n)\hat{Y}_t$. Eq. (26) shows that Eq. (19) is a general formulation of the NKPC.⁹

2.6 Calibration

For the ensuing analysis, an empirically plausible calibration of the model is used. The benchmark calibration for the quarterly model is presented in Table 1. Coibion and Gorodnichenko (2011) is followed to set the subjective discount factor at $\beta = 0.99$, the inverse of the elasticity of labor supply at $\sigma_n = 1$, the parameter governing the price elasticity of demand at $\theta = 10$,

⁹For the so-called generalized NKPC, see the literature review by Ascari and Sbordone (2014).

which implies a markup of 11 percent at a zero rate of trend inflation, and the probability of no price change at $\alpha = 0.55$, which implies that prices change on average every 6.7 months.

The parameter governing the curvature of demand curves is chosen to target a small slope of the NKPC (26) (i.e., the one (19) with $\pi = 1$) consistent with that reported in the empirical literature on the NKPC, such as Galí and Gertler (1999), Galí et al. (2001), Sbordone (2002), and Eichenbaum and Fisher (2007).¹⁰ In particular, the parameter is set at $\epsilon = -9$, which, along with the calibration of the other parameters, implies that the slope of the NKPC (26) takes the value of 0.034. This value is the same as implied by the model and calibration of Coibion and Gorodnichenko (2011). The thick line in Fig. 1 displays the demand curve under the benchmark calibration. Note that to meet Assumption 1 under the calibration presented above, the annualized trend inflation rate needs to be greater than -2.96 percent.

3 Natural rate hypothesis

This section examines implications of a smoothed-off kink in demand curves for the NRH in the Calvo model. Specifically, the (non-linear) steady-state relationship between output and inflation is analyzed to show that the kink ensures that the violation of the NRH is minor.

3.1 Implications of a smoothed-off kink in demand curves

In the model, steady-state output is influenced by trend inflation through its effects on the average markup and the relative price distortion. In a steady state, Eq. (11) implies that

$$Y = \left(\frac{1}{mc} \right)^{-\frac{1}{1+\sigma_n}} \left(\frac{s + \epsilon}{1 + \epsilon} \right)^{-\frac{\sigma_n}{1+\sigma_n}},$$

so that output declines if the average markup (i.e., the inverse of real marginal cost) or the relative price distortion (i.e., $(s + \epsilon)/(1 + \epsilon)$) increases. Then, a rise in trend inflation has

¹⁰For a discussion of this empirical literature, see footnote 34 in Woodford (2005).

offsetting effects on the average markup by increasing price-adjusting firms' markups and eroding non-adjusting firms' markups, as indicated by King and Wolman (1996). The relative price distortion increases with higher trend inflation in the presence of staggered price setting.¹¹ Combining the above equation with Eqs. (6), (13), (14), (15), and (16) at a steady state yields the relationship between steady-state output Y and trend inflation π

$$Y = \left[\frac{\frac{1}{1+\epsilon} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{1}{\gamma-1}} + \frac{\epsilon}{1+\epsilon} \frac{1-\alpha}{1-\alpha\pi^{-1}}}{\frac{\gamma-1}{\gamma} \frac{1-\alpha\beta\pi^\gamma}{1-\alpha\beta\pi^{\gamma-1}} - \frac{\epsilon}{\gamma} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{\frac{\gamma}{\gamma-1}} \frac{1-\alpha\beta\pi^\gamma}{1-\alpha\beta\pi^{-1}}} \right]^{-\frac{1}{1+\sigma_n}} \left[\frac{\frac{1-\alpha}{1-\alpha\pi^\gamma} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{\gamma}{\gamma-1}} + \epsilon}{1+\epsilon} \right]^{-\frac{\sigma_n}{1+\sigma_n}}.$$

In the absence of Calvo staggered price setting, the (steady-state) natural rate of output can be obtained as

$$Y^n = \left(\frac{\theta - 1}{\theta} \right)^{\frac{1}{1+\sigma_n}}. \quad (27)$$

The deviation of steady-state output from its natural rate (i.e., the steady-state output gap) is thus given by

$$\log Y - \log Y^n = -\frac{1}{1+\sigma_n} \log \frac{\frac{\theta-1}{\theta} \left[\frac{1}{1+\epsilon} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{1}{\gamma-1}} + \frac{\epsilon}{1+\epsilon} \frac{1-\alpha}{1-\alpha\pi^{-1}} \right]}{\frac{\gamma-1}{\gamma} \frac{1-\alpha\beta\pi^\gamma}{1-\alpha\beta\pi^{\gamma-1}} - \frac{\epsilon}{\gamma} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{\frac{\gamma}{\gamma-1}} \frac{1-\alpha\beta\pi^\gamma}{1-\alpha\beta\pi^{-1}}} - \frac{\sigma_n}{1+\sigma_n} \log \frac{\frac{1-\alpha}{1-\alpha\pi^\gamma} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{\gamma}{\gamma-1}} + \epsilon}{1+\epsilon}. \quad (28)$$

Note that at a zero rate of trend inflation (i.e., $\pi = 1$), steady-state output Y is equal to the natural rate of output Y^n and hence the steady-state output gap is zero.

The steady-state output gap is highly sensitive to trend inflation in the case of no kink in demand curves (i.e., $\epsilon = 0$). The effect of the annualized trend inflation rate on the steady-state output gap (28) is illustrated by the thin line in Fig. 2. This line is obtained by choosing the probability of no price change of $\alpha = 0.83$ so as to set the slope of the NKPC (26) at the same size as that implied by the benchmark calibration presented in Table 1 (i.e., 0.034) along with the values of β , σ_n , and θ in the calibration.¹² The thin line shows that the steady-state

¹¹King and Wolman (1999) provide a detailed discussion of the effects of trend inflation on the average markup and the relative price distortion in the Taylor model of staggered price setting (with no kink in demand curves).

¹²The value of $\alpha = 0.83$ implies that prices change on average every 18 months, which—as the empirical

output gap declines exponentially with higher trend inflation, as pointed out by Ascari (2004), Levin and Yun (2007), and Yun (2005). As reported in Panel C of Table 2, a rise in the annualized trend inflation rate from three percent to six percent decreases the steady-state output gap from -1.20 percent to -14.42 percent. Moreover, both components of the steady-state output gap—the average markup and the relative price distortion—make a substantial contribution to the size of the gap. Higher trend inflation causes firms to choose a higher markup when they adjust prices, and erodes non-adjusting firms’ markups more severely. The average markup then increases, because the effect of price-adjusting firms’ higher markups induced by higher trend inflation dominates that of non-adjusting firms’ more severely eroding markups.¹³ Higher trend inflation thus increases the average markup and decreases the steady-state output gap. Moreover, it widens the dispersion of relative prices of goods, because of adjusting firms’ increasing relative prices and non-adjusting firms’ eroding relative prices. Higher trend inflation thus expands the dispersion of demand for goods, thereby raising the relative price distortion and lowering the steady-state output gap. Therefore, at an empirically plausible value for the slope of the NKPC (26), the Calvo model with no kink in demand curves is characterized by a large violation of the NRH.

This large violation of the NRH is prevented by a smoothed-off kink in demand curves. The thick line in Fig. 2 represents the steady-state output gap (28) under the benchmark calibration presented in Table 1. It shows a small positive steady-state output gap even under high trend inflation. As reported in Panel A of Table 2, the output gap is 0.37 percent at an annualized trend inflation rate of three percent and 0.88 percent at the rate of six percent. According to this panel, there are two reasons why the kink in demand curves substantially reduces the

literature on the NKPC stresses in the absence of strategic complementarity in price setting—is much longer than micro evidence indicates.

¹³This also holds in the Taylor model of staggered price setting, as shown by King and Wolman (1999).

steady-state output gap. First, both the average markup and relative price distortions are less sensitive to trend inflation. The average markup contributes 0.38 percent to the output gap at an annualized trend inflation rate of three percent and 0.91 percent at the rate of six percent, and the relative price distortion is tiny even under high trend inflation. Second, trend inflation has offsetting effects on the two distortions. As trend inflation rises, the relative price distortion decreases the steady-state output gap, while the average markup *increases* it. For these reasons, the violation of the NRH is minor in the presence of the kink in demand curves.

An intuition for this result is as follows. As noted above, in the case of no kink in demand curves, higher trend inflation increases both the average markup and relative price distortion, thereby decreasing the steady-state output gap. The kink in demand curves causes the relative demand for a good to become more price-elastic for an increase in the relative price of the good. Higher trend inflation then decreases the average markup, because the kinked demand curves cause the effect on the average markup of non-adjusting firms' eroding markups to dominate that of adjusting firms' increasing markups, by dampening the increase in adjusting firms' markup. Therefore, the output distortion associated with the average markup is mitigated in the presence of the kink in demand curves. Moreover, the kink causes the relative demand for a good to become less price-elastic for a decline in the relative price of the good. Consequently, as higher trend inflation increases the dispersion of relative prices, the associated increase in the dispersion of relative demand is mitigated. Therefore, the kinked demand curves mitigate the output loss associated with the relative price distortion. Because of these effects, the kink in demand curves ensures that the violation of the NRH is minor.

3.2 Robustness exercise regarding the curvature of demand curves

The finding that the violation of the NRH is minor in the presence of a smoothed-off kink in demand curves is robust to a wide range of values for the parameter governing the curvature of demand curves ϵ . Evidence on the degree of the curvature is scant. Klenow and Willis (2006) estimate an industry equilibrium model with kinked demand curves and find that very large firm-level productivity shocks are required to fit micro data on price changes, casting doubt on the plausibility of high degrees of the curvature. Dossche et al. (2010) directly estimate the curvature of demand curves and obtain a result that favors a kink, albeit a smaller one than used in most macroeconomic studies. Specifically, their result points to a value of $\epsilon = -1.4$, which is depicted by the dashed line in Fig. 1.¹⁴ Panel B of Table 2 shows the steady-state output gap under the calibration with the small kink in demand curves. The output gap is 0.06 percent and 0.17 percent at an annualized trend inflation rate of three percent and six percent, respectively. Strikingly, the output gap remains even smaller than under the benchmark calibration, as trend inflation increases. The reason is that the average markup is even less sensitive to trend inflation, as the effect of non-adjusting firms' eroding markups is offset to a greater extent by that of adjusting firms' increasing markups. The relative price distortion is only slightly more sensitive to trend inflation than under the benchmark calibration.

¹⁴Dossche et al. (2010) argue that “a very sensible value to choose for the curvature would be around 4” (p. 740). They define curvature as the steady-state elasticity of the price elasticity of demand with respect to relative prices, which corresponds to $-\gamma$ in our model. With our calibration of the price elasticity of demand parameter of $\theta = 10$, their argument implies that $\epsilon = -1.4$.

4 Equilibrium determinacy

This section analyzes implications of a smoothed-off kink in demand curves for determinacy of equilibrium in the log-linearized model consisting of Eqs. (18)–(25). It shows that the kink prevents indeterminacy caused by high trend inflation. It also sheds light on the veiled relationship between the NRH and the long-run version of the Taylor principle.

4.1 Implications of a smoothed-off kink in demand curves

At an annualized trend inflation rate of zero, three, and six percent, Fig. 3 displays regions of the Taylor rule’s coefficients (ϕ_π, ϕ_y) that guarantee equilibrium determinacy under the benchmark calibration presented in Table 1 and under the calibration in the case of no kink in demand curves (i.e., $\epsilon = 0$, $\alpha = 0.83$). Note that the coefficients estimated by Taylor (1993) are $(\phi_\pi, \phi_y) = (1.5, 0.5/4) = (1.5, 0.125)$ —which is marked by “ \times ” in each panel of the figure—and thus it is reasonable to consider the range of $0 \leq \phi_\pi \leq 1.5 \times 3 = 4.5$ and $0 \leq \phi_y \leq 0.125 \times 3 = 0.375$.

In the presence of a smoothed-off kink in demand curves, the left column of Fig. 3 shows only one region of determinacy within the coefficient range considered, at each rate of trend inflation. This region is characterized by

$$\phi_\pi + \phi_y \epsilon_y > 1, \quad (29)$$

where ϵ_y represents the long-run inflation elasticity of output, given by

$$\epsilon_y = \frac{\alpha\pi^{\gamma-1}[1-\tilde{\epsilon}\gamma/(\gamma-1-\tilde{\epsilon})]}{(1+\sigma_n)(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^{\gamma-1})} \times \left[\begin{aligned} & 1 - \beta - \frac{\gamma(\pi-1)[\beta(1-\alpha\pi^{\gamma-1})(1-\alpha\pi^\gamma) + \sigma_n(1-\alpha\beta\pi^{\gamma-1})(1-\alpha\beta\pi^\gamma)s/(s+\epsilon)]}{(1-\alpha\pi^\gamma)(1-\alpha\beta\pi^\gamma)[1-\tilde{\epsilon}\gamma/(\gamma-1-\tilde{\epsilon})]} \\ & - \frac{\tilde{\epsilon}(\pi^\gamma-1)(1-\alpha\pi^{\tilde{\theta}-1})\{[(1-\alpha\beta\pi^{\gamma-1})[\beta(1-\alpha\pi^{-1})^2 + (\gamma-1-\tilde{\epsilon})(1-\alpha\beta\pi^{-1})^2] + \tilde{\epsilon}\beta(1-\alpha\pi^{\gamma-1})(1-\alpha\pi^{-1})(1-\alpha\beta\pi^{-1})\}]}{\pi^\gamma(1-\alpha\pi^{-1})(1-\alpha\beta\pi^{-1})[\gamma-1-\tilde{\epsilon}(\gamma+1)][(1-\alpha\beta\pi^{\gamma-1})(1-\alpha\pi^{-1}) + \tilde{\epsilon}(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^{-1})]} \end{aligned} \right].$$

Condition (29) can be interpreted as the long-run version of the Taylor principle—in the long run the interest rate should be raised by more than the increase in inflation—and its boundary

is illustrated by the dashed line in each panel of Fig. 3.¹⁵ The left column of the figure demonstrates that in the presence of the kink in demand curves, determinacy is guaranteed even under high trend inflation if the Taylor principle (29) is met and that this condition is not restrictive because the coefficient estimates by Taylor (1993), i.e., $(\phi_\pi, \phi_y) = (1.5, 0.125)$, ensure determinacy at any trend inflation rate considered.

This result contrasts starkly with that obtained by Ascari and Ropele (2009) and Kurozumi (2014), who consider the case of no kink in demand curves. In this case, the right column of Fig. 3 shows that indeterminacy is more likely with higher trend inflation. In each panel of the figure, there is only one region of determinacy within the coefficient range considered. This region is characterized not only by the Taylor principle (29) but also by another condition.¹⁶ The latter condition induces lower bounds on the inflation and output coefficients ϕ_π, ϕ_y , but it becomes irrelevant in the presence of the kink in demand curves. The Taylor principle (29) generates an upper bound on the output coefficient and is more likely to be satisfied for the Taylor rule's coefficients $\phi_\pi, \phi_y \geq 0$ as the long-run inflation elasticity of output ϵ_y is larger. Even in the presence of the kink, the Taylor principle (29) remains a relevant condition for determinacy, although it yields lower bounds on the inflation and output coefficients.¹⁷ Therefore, the kink can prevent indeterminacy caused by high trend inflation.

¹⁵From the log-linearized equilibrium conditions (19)–(24), it follows that a one percentage point permanent increase in inflation yields an ϵ_y percentage points permanent change in output. The Taylor rule (25) then implies a $(\phi_\pi + \phi_y \epsilon_y)$ percentage points permanent change in the interest rate in response to a one percentage point permanent increase in inflation.

¹⁶At a zero rate of trend inflation, the region of determinacy defined by $\phi_\pi, \phi_y \geq 0$ is characterized only by the Taylor principle (29).

¹⁷In the case of no kink in demand curves, higher trend inflation makes the long-run inflation elasticity of output ϵ_y decline exponentially, as shown in Panel B of Table 2. A rise in the annualized trend inflation rate from zero to three percent and six percent reduces the elasticity from 0.14 to -4.74 and -53.04 , respectively. This exponential decline in the elasticity caused by higher trend inflation is reversed in the presence of the kink in demand curves. As reported in Panel A of the table, the elasticity increases from 0.15 to 0.67 and 0.75 respectively when the trend inflation rate increases from zero to three percent and six percent.

4.2 Relationship between the natural rate hypothesis and the long-run version of the Taylor principle

The ability of the smoothed-off kink in demand curves to prevent large violations of the NRH is closely related to its ability to prevent indeterminacy of equilibrium under the Taylor rule.¹⁸ As noted above, the Taylor principle (29) is more likely to be satisfied for the Taylor rule's coefficients $\phi_\pi, \phi_y \geq 0$ when the long-run inflation elasticity of output ϵ_y is larger. By definition, this elasticity—the permanent percentage change in output in response to a one percentage point permanent increase in inflation—is given by $\epsilon_y = d \log Y / d \log \pi$. Because the natural rate Y^n is constant with respect to the trend inflation rate, the derivative of the steady-state output gap with respect to that rate equals the long-run inflation elasticity of output (i.e., $d(\log Y - \log Y^n) / d \log \pi = d \log Y / d \log \pi = \epsilon_y$). Thus, a given change in the derivative is associated with the same change in the elasticity.

It follows that a rise in trend inflation is more likely to induce indeterminacy of equilibrium under the Taylor rule, by lowering the upper bound on the rule's coefficient on output, if and only if such a rise reduces the steady-state output gap at an increased rate. By mitigating the influence of high trend inflation on aggregate output through the average markup and the relative price distortion, the kink in demand curves increases the size of the derivative of the steady-state output gap with respect to the trend inflation rate and thus ensures that the violation of the NRH is minor. At the same time, the kink increases the size of the long-run inflation elasticity of output and thus prevents higher trend inflation from inducing indeterminacy of equilibrium under the Taylor rule.

¹⁸The link between the steady-state relationship of output and inflation and the long-run inflation elasticity of output in the long-run version of the Taylor principle is also indicated by Ascari and Ropele (2009). In the Calvo model with no kink in demand curves, they stress that when trend inflation becomes sufficiently high, the elasticity changes its sign along the lines of the steady-state relationship between output and inflation.

4.3 Some robustness exercises

The finding that a smoothed-off kink in demand curves prevents equilibrium indeterminacy caused by high trend inflation survives under alternative values of the parameters regarding the probability of no price change and the price elasticity of demand. Fig. 4 displays regions of the Taylor rule's coefficients that guarantee determinacy when the probability of no price change is $\alpha = 0.75$, which implies that prices change once a year on average in line with the evidence of Kehoe and Midrigan (forthcoming). Fig. 5 illustrates the case of the price elasticity of demand parameter of $\theta = 6$, which implies a markup of 20 percent at a zero rate of trend inflation. In each of these figures the left column shows the implications of introducing a small kink in demand curves, by setting ϵ such that $\gamma = -4$, to illustrate that the determinacy region is not sensitive to the degree of the curvature of demand curves. The sharp contrast between the left and right columns of each figure shows that even a small kink in demand curves makes the determinacy region insensitive to the rate of trend inflation in both the cases of a high degree of price rigidity and a small price elasticity of demand.

5 Comparison with the model with firm-specific labor

A smoothed-off kink in demand curves gives rise to strategic complementarity in price setting. This section shows that firm-specific labor—which is another source of the complementarity analyzed in previous literature—does not share the desirable properties of the kink in terms of preventing both large violations of the NRH and equilibrium indeterminacy caused by high trend inflation. It also demonstrates that introducing such a kink in the Calvo model with firm-specific labor once again ensures that the violation of the NRH is minor and that indeterminacy caused by high trend inflation is prevented.

5.1 On the natural rate hypothesis

A description of the Calvo model with firm-specific labor and a smoothed-off kink in demand curves is provided in Appendix. The equilibrium conditions consist of not only Eqs. (2), (6), (8), (14), (15), and (17), given in Section 2, but also Eq. (32), which can be found in Appendix. In the model the relationship between the steady-state output gap and trend inflation can only be traced for integer values of σ_n —otherwise some infinite sums cannot be reduced—so that the analysis is restricted to the case of $\sigma_n = 1$. For this case, the relationship between steady-state output and trend inflation is given by

$$Y = \left\{ \frac{\frac{\gamma-1}{\gamma} \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^{\gamma-1}} - \frac{\epsilon}{\gamma} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{\frac{\gamma}{\gamma-1}} \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^{-1}}}{\left[\frac{1}{1+\epsilon} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{1}{\gamma-1}} + \frac{\epsilon}{1+\epsilon} \frac{1-\alpha}{1-\alpha\pi^{-1}} \right] \left[\frac{1}{1+\epsilon} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{\gamma}{\gamma-1}} + \frac{\epsilon}{1+\epsilon} \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^{\gamma}} \right]} \right\}^{\frac{1}{2}}.$$

In the absence of Calvo staggered price setting, the (steady-state) natural rate of output can be obtained as (27). The steady-state output gap is thus given by

$$\log Y - \log Y^n = -\frac{1}{2} \log \frac{\frac{\theta-1}{\theta} \left[\frac{1}{1+\epsilon} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{1}{\gamma-1}} + \frac{\epsilon}{1+\epsilon} \frac{1-\alpha}{1-\alpha\pi^{-1}} \right] \left[\frac{1}{1+\epsilon} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{-\frac{\gamma}{\gamma-1}} + \frac{\epsilon}{1+\epsilon} \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^{\gamma}} \right]}{\frac{\gamma-1}{\gamma} \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^{\gamma-1}} - \frac{\epsilon}{\gamma} \left(\frac{1-\alpha}{1-\alpha\pi^{\gamma-1}} \right)^{\frac{\gamma}{\gamma-1}} \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^{-1}}}. \quad (30)$$

In the presence of firm-specific labor, the steady-state output gap contains only the average markup distortion and no relative price distortion. Note that at a zero rate of trend inflation (i.e., $\pi = 1$), steady-state output Y is equal to the natural rate of output Y^n and hence the steady-state output gap is zero.

If firm-specific labor is the only source of strategic complementarity in price setting, the violation of the NRH is much larger than if the complementarity arises solely from the smoothed-off kink in demand curves, under comparable calibrations. In Fig. 6, the thin line displays the effect of the annualized trend inflation rate on the steady-state output gap (30) in the model with only firm-specific labor under the benchmark calibration summarized in Table 1 except

for $\epsilon = 0$. Under this calibration, the NKPC at a zero rate of trend inflation (i.e., Eq. (31) with $\epsilon = 0$ and $\pi = 1$) has a slope of 0.034, which is the same as that in the model with only the kinked demand curves under the benchmark calibration used in the preceding sections. Thus, because both calibrations target the same size of the slope of the NKPC at a zero rate of trend inflation and contain the same probability of no price change (i.e., $\alpha = 0.55$), the degree of strategic complementarity in both calibrated models is comparable. The thin line in the figure illustrates that the steady-state output gap is much more sensitive to trend inflation if firm-specific labor is the only source of the complementarity than if the kink is the only source, as the latter generates the thick line in Fig. 2. Indeed, at an annualized trend inflation rate of three percent and six percent, the steady-state output gap is -0.91 percent and -5.26 percent in the former model, whereas the gap is 0.37 percent and 0.88 percent in the latter model. The reason for the much larger violation of the NRH in the model with only firm-specific labor is that the influence of high trend inflation on aggregate output is mitigated in the case of only the kinked demand curves, whereas this mitigating effect is absent in the case of only firm-specific labor.¹⁹

Adding a smoothed-off kink in demand curves to the model with firm-specific labor ensures that the violation of the NRH is minor. The thick line in Fig. 6 illustrates the effect of the annualized trend inflation rate on the steady-state output gap (30) in the model with both firm-specific labor and the kinked demand curves. This line uses a calibration that keeps the slope of the NKPC unchanged. Specifically, the value of $\epsilon = -1.4$ —which is consistent with the available evidence on the curvature of demand curves as noted above—is adopted and the probability of no price change is then set at $\alpha = 0.53$ to target the same slope of 0.034 in the

¹⁹Specifically, although firm-specific factors dampen the size of firms' price changes, higher trend inflation causes price-adjusting firms to choose a higher markup. Consequently, it exponentially increases the average markup and hence exponentially decreases the steady-state output gap.

NKPC (i.e., Eq. (31) with $\pi = 1$) as before. The figure illustrates that even a small kink in demand curves can prevent a large violation of the NRH. Indeed, the steady-state output gap takes a modest positive value of 0.37 and 1.15 at an annualized trend inflation rate of three percent and six percent, respectively. Thus, a smoothed-off kink in demand curves ensures that the violation of the NRH is minor, regardless of whether labor is homogeneous or firm-specific.

5.2 On equilibrium determinacy

The violation of the NRH suggests, from the relationship between the NRH and the long-run version of the Taylor principle, that indeterminacy of equilibrium under the Taylor rule is much more likely in the model with only firm-specific labor than in the model with only the kinked demand curves or with both sources of strategic complementarity.

To analyze equilibrium determinacy the model is log-linearized, restricting again attention to the case of $\sigma_n = 1$. Rearranging the resulting equations leads to Eqs. (18), (22), (25), and

$$\begin{aligned} \hat{\pi}_t = & \frac{\beta \left\{ 1 + \left[e + (1 - \alpha\pi^{\gamma-1}) \left(e + e\tilde{\epsilon} \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^\gamma} \frac{1-\alpha\beta\pi^{-1}}{1-\alpha\beta\pi^{\gamma-1}} - \gamma \right) \right] - \frac{\tilde{\epsilon}(1+\gamma\alpha\pi^{\gamma-1})}{\gamma-1} \right\}}{1 + e - \frac{\tilde{\epsilon}(1+\gamma)}{\gamma-1}} E_t \hat{\pi}_{t+1} \\ & + \frac{2(1 - \alpha\pi^{\gamma-1})(1 - \alpha\beta\pi^{2\gamma})e \left(1 + \tilde{\epsilon} \frac{1-\alpha\beta\pi^{-1}}{1-\alpha\beta\pi^{\gamma-1}} \right)}{\alpha\pi^{\gamma-1}\gamma \left(1 + e - \frac{\tilde{\epsilon}(1+\gamma)}{\gamma-1} \right)} \hat{Y}_t - \frac{1}{\alpha\pi^{\gamma-1}} \left(\hat{d}_{1t} - \alpha\beta\pi^{\gamma-1} E_t \hat{d}_{1t+1} \right) + \hat{d}_{1t-1} - \alpha\beta\pi^{\gamma-1} \hat{d}_{1t} \\ & + \frac{(1 - \alpha\pi^{\gamma-1}) \left[\gamma(1 - \alpha\beta\pi^{\gamma-1}) + e(1 - \alpha\beta\pi^{2\gamma}) \left(2 + \tilde{\epsilon} \frac{1-\alpha\beta\pi^{-1}}{1-\alpha\beta\pi^{\gamma-1}} \right) \right]}{\alpha\pi^{\gamma-1} \left(1 + e - \frac{\tilde{\epsilon}(1+\gamma)}{\gamma-1} \right)} \hat{d}_{1t} + \xi_{1t} + \xi_{2t} + \psi_t, \end{aligned} \quad (31)$$

$$\xi_{1t} = \alpha\beta\pi^{2\gamma} E_t \xi_{1t+1} + \frac{2e\beta(\pi^{1+\gamma} - 1)(1 - \alpha\pi^{\gamma-1})}{1 + e - \frac{\tilde{\epsilon}(1+\gamma)}{\gamma-1}} \left[E_t \hat{\pi}_{t+1} + \frac{1 - \alpha\beta\pi^{2\gamma}}{\gamma} E_t \hat{Y}_{t+1} + (1 - \alpha\beta\pi^{2\gamma}) E_t \hat{d}_{1t+1} \right],$$

$$\xi_{2t} = \alpha\beta\pi^\gamma E_t \xi_{2t+1} + \frac{\beta\theta\tilde{\epsilon}(\pi - 1)(1 - \alpha\pi^{\gamma-1})(1 - \alpha\beta\pi^{2\gamma})(1 - \alpha\beta\pi^{-1})}{(1 - \alpha\beta\pi^{\gamma-1}) \left(1 + e - \frac{\tilde{\epsilon}(1+\gamma)}{\gamma-1} \right)} \left[\frac{1}{1 - \alpha\beta\pi^\gamma} E_t \hat{\pi}_{t+1} + \frac{2}{\gamma} E_t \hat{Y}_{t+1} + E_t \hat{d}_{1t+1} \right],$$

$$\psi_t = \alpha\beta\pi^{-1} E_t \psi_{t+1} + \frac{\tilde{\epsilon}\beta(\pi^\gamma - 1)(1 - \alpha\pi^{\gamma-1})}{\pi^\gamma [(1 + e)(\gamma - 1) - \tilde{\epsilon}(1 + \gamma)]} E_t \hat{\pi}_{t+1},$$

where $e = \gamma[1 - \tilde{\epsilon}/(\gamma - 1)]/\{1 + \tilde{\epsilon}(1 - \alpha\beta\pi^{2\gamma})(1 - \alpha\beta\pi^{-1})/[(1 - \alpha\beta\pi^\gamma)(1 - \alpha\beta\pi^{\gamma-1})]\}$.

The strategic complementarity arising from both firm-specific labor and the kinked demand curves reduces the slope of the NKPC (31) by a factor $e[1 + \tilde{\epsilon}(1 - \alpha\beta\pi^{-1})/(1 - \alpha\beta\pi^{\gamma-1})]/\{\gamma[1 + e - \tilde{\epsilon}(1 + \gamma)/(\gamma - 1)]\}$. Note that in the case of no kink in demand curves (i.e., $\epsilon = 0$), we have $\tilde{\epsilon} = 0$ and $e = \theta$, and thus the factor in the slope of the NKPC is $1/(1 + \theta)$. Therefore, the degree of the complementarity generated by firm-specific labor increases with the price elasticity of demand. If $\epsilon < 0$ then a combination of both sources of the complementarity allows to reconcile the Calvo model with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC.

In the absence of the kink in demand curves, indeterminacy in the model with firm-specific labor is more likely with higher trend inflation, in line with Coibion and Gorodnichenko (2011) and Kurozumi and Van Zandweghe (2012). At an annualized trend inflation rate of zero, three, and six percent, the right column of Fig. 7 displays regions of the Taylor rule's coefficients (ϕ_π, ϕ_y) that guarantee determinacy of equilibrium under the calibration presented in the preceding subsection (i.e., the benchmark calibration except for $\epsilon = 0$). The dashed line represents the long-run version of the Taylor principle (29), where the long-run inflation elasticity of output (at $\sigma_n = 1$) is now given by

$$\epsilon_y = \frac{\left[\begin{aligned} & (1 - \beta)(1 + e) - \frac{2e\beta(\pi^{1+\gamma}-1)(1-\alpha\pi^{\gamma-1})}{1-\alpha\beta\pi^{2\gamma}} - \beta(1 - \alpha\pi^{\gamma-1}) \left(e + \tilde{\epsilon}e \frac{1-\alpha\beta\pi^{2\gamma}}{1-\alpha\beta\pi^\gamma} \frac{1-\alpha\beta\pi^{-1}}{1-\alpha\beta\pi^{\gamma-1}} - \gamma \right) \right. \\ & + \frac{\tilde{\epsilon}(1-\alpha\beta\pi^{\gamma-1})[\beta-\pi^\gamma(1+\gamma)(1-\alpha\beta\pi^{-1})]}{\pi^\gamma(\gamma-1)(1-\alpha\beta\pi^{-1})} - \frac{\tilde{\epsilon}\beta\theta(\pi-1)(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^{2\gamma})(1-\alpha\beta\pi^{-1})}{(1-\alpha\beta\pi^\gamma)(1-\alpha\beta\pi^\gamma)^2} \\ & + \frac{\tilde{\epsilon}(\pi^\gamma-1)(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^{-1})(1-\alpha\beta\pi^{\gamma-1})(e+\gamma-1+\frac{\tilde{\epsilon}(1+\gamma)}{\gamma-1})}{\pi^\gamma(1-\alpha\pi^{-1})[(1-\alpha\beta\pi^{\gamma-1})(1-\alpha\pi^{-1})+\tilde{\epsilon}(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^{-1})]} \\ & \left. + \frac{\tilde{\epsilon}^2(\pi^\gamma-1)(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^{2\gamma})(1-\alpha\beta\pi^{-1})^2[e(1-\alpha\beta\pi^\gamma)+\alpha\beta\theta\pi^{\gamma-1}(\pi-1)]}{\pi^\gamma(1-\alpha\pi^{-1})(1-\alpha\beta\pi^{\gamma-1})(1-\alpha\beta\pi^\gamma)[(1-\alpha\beta\pi^{\gamma-1})(1-\alpha\pi^{-1})+\tilde{\epsilon}(1-\alpha\pi^{\gamma-1})(1-\alpha\beta\pi^{-1})]} \right]}{\frac{2(1-\alpha\pi^{\gamma-1})}{\alpha\pi^{\gamma-1}\gamma} \left[e(1 - \alpha\beta\pi^{\gamma-1}) + \frac{\tilde{\epsilon}(1-\alpha\beta\pi^{2\gamma})(1-\alpha\beta\pi^{-1})[e(1-\alpha\beta\pi^\gamma)+\alpha\beta\theta\pi^{\gamma-1}(\pi-1)]}{(1-\alpha\beta\pi^{\gamma-1})(1-\alpha\beta\pi^\gamma)} \right]}.$$

At each rate of trend inflation, there is only one region of determinacy within the coefficient range considered. This region is characterized by the Taylor principle (29), which generates an upper bound on the output coefficient ϕ_y . At an annualized trend inflation rate of three percent and six percent, it is also featured by the other condition, which induces lower bounds on the

inflation and output coefficients ϕ_π, ϕ_y . As noted above, the Taylor principle (29) is more likely to be satisfied for the Taylor rule's coefficients $\phi_\pi, \phi_y \geq 0$ as the long-run inflation elasticity of output ϵ_y is larger. Yet higher trend inflation exponentially reduces this elasticity: its value is 0.15, -3.01 , and -10.24 respectively at an annualized trend inflation rate of zero, three, and six percent. Hence the Taylor principle (29) induces a more severe upper bound on the output coefficient ϕ_y as trend inflation rises. The other condition for determinacy generates more severe lower bounds on the inflation and output coefficients ϕ_π, ϕ_y for higher trend inflation. Consequently, indeterminacy is much more likely in the model with only firm-specific labor than in the model with only the kinked demand curves, as the relationship between the NRH and the long-run version of the Taylor principle implies.

Adding a smoothed-off kink in demand curves to the model with firm-specific labor ensures that equilibrium determinacy is no longer sensitive to trend inflation. The left column of Fig. 7 shows regions of the Taylor rule's coefficients that guarantee determinacy at the different rates of trend inflation, under the calibration that keeps the slope of the NKPC unchanged by setting $\epsilon = -1.4$ and $\alpha = 0.53$. This figure illustrates that even a small kink in demand curves can prevent indeterminacy caused by high trend inflation. Correspondingly, the long-run inflation elasticity of output ϵ_y takes a moderate positive value of 0.82 and 1.34 at an annualized trend inflation rate of three percent and six percent, respectively. Thus, a smoothed-off kink in demand curves prevents equilibrium indeterminacy caused by high trend inflation, regardless of whether labor is homogeneous or firm-specific.

5.3 Revisiting the role of trend inflation for the U.S. shift from the Great Inflation era to the Great Moderation era

Coibion and Gorodnichenko (2011) use the Calvo model with firm-specific labor but no kink in demand curves to emphasize the importance of a decline in trend inflation for the U.S. economy's shift from the Great Inflation era to the Great Moderation era. This subsection revisits their conclusion using the models with a smoothed-off kink in demand curves. It shows that the models both suggest that trend inflation did not play a role in the shift.

Coibion and Gorodnichenko (2011) estimate versions of the Federal Reserve's reaction function in the pre-1979 and the post-1982 periods and use the model with (only) firm-specific labor to assess how the probability of equilibrium determinacy for the U.S. economy is affected by estimated coefficients in the reaction function and the trend inflation rate. In addition to the policy responses to inflation and output (or equivalently, the output gap) in the Taylor rule (17) considered so far, the estimated reaction function allows for policy responses to output growth and lags of the policy rate. Using real-time data, the policy rate in the reaction function responds to either a nowcast for the current quarter (contemporaneous Taylor rule), a forecast of the average over the next two quarters (forward-looking Taylor rule), or a mix of the forecast for inflation and the nowcast for the output gap and output growth (mixed Taylor rule). Drawing from the distribution of the estimated coefficients in the reaction function, the probability of equilibrium determinacy for each version of the reaction function is calculated as the fraction of draws that ensure determinacy at an annualized trend inflation rate of three percent and six percent, the average inflation rate in the pre-1979 and the post-1982 periods, respectively. Panel C of Table 3 summarizes their results, which are obtained using the model with firm-specific labor under the benchmark calibration except for $\epsilon = 0$.²⁰ For each ver-

²⁰Recall that this calibration is the same as used by Coibion and Gorodnichenko (2011). These authors'

sion of the reaction function, the switch in its coefficients from the pre-1979 estimates to the post-1982 estimates raises the probability of determinacy substantially. However, while the post-1982 estimates are consistent with determinacy at an annualized trend inflation rate of three percent (the probability ranges between 0.71–0.99), they are only marginally consistent with determinacy at the rate of six percent (0.12–0.63). This suggests that the decline in trend inflation as well as the switch in coefficients of the reaction function was important to the shift from the Great Inflation era to the Great Moderation era.

Panel A of Table 3 conducts the same exercise after adding a smoothed-off kink in demand curves to the model with firm-specific labor, using the benchmark calibration except for $\epsilon = -1.4$ and $\alpha = 0.53$. The probability of determinacy is higher than in the case of no kink in demand curves, regardless of the version of the reaction function or the time period considered, which is in line with the analysis in the preceding subsection. As is the case with no kink, the switch from the pre-1979 estimates to the post-1982 estimates raises the probability of determinacy for each version of the reaction function. It rises from a probability that is fairly consistent with determinacy (0.57–0.90) to one that is highly consistent with determinacy (0.93–1.00). In contrast with the case of no kink, however, the probability of determinacy for each version of the reaction function is barely affected by trend inflation. In fact, the probability is slightly higher at an annualized trend inflation rate of six percent than at the rate of three percent, in line with the analysis in the preceding subsection. The model with the kinked demand curves and firm-specific labor thus indicates that the switch in the reaction function's coefficients played a key role for the shift from the Great Inflation era to the Great Inflation

program Table1.m, which is available from www.aeaweb.org, was used to replicate Table 1 of the paper. This code has some typos (on lines 75 and 181), which account for a different probability obtained with the forward-looking Taylor rule. In addition, Table 1 of their paper contains a typo of a fraction obtained with the mixed Taylor rule. Hence there are the minor differences between their Table 1 and our Panel C of Table 3.

era, regardless of trend inflation.

Likewise, Panel B of Table 3 presents the probability of equilibrium determinacy in the model with the kinked demand curves and homogeneous labor under the baseline calibration listed in Table 1. For each version of the reaction function and each time period, the probability exceeds that in the model with firm-specific labor and no kink in demand curves (Panel C), but is not so high as that in the model with firm-specific labor and the kinked demand curves (Panel A). For each version of the reaction function, the switch from the pre-1979 estimates to the post-1982 estimates raises the probability of determinacy substantially. It rises from a probability that is only marginally consistent with determinacy (0.33–0.76) to one that is consistent with determinacy (0.89–1.00). Moreover, the probability of determinacy for each version of the reaction function is barely affected by trend inflation. Therefore, the model with the kinked demand curves and homogeneous labor reaches the same conclusion as that obtained using the model with the kinked demand curves and firm-specific labor.

6 Concluding remarks

This paper has examined implications of a smoothed-off kink in demand curves for the NRH and macroeconomic stability in the Calvo model. An empirically plausible calibration of the model has shown that such a kink mitigates the influence of high trend inflation on aggregate output through the average markup (and the relative price distortion in the case of homogeneous labor), thereby ensuring that the violation of the NRH is minor and preventing equilibrium indeterminacy caused by high trend inflation.

For the U.S. economy's shift from the Great Inflation era to the Great Moderation era, the previous literature, including Clarida et al. (2000) and Lubik and Schorfheide (2004), has stressed the key role played by the Federal Reserve's switch from a passive to an active policy

response to inflation. In addition, Coibion and Gorodnichenko (2011) argue for the importance of a decline in trend inflation for the shift, using the Calvo model with firm-specific labor and no kink in demand curves. However, the present paper has demonstrated that such a model induces a large loss in steady-state output relative to its natural rate during the Great Inflation era. The Calvo model with a smoothed-off kink in demand curves yields a minor violation of the NRH and supports the view of the previous literature.

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A The Calvo model with firm-specific labor and a smoothed-off kink in demand curves

This appendix presents the Calvo model with firm-specific labor and a smoothed-off kink in demand curves. The descriptions of the monetary authority and the representative final-good firm are the same as in Section 2. The representative household and the intermediate-good firms are described in turn below.

The representative household supplies $\{N_t(f)\}$ labor specific to each intermediate-good firm f and chooses consumption and bond holdings to maximize the utility function $E_0 \sum_{t=0}^{\infty} \beta^t \{\log C_t - \int_0^1 (N_t(f))^{1+\sigma_n} / (1 + \sigma_n) df\}$ subject to the budget constraint $P_t C_t + B_t = \int_0^1 P_t W_t(f) N_t(f) df + i_{t-1} B_{t-1} + T_t$, where $W_t(f)$ and $N_t(f)$ are the firm-specific real wage and labor supply. Combining first-order conditions for utility maximization with respect to consumption, labor supply, and bond holdings yields the Euler equation (2) and the labor supply curve $W_t(f) = C_t (N_t(f))^{\sigma_n}$.

Each intermediate-good firm f produces one kind of differentiated good $Y_t(f)$ under monopolistic competition using the production function (9). Given the real wage $W_t(f)$, the first-order condition for minimization of production cost determines firm f 's real marginal cost $mc_t(f) = W_t(f)$. Facing this marginal cost and the final-good firm's demand (4), intermediate-good firms choose prices of their products subject to Calvo staggered price setting. Each period a fraction $1 - \alpha$ of firms sets the price $P_t(f)$ to maximize the profit function

$$E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \left(\frac{P_t(f)}{P_{t+j}} - mc_{t+j}(f) \right) \frac{1}{1 + \epsilon} \left[\left(\frac{P_t(f)}{P_{t+j} d_{1t+j}} \right)^{-\gamma} + \epsilon \right] Y_{t+j}.$$

The Calvo model with firm-specific labor imposes the following assumption instead of Assumption 1 in order for the profit function to be well-defined.

Assumption 2 *The four inequalities $\alpha\beta\pi^{\gamma-1} < 1$, $\alpha\beta\pi^{\gamma} < 1$, $\alpha\beta\pi^{-1} < 1$, and $\alpha\beta\pi^{\gamma(1+\sigma_n)} < 1$ hold.*

The first-order condition for Calvo staggered price setting leads to

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \prod_{k=1}^j \pi_{t+k}^{\gamma} \left(\left\{ p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} - \frac{\gamma Y_{t+j}^{1+\sigma_n} \left[\left(\frac{p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} \right)^{-\gamma} + \epsilon \right]^{\sigma_n}}{1 + \epsilon} \right\} d_{1t+j}^{\gamma} - \frac{\epsilon}{\gamma - 1} \left(p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} \right)^{1+\gamma} \right) = 0. \quad (32)$$

The price dispersion equations (5) and (7) can be reduced to (14) and (15), respectively.

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Table 1: Calibration for the quarterly model

β	Subjective discount factor	0.99
σ_n	Inverse of the elasticity of labor supply	1
α	Probability of no price change	0.55
θ	Parameter governing the price elasticity of demand	10
ϵ	Parameter governing the curvature of demand curves	-9

Table 2: Relationship between steady-state output and trend inflation

Annualized trend inflation rate (%)	0	3	6
A. Kink in demand curves ($\epsilon = -9$)			
Steady-state output gap (%)	0	0.37	0.88
Average markup (%)	0	0.38	0.91
Relative price distortion (%)	0	-0.01	-0.03
Long-run inflation elasticity of output	0.15	0.67	0.75
B. Small kink in demand curves ($\epsilon = -1.4$)			
Steady-state output gap (%)	0	0.06	0.17
Average markup (%)	0	0.09	0.30
Relative price distortion (%)	0	-0.03	-0.12
Long-run inflation elasticity of output	0.03	0.12	0.20
C. No kink in demand curves ($\epsilon = 0, \alpha = 0.83$)			
Steady-state output gap (%)	0	-1.20	-14.42
Average markup (%)	0	-0.48	-5.87
Relative price distortion (%)	0	-0.73	-8.55
Long-run inflation elasticity of output	0.14	-4.74	-53.04

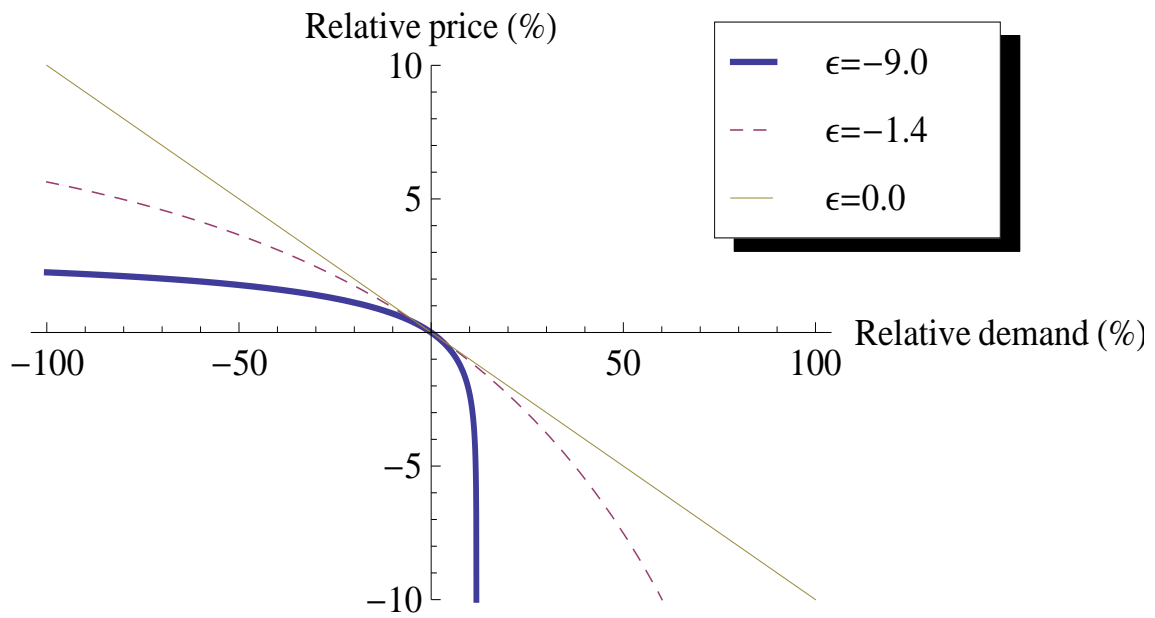
Note: The lines “Average markup” and “Relative price distortion” show the contribution of each distortion to the steady-state output gap.

Table 3: Probability of equilibrium determinacy

	Contemporaneous Taylor rule		Forward-looking Taylor rule		Mixed Taylor rule	
	pre-1979	post-1982	pre-1979	post-1982	pre-1979	post-1982
A. Kinked demand curves and firm-specific labor						
3 percent inflation	Yes	Yes	Yes	Yes	Yes	Yes
Fraction at 3 percent	0.569	0.928	0.804	0.999	0.814	1.000
6 percent inflation	Yes	Yes	Yes	Yes	Yes	Yes
Fraction at 6 percent	0.754	0.952	0.832	1.000	0.904	1.000
B. Kinked demand curves and homogeneous labor						
3 percent inflation	No	Yes	Yes	Yes	Yes	Yes
Fraction at 3 percent	0.331	0.889	0.760	0.998	0.652	0.999
6 percent inflation	No	Yes	Yes	Yes	Yes	Yes
Fraction at 6 percent	0.346	0.891	0.762	0.998	0.662	0.999
C. Firm-specific labor and no kink in demand curves						
3 percent inflation	No	Yes	No	Yes	No	Yes
Fraction at 3 percent	0.012	0.712	0.480	0.977	0.075	0.994
6 percent inflation	No	No	No	Yes	No	Yes
Fraction at 6 percent	0.000	0.123	0.077	0.494	0.000	0.633

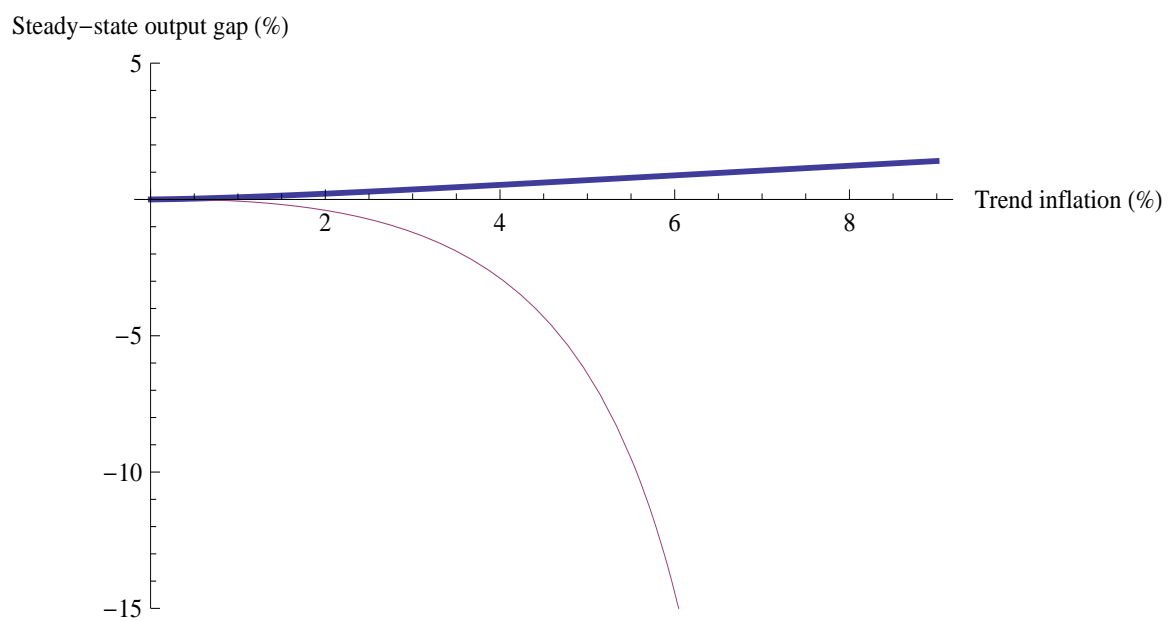
Notes: Like Table 1 of Coibion and Gorodnichenko (2011), this table reports whether their estimated coefficients in each version of the Federal Reserve’s reaction function, along with the calibration of the other parameters in each model, are consistent with a determinate equilibrium at an annualized trend inflation rate of 3 percent and 6 percent. “Yes”/“No” presents whether there is a determinate equilibrium under their point estimates of the coefficients. Fraction at x percent shows the fraction of draws from the distribution of their estimated coefficients that ensure determinacy at the specified rate of trend inflation.

Figure 1: Smoothed-off kink in demand curve.



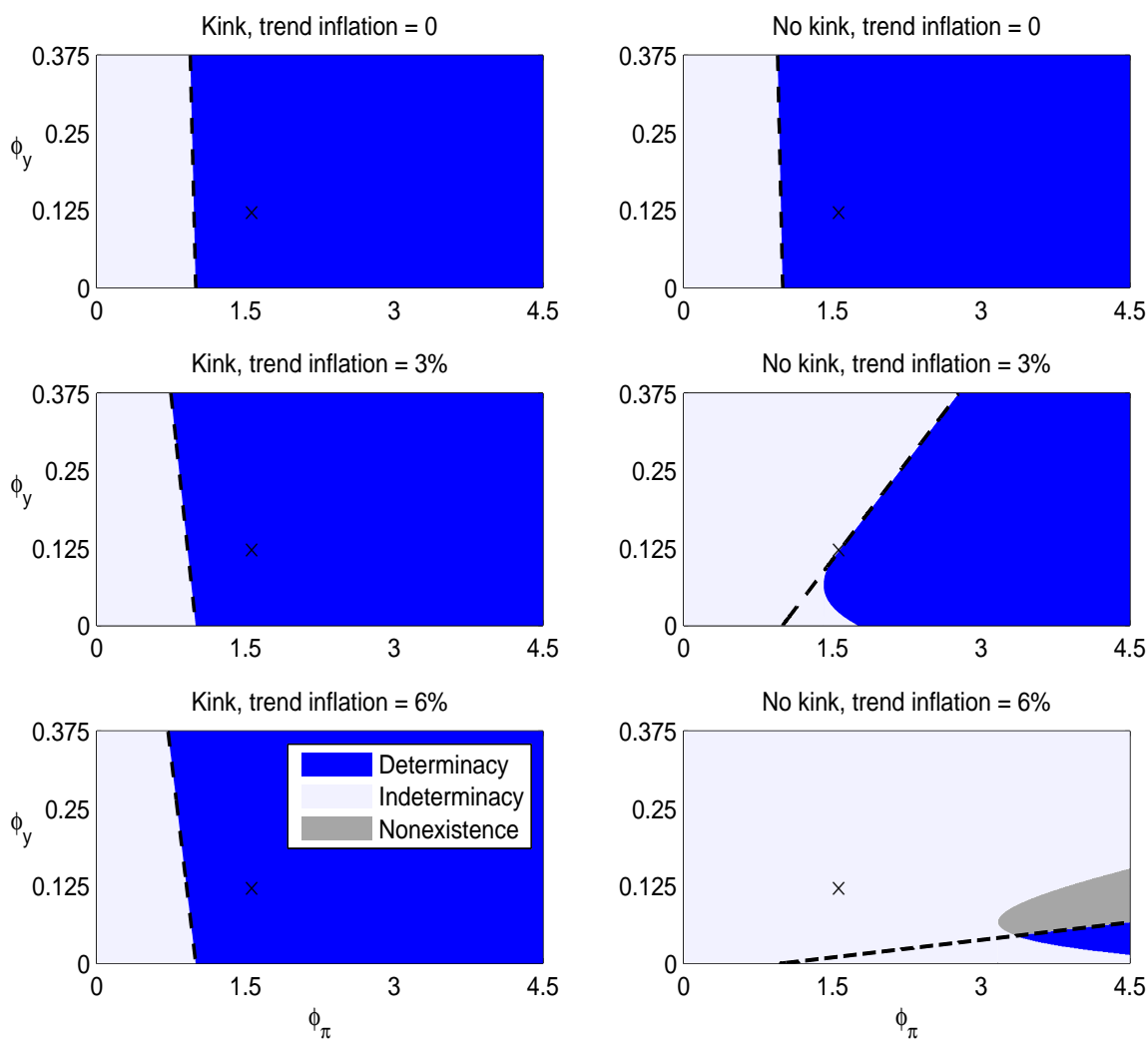
Note: In each case of the value of the parameter governing the curvature of demand curves, ϵ , this figure uses the calibration of the other model parameters presented in Table 1.

Figure 2: Effect of trend inflation on steady-state output gap.



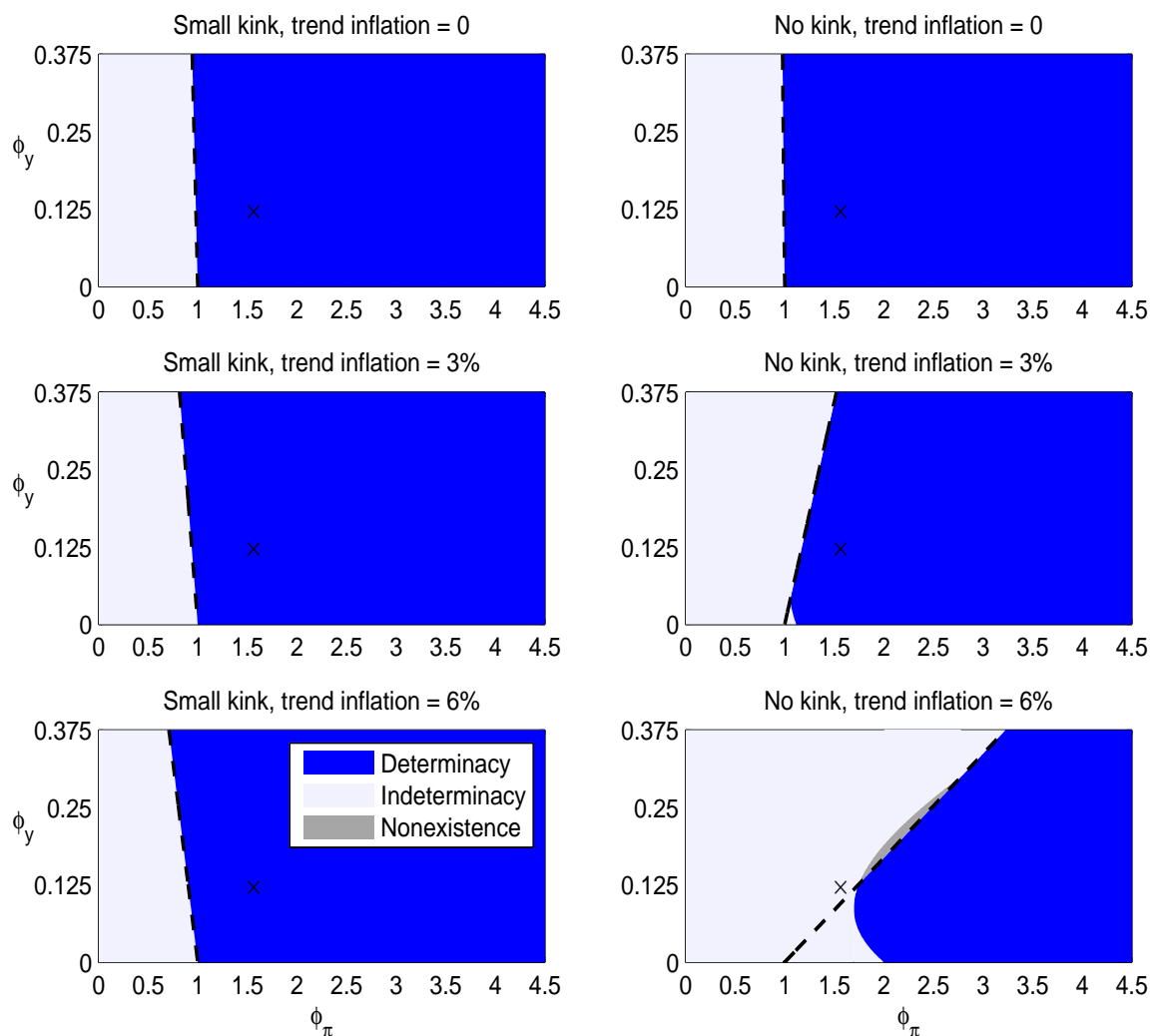
Note: The thick line shows the case of a smoothed-off kink in demand curves and the thin line shows the case of no kink (i.e., $\epsilon = 0$, $\alpha = 0.83$).

Figure 3: Regions of the Taylor rule's coefficients (ϕ_π, ϕ_y) that guarantee equilibrium determinacy.



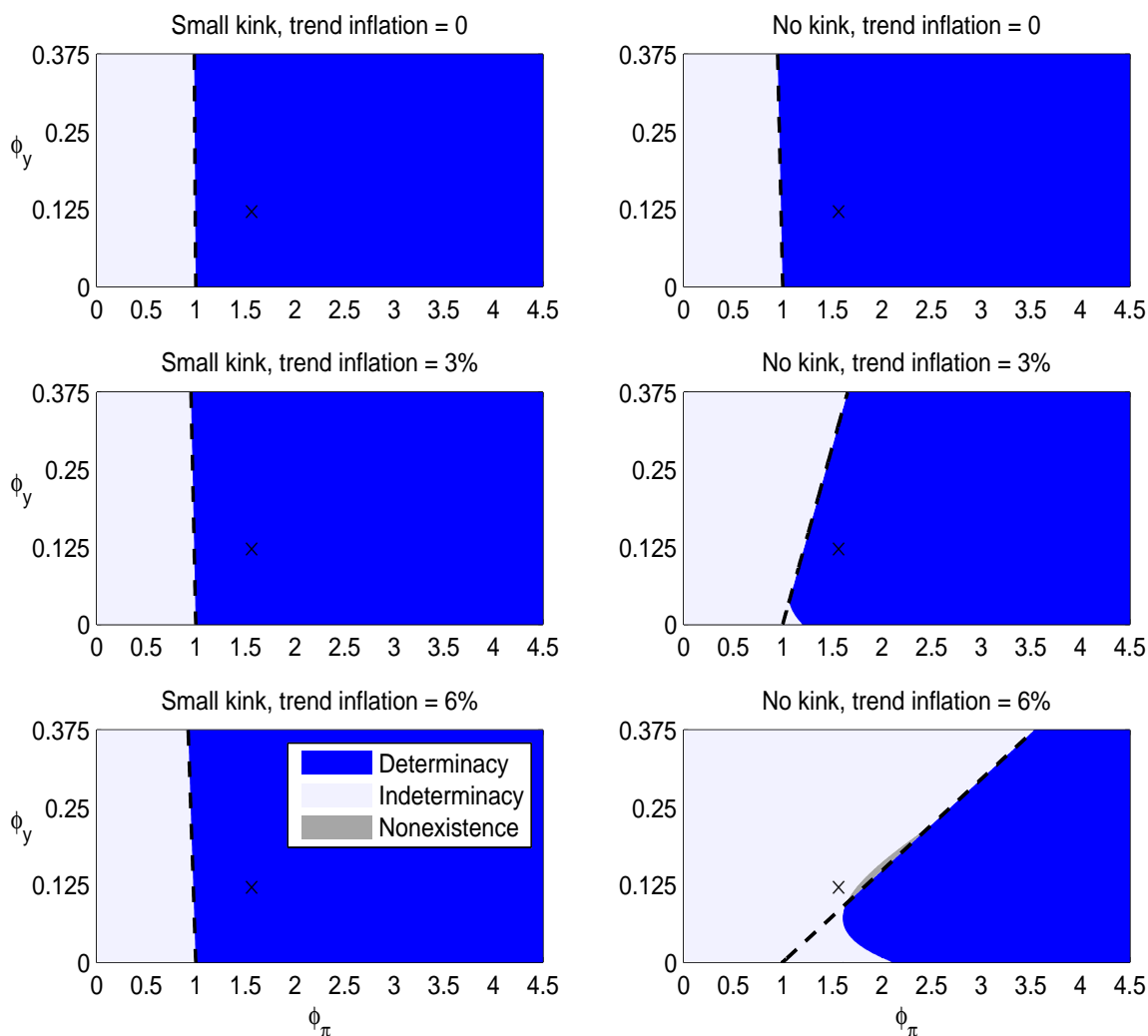
Notes: The first column presents results of the benchmark calibration at an annualized trend inflation rate of zero, three, and six percent. The second column presents results of the calibration with no kink in demand curves (i.e. $\epsilon = 0$, $\alpha = 0.83$). In each panel the mark "x" shows Taylor (1993)'s estimates $(\phi_\pi, \phi_y) = (1.5, 0.5/4)$ and the dashed line represents the boundary defined by the long-run version of the Taylor principle (29).

Figure 4: Regions of the Taylor rule's coefficients (ϕ_π, ϕ_y) that guarantee equilibrium determinacy: robustness with respect to the degree of price rigidity.



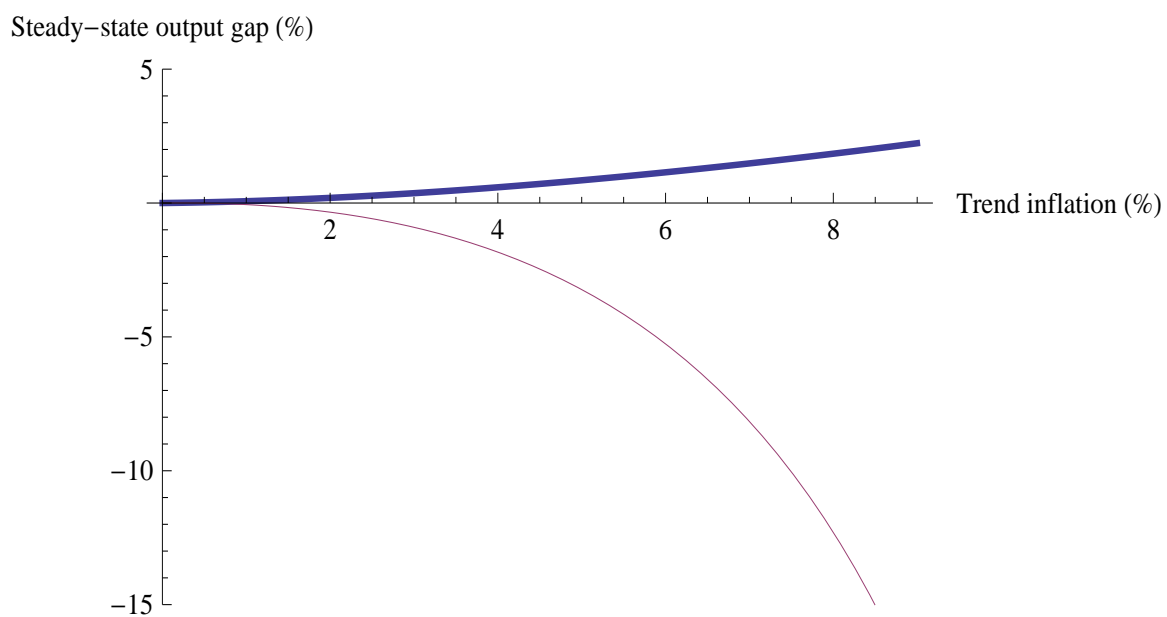
Notes: The first column presents results of the benchmark calibration, except for a small kink in demand curves and a high degree of price rigidity ($\epsilon = -1.4$, $\alpha = 0.75$), at an annualized trend inflation rate of zero, three, and six percent. The second column presents results of the calibration with no kink in demand curves (i.e. $\epsilon = 0$, $\alpha = 0.75$). In each panel the mark “x” shows Taylor (1993)’s estimates $(\phi_\pi, \phi_y) = (1.5, 0.125)$ and the dashed line represents the boundary defined by the long-run version of the Taylor principle (29).

Figure 5: Regions of the Taylor rule's coefficients (ϕ_π, ϕ_y) that guarantee equilibrium determinacy: robustness with respect to the price elasticity of demand.



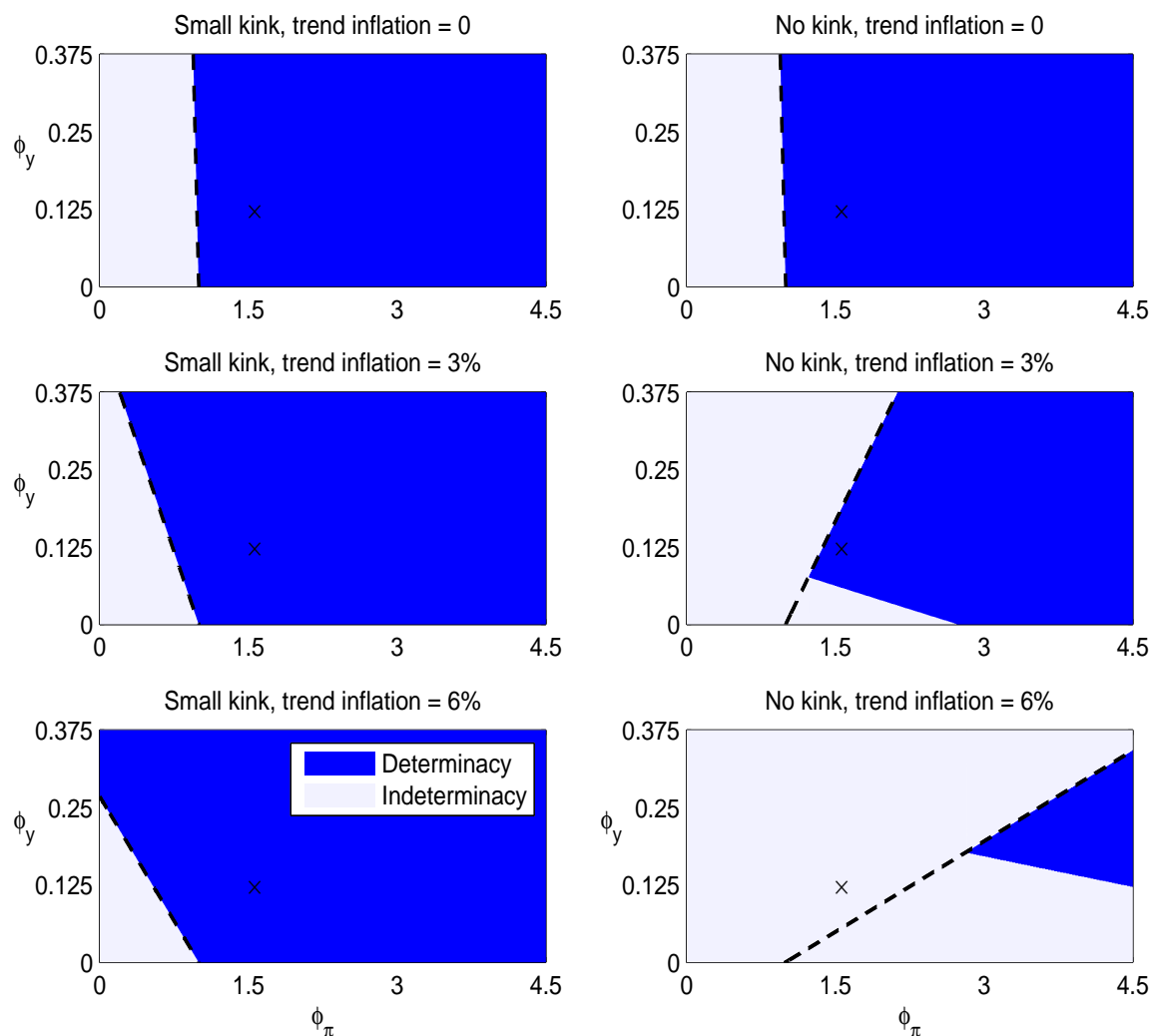
Notes: The first column presents results of the benchmark calibration, except for a small kink in demand curves and a small price elasticity of demand ($\epsilon = -1.67, \theta = 6$), at an annualized trend inflation rate of zero, three, and six percent. The second column presents results of the calibration with no kink in demand curves (i.e. $\epsilon = 0, \theta = 6, \alpha = 0.83$). In each panel the mark "x" shows Taylor (1993)'s estimates $(\phi_\pi, \phi_y) = (1.5, 0.5/4)$ and the dashed line represents the boundary defined by the long-run version of the Taylor principle (29).

Figure 6: Effect of trend inflation on steady-state output gap: model with firm-specific labor.



Note: The thick line shows the case of a smoothed-off kink in demand curves (i.e. $\epsilon = -1.4$, $\alpha = 0.53$) and the thin line shows the case of no kink (i.e., $\epsilon = 0$).

Figure 7: Regions of the Taylor rule's coefficients (ϕ_π, ϕ_y) that guarantee equilibrium determinacy: model with firm-specific labor.



Notes: The first column presents results of the benchmark calibration, except for a small kink in demand curves and a reduced probability of no price change ($\epsilon = -1.4$, $\alpha = 0.53$), at an annualized trend inflation rate of zero, three, and six percent. The second column presents results of the calibration with no kink in demand curves (i.e. $\epsilon = 0$). In each panel the mark “x” shows Taylor (1993)’s estimates $(\phi_\pi, \phi_y) = (1.5, 0.5/4)$ and the dashed line represents the boundary defined by the long-run version of the Taylor principle (29).