

Measuring the Stance of Monetary Policy on and off the Zero Lower Bound

By Taeyoung Doh and Jason Choi

In December 2008, the Federal Open Market Committee (FOMC) lowered its target range for the federal funds rate to 0–25 basis points, effectively hitting the zero lower bound (ZLB). At this point, policymakers were constrained from lowering the target federal funds rate further. As a result, the FOMC turned to alternative tools, such as large-scale purchases of long-term Treasury bonds and mortgage-backed securities and forward guidance about future policy actions, to provide necessary stimulus to the economy. The economy recovered slowly from the depth of the recession, and the FOMC lifted the federal funds rate target to 25–50 basis points in December 2015.

The prolonged period at the zero lower bound raises the question of how to measure the overall stance of monetary policy when constraints prevent the FOMC from using its traditional policy tool. A proper measure of the monetary policy stance can help policymakers identify the degree to which monetary policy was constrained by the ZLB and the extent to which alternative monetary policy tools were effective.

A natural way to calibrate the stance of monetary policy during the ZLB period is to look for proxy variables correlated with the federal funds rate that were not constrained by the ZLB. Longer-term interest rates such as the 10-year Treasury bond yield are good candidates. Since current and expected future short-term interest rates systematically

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influence the current level of long-term interest rates, we can construct a “shadow short-term interest rate”—in other words, the rate that observed long-term interest rates would imply if the short-term interest rate could go below the ZLB. Several estimates of the shadow short-term interest rate are based on this idea. Lombardi and Zhu; Krippner; and Wu and Xia use long-term government bond yields in their estimates of the shadow short-term interest rate.

In this article, we propose a new shadow rate that reflects both government and private-sector borrowing conditions. We separate common factors affecting all interest rates from idiosyncratic factors affecting only individual interest rates and construct the shadow short-term interest rate implied by these common factors. Fluctuations in our measure are qualitatively similar to the existing shadow rate measures. Quantitatively, however, our measure is much less negative during the ZLB period than other measures that do not separate out idiosyncratic factors in explaining the variations in observed interest rates.

Our measure of the shadow rate closely tracks the effective federal funds rate during the period before the ZLB. Not only are both interest rates highly correlated during the period before the ZLB, but their effects on macroeconomic variables are also comparable. Our analysis shows that the responses of inflation and unemployment to the shadow rate since the onset of the ZLB are qualitatively similar to those obtained during the period before the ZLB using the effective federal funds rate. Quantitatively, the response of inflation is similar, but the response of the unemployment rate is stronger after the ZLB.

Our shadow rate measure suggests that policy was tighter in the ZLB period up to September 2012 than the policy prescription from a policy rule estimated during the pre-ZLB period. After September 2012, when the third round of asset purchases (QE3) began, our shadow rate indicates that policy was easier than the policy prescription.

I. Measuring the Stance of Monetary Policy during the ZLB Period

The federal funds rate has been the primary tool of monetary policy since the early 1980s. As such, researchers typically use the effective federal funds rate to identify the stance of policy and estimate how policy-makers change the stance of policy in response to economic conditions (Bernanke and Mihov).

When the ZLB became a binding constraint in 2008, the effective federal funds rate ceased to respond to macroeconomic variables, making it impossible to identify the stance of policy through the funds rate alone. Longer-term interest rates, however, were not constrained by the ZLB. In fact, they responded to macroeconomic news in a manner consistent with their responses prior to the ZLB (Swanson and Williams). This suggests long-term interest rates may provide a way to consistently measure the stance of monetary policy on and off the ZLB.

Wu and Xia, for example, compute such a measure by backing out a potentially negative shadow federal funds rate from various long-term Treasury bond yields. Long-term Treasury bond yields are tightly linked to expectations of future short-term interest rates through a no-arbitrage relationship. For example, if long-term interest rates are higher than the risk-adjusted, expected short-term interest rates averaged over the time to maturity, investors can gain riskless profit by buying long-term bonds and selling short-term bonds.¹ The no-arbitrage assumption is that investors engage in this activity on an ongoing basis so that there is no opportunity to profit from such trades for any sustainable period of time.

However, imposing no-arbitrage restrictions on bond yields of different maturities might be restrictive, especially during the financial crisis. In such a period, investors' heightened risk aversion may hamper their ability to eliminate arbitrage opportunities through trading. In addition, by considering information from only Treasury bond yields, the Wu-Xia measure ignores private borrowing conditions that might have been influenced by unconventional monetary policies. The main goal of purchasing mortgage-backed securities, for example, was to directly ease private-sector financing conditions. Evaluating the efficacy of these unconventional monetary policies requires a shadow rate that incorporates private-sector financing conditions as well as government financing conditions.

As such, we construct a new measure of the shadow rate that includes information from various interest rates representing both government and private-sector financing conditions (details of how we construct our measure are available in the Appendix). We use common statistical factors moving various interest rates to construct a measure of the shadow rate. In this analysis, the variation in each interest rate can

be decomposed into the variation correlated with common factors and the idiosyncratic variation uncorrelated with common factors. Since we do not impose no-arbitrage restrictions, we do not directly link long-term interest rates to expectations of future short-term interest rates (as Wu and Xia do).

Table 1 lists the 12 interest rate variables we use to extract these factors: eight level variables and four spread variables. The eight level variables represent borrowing conditions for both the government and the private sector. The four spread variables represent the additional compensation that investors demand to hold assets with longer-term maturities or with credit risks.

Interest rates have trended down since the early 1980s, while interest rate spreads have been volatile. Chart 1 shows various interest rates trending down and closely following each other. Chart 2 shows that interest rate spreads are more volatile, show less co-movement, and spike during recessionary periods. Beyond the common variations in the two groups of variables, individual variables exhibit some idiosyncratic month-to-month variations. For example, since September 2012, when forward guidance about the future path of the policy rate was extended beyond a two-year horizon, the two-year Treasury note yield has occasionally moved in the opposite direction of the 10-year Treasury note (Chart 3).²

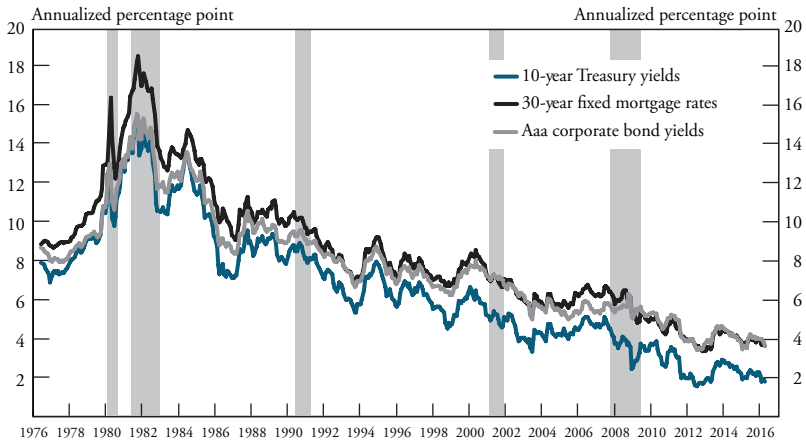
Since our main goal is to isolate the stance of monetary policy that affects co-movements of different interest rates, it is important to isolate common factors from factors specific to each individual variable. Our statistical method is similar to that of Lombardi and Zhu, but excludes Federal Reserve balance sheet variables and instead uses private-sector borrowing conditions. Since the FOMC indicated asset purchases were intended to put downward pressure on interest rates, including those related to private-sector borrowing conditions, the effect of balance sheet variables can be equally well captured by broad financial market borrowing conditions (Board of Governors of the Federal Reserve System).³

We construct the new measure of the shadow rate in two steps. First, we extract three principal components that explain most of the time-variation in eight interest rates and four interest rate spreads capturing the term premium and credit risk premium. The extracted principal components are averages of the 12 variables weighted to

Table 1
Data Description

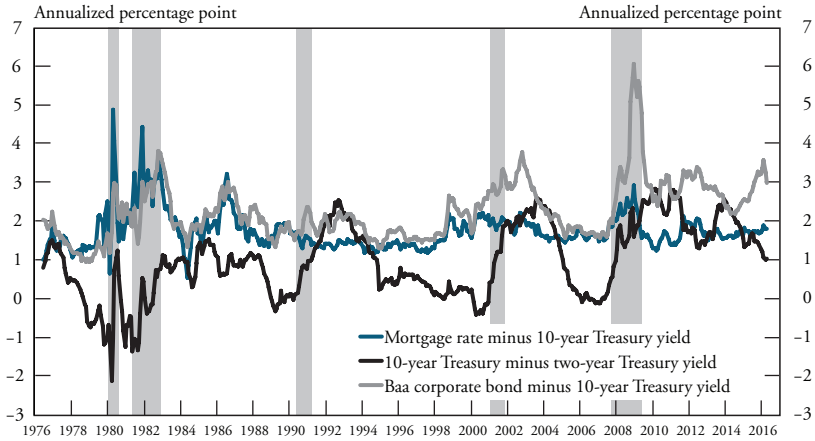
Variables	Description	Source
Level variables	Contract rates on commitments: conventional 30-year mortgages, Federal Home Loan Mortgage Corporation (percent)	Federal Reserve Board
	Two-year Treasury note yield at constant maturity (percent, annualized)	Federal Reserve Board
	Five-year Treasury note yield at constant maturity (percent, annualized)	Federal Reserve Board
	Seven-year Treasury note yield at constant maturity (percent, annualized)	Federal Reserve Board
	10-year Treasury note yield at constant maturity (percent, annualized)	Federal Reserve Board
	Bond buyer index: state/local bonds, 20-year, general obligation (percent, annualized)	Federal Reserve Board
	Moody's seasoned Aaa corporate bond yield (percent, annualized)	Federal Reserve Board
	Moody's seasoned Baa corporate bond yield (percent, annualized)	Federal Reserve Board
Spread variables	Mortgage rates and 10-year Treasury spread	Authors' calculations
	Two-year and 10-year Treasury spread	Authors' calculations
	Aaa corporate bond yield and 10-year Treasury spread	Authors' calculations
	Baa corporate bond yield and 10-year Treasury spread	Authors' calculations

Chart 1
Interest Rate Level Variables



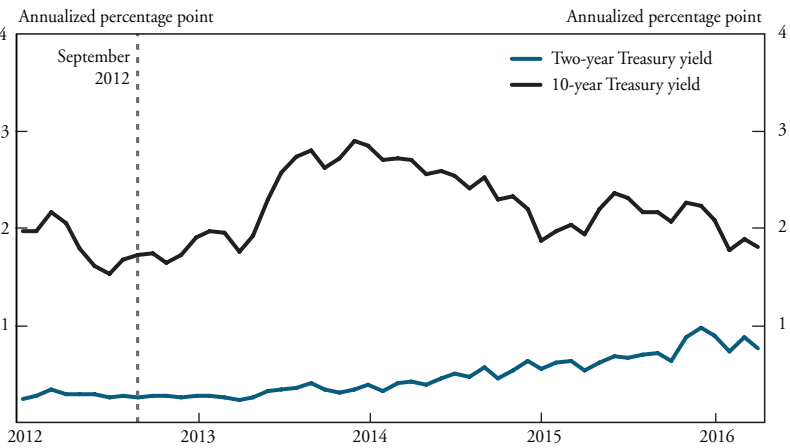
Notes: Gray bars denote NBER-defined recessions.
Source: Federal Reserve Board.

Chart 2
Interest Rate Spread Variables



Notes: Gray bars denote NBER-defined recessions.
Source: Federal Reserve Board.

Chart 3
Movements in Two-Year and 10-Year Treasury Yields



Sources: Federal Reserve Board and authors' calculations.

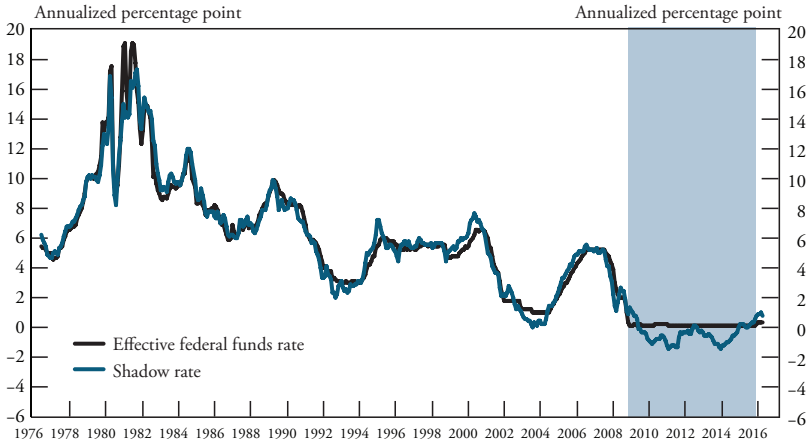
explain co-movements of different variables. Next, we regress the effective federal funds rate on a constant term and the three principal components over the pre-ZLB period. The resulting linear combination of the three factors is the shadow rate.

To be relevant as a measure of the monetary policy stance, a shadow rate must be highly correlated with the effective federal funds rate during the pre-ZLB period. Chart 4 shows that this is indeed the case for our shadow rate (black line), which closely tracks the effective federal funds rate (blue line) until 2008. The coefficient of correlation between the effective federal funds rate and our measure of the shadow rate is 0.95. Moreover, the variation in the shadow rate explains 91 percent of the variation in the effective federal funds rate for the pre-ZLB period. Similarly, Wu and Xia find that their shadow rate measure is highly correlated with the effective federal funds rate during the pre-ZLB period. Furthermore, both our measure and the Wu-Xia measure decline into negative territory for about 30 months after hitting the ZLB.

However, our estimates of the shadow rate differ noticeably from those of Wu and Xia during the ZLB period. For the most part, these differences can be explained by our relaxed no-arbitrage restriction. For example, Chart 5 shows that our shadow rate estimates start to sharply increase in August 2011 and stay at a relatively elevated level until August 2012 when the FOMC signaled an additional round of asset purchases; the Wu and Xia shadow rate, on the other hand, changes little over the same period.⁴ The reason for this difference is the substantial decline in long-term Treasury yields due to safe haven demand in the midst of fiscal uncertainty and the Eurozone crisis. While the Wu-Xia measure interprets the huge drop in the long-term Treasury bond yield mostly by the corresponding decline in the common risk factor affecting the entire yield curve, our model suggests that some of these variations were specific to long-term maturity bonds and not driven by common risk factors. The finding is consistent with our earlier discussion on the idiosyncratic variation of the two-year Treasury yield for this period. In addition, our shadow rate measure begins to increase in December 2013, when then-Federal Reserve Chairman Ben Bernanke announced the tapering of asset purchases, while the Wu and Xia measure does not begin to increase until August 2014 when expectations of the first rate hike drove up near-term Treasury note yields.⁵

Chart 4

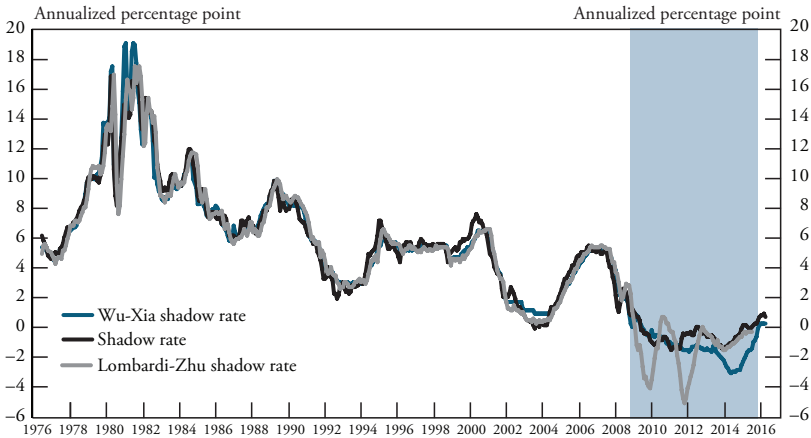
Shadow Rate Compared with the Effective Federal Funds Rate



Note: Blue bar denotes zero lower bound period.
Sources: Federal Reserve Board and authors' calculations.

Chart 5

Comparison of Different Shadow Rate Estimates



Note: Blue bar denotes zero lower bound period.
Sources: FRB Atlanta, Lombardi and Zhu, and authors' calculations.

Our findings suggest that imposing tight no-arbitrage restrictions on the entire Treasury yield curve may overstate the degree of policy accommodation during the ZLB period. In particular, policymakers must be careful in distinguishing declines in Treasury bond yields due to the movement in the common risk factor from those due to the maturity-specific risk factor.

II. Responses of Inflation and Unemployment to Monetary Policy Shocks during the ZLB Period

For us to consider our shadow rate a consistent measure of the stance of monetary policy on and off the ZLB, an unanticipated decline in the shadow rate must have the same effects on macroeconomic variables during the ZLB period as a similar decline in the effective federal funds rate does during the pre-ZLB period.

To examine whether the shadow rate preserves the systematic relationship between macroeconomic variables and the stance of monetary policy during the ZLB period, we estimate a statistical model describing the dynamic relationships of these variables using data from both before and after the onset of the ZLB. More specifically, we regress inflation (measured by core PCE inflation), the unemployment rate, and our measure of the shadow rate on a constant term and lagged values of inflation, the unemployment rate, and the shadow rate. Table 2 shows the estimated coefficients of the vector autoregression (VAR) model do not vary much over the two subsample periods, suggesting the systematic relationship between macroeconomic variables and the stance of monetary policy has not changed dramatically since the ZLB. This finding is in line with Stock and Watson, who argue the Great Recession might have resulted from the same set of shocks that buffeted the economy in the past, albeit at a larger magnitude. This finding is also in line with Bundick, who shows the monetary policy rule perceived by private-sector forecasters during the ZLB period was similar to the rule they perceived during the pre-ZLB period.

While some unconventional monetary policies were expected after the onset of the ZLB, certain components of these policies were unexpected. How, then, can we isolate the macroeconomic effect of unanticipated monetary policy actions? Our estimated VAR allows us to quantify the effect of an unanticipated monetary policy shock on

Table 2

Vector Autoregressions in the Pre- and Post-ZLB Sample Periods

	Pre-ZLB			Post-ZLB		
	Inflation	Unemployment rate	Shadow rate	Inflation	Unemployment rate	Shadow rate
Inflation (-1)	1.1837*** (-0.0931)	0.0922 (-0.0818)	-0.1243 (-0.2977)	0.7265*** (-0.1334)	-0.2393 (-0.1984)	0.3034 (-0.1845)
Inflation (-2)	-0.1329 (-0.1006)	-0.0953 (-0.0884)	0.4382 (-0.3216)			
Unemployment rate (-1)	-0.2162** (-0.0999)	1.6600*** (-0.0877)	-1.2635*** (-0.3192)	-0.0078 (-0.0251)	1.0880*** (-0.0374)	-0.1129*** (-0.0348)
Unemployment rate (-2)	0.1407 (-0.0920)	-0.7069*** (-0.0807)	1.0756*** (-0.2939)			
Shadow rate (-1)	-0.0516 (-0.0319)	0.0495* (-0.0280)	0.9006*** (-0.1019)	-0.0948 (-0.0610)	0.4920*** (-0.0908)	0.7250*** (-0.0844)
Shadow rate (-2)	0.0295 (-0.0309)	-0.0301 (-0.0272)	-0.0430 (-0.0989)			
Constant	0.3870*** (-0.1209)	0.1805* (-0.1061)	0.8700** (-0.3863)	0.4236 (-0.2905)	-0.1948 (-0.4321)	0.3081 (-0.4017)
R ²	0.9874	0.9764	0.9513	0.5635	0.9730	0.8342

*** Significant at the 1 percent level

** Significant at the 5 percent level

* Significant at the 10 percent level

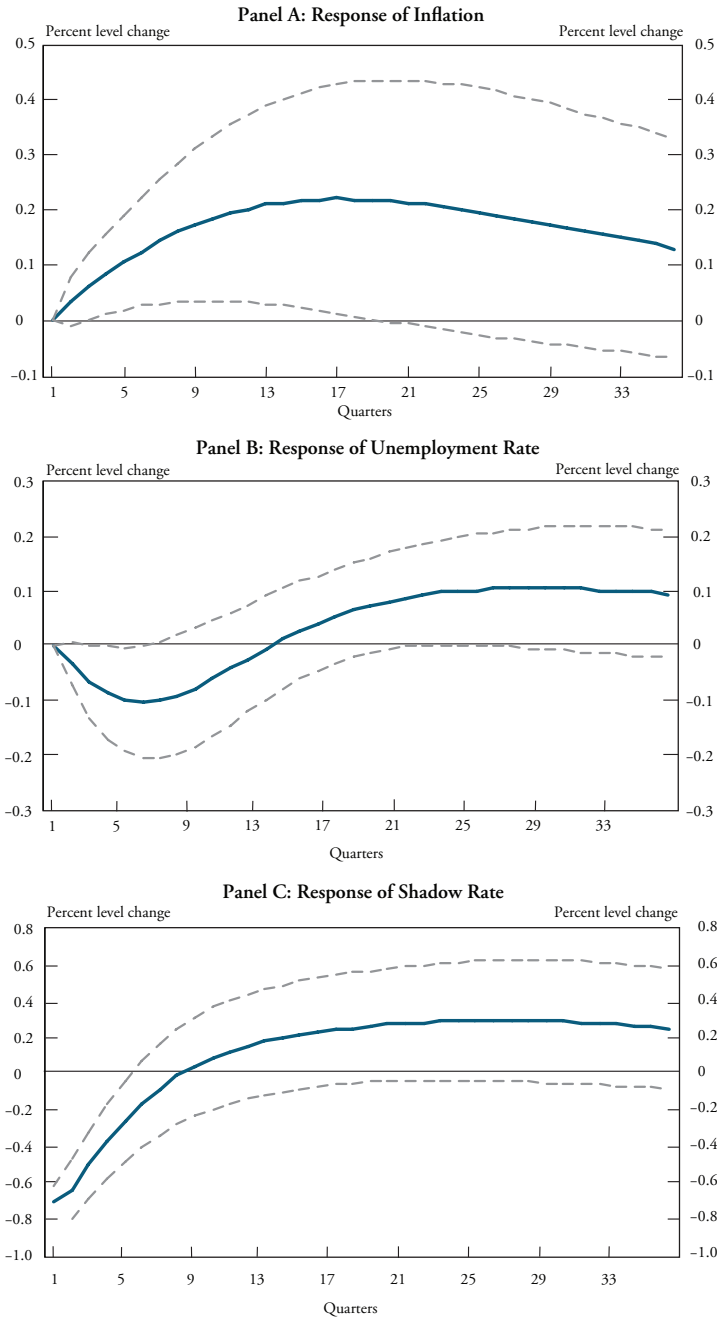
Note: Standard errors are in parentheses.

inflation and the unemployment rate with some additional assumptions. Residuals in observed variables that the VAR cannot predict are unanticipated shocks; however, because they are correlated with each other, they do not have economic interpretations by themselves. We thus impose restrictions on the contemporaneous responses of inflation and the unemployment rate to a monetary policy shock to identify them from the VAR residuals. We implement this restriction by ordering the variable for the shadow rate last in the VAR. Under this ordering, we can decompose VAR residuals in the equation for the shadow rate into monetary policy shocks and non-monetary policy shocks.

Charts 6 and 7 show that the dynamic responses of inflation and unemployment to a monetary policy shock since the ZLB period are generally similar to those during the pre-ZLB period. In addition, Chart 8 shows that the responses of macroeconomic variables to a monetary policy shock during the pre-ZLB period are essentially the same whether or not we use our shadow rate measure or the effective federal funds rate to identify the shock. A contractionary monetary policy shock decreases

Chart 6

Impulse Response to a Shock to the Shadow Rate: Pre-ZLB Sample

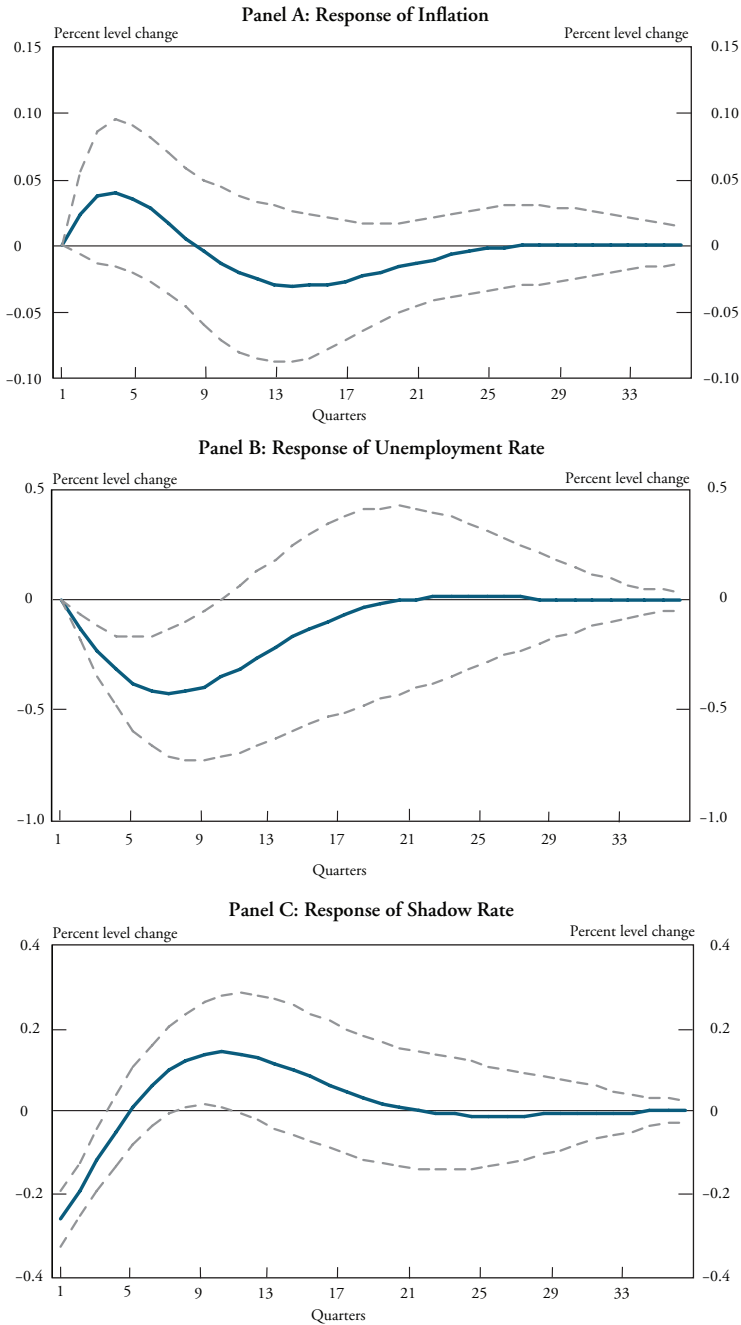


Notes: Blue line indicates the evolution of each variable when there is a one-time 1 standard deviation increase in the shadow rate. The dashed lines indicate ± 2 standard error bands around the responses.

Source: authors' calculations.

Chart 7

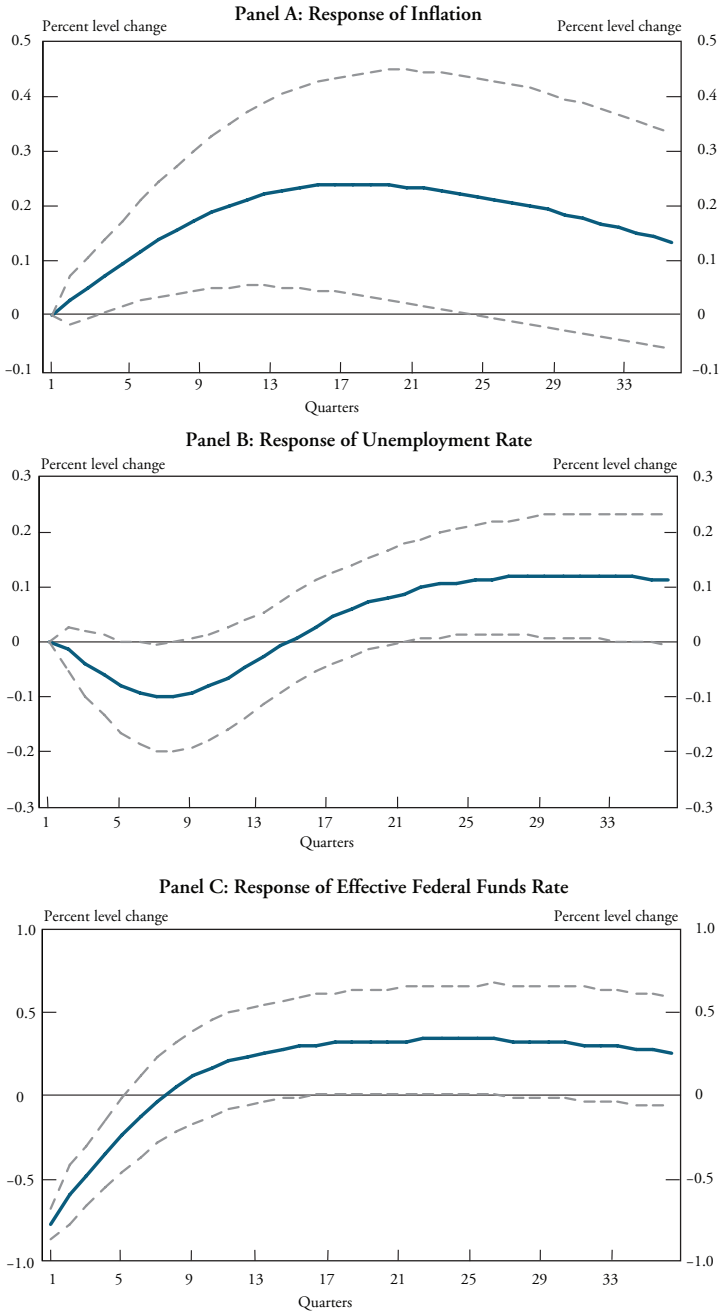
Impulse Response to a Shock to the Shadow Rate: Post-ZLB Sample



Notes: Blue line indicates the evolution of each variable when there is a one-time 1 standard deviation increase in the shadow rate. The dashed lines indicate ± 2 standard error bands around the responses.
 Source: Authors' calculations.

Chart 8

Impulse Response to a Shock to the Effective Federal Funds Rate: Pre-ZLB Sample



Notes: Blue line indicates the evolution of each variable when there is a one-time 1 standard deviation increase in the shadow rate. The dashed lines indicate ± 2 standard error bands around the responses.
Sources: Authors' calculations.

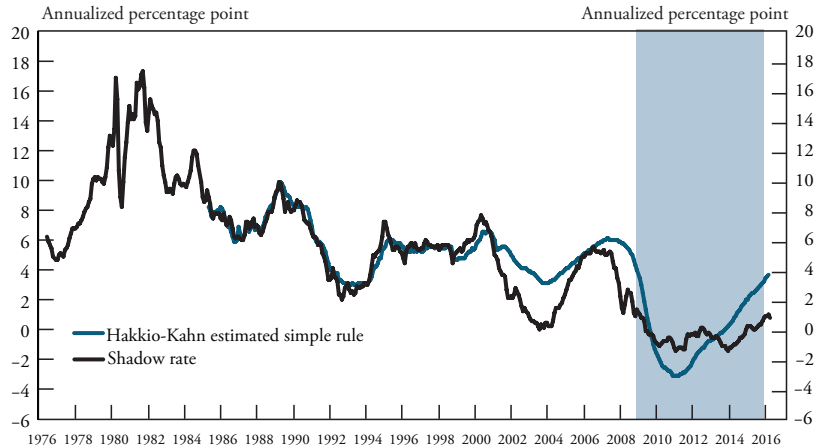
inflation but increases the unemployment rate over time. However, the timing and magnitude of the peak responses differ across the two periods. Inflation responds to the same amount of policy tightening more dramatically during the pre-ZLB period. The inflation response peaks 14 quarters after the shock, while the unemployment rate response peaks after six quarters. In contrast, after the onset of the ZLB, the responses of both inflation and the unemployment rate peak within six quarters of the shock. The inflation response is much more muted after the onset of the ZLB, while the unemployment rate response is amplified. However, uncertainty surrounding these responses increases after the onset of the ZLB due to a much shorter subsample period.⁶

Despite these differences, our VAR results suggest that an unexpected decline in our measure of the shadow rate might have provided stimulus to the economy after the onset of the ZLB. To evaluate whether the stimulus was sufficient to overcome the constraint on policy that the ZLB imposed, we compare our estimated shadow rate with the rate prescribed by a policy rule estimated from 1985 to 2001, a period of relatively favorable economic performance (Hakkio and Kahn). We plot our shadow rate measure alongside the policy prescription implied by Hakkio and Kahn, extending their analysis to a more recent period (Chart 9).

Comparing our shadow rate measure with the policy prescription from Hakkio and Kahn shows the stance of monetary policy was tighter than their policy rule prescribed from the end of QE1 (March 2010) to the beginning of QE3 (September 2012) and easier than their policy rule prescribed since then. This finding suggests that at the onset of the ZLB, policymakers were slow to recognize the depth of stimulus necessary to provide appropriate policy accommodation as prescribed by the estimated rule. Our results also show that policymakers might have compensated for this interest rate gap later by delaying liftoff and engaging in additional asset purchases. However, while this compensatory deviation might have offset the tighter-than-prescribed policy during the early period of the ZLB, the cumulative deviation was almost closed as of February 2016. This suggests compensating for past misses is no longer necessary—instead, a gradual return to more conventional monetary policy may be more appropriate.

Chart 9

Shadow Rate Compared with the Estimated Simple Rule



Note: The Hakkio-Kahn measure estimates a simple Taylor rule from April 1985 to March 2001 and uses these coefficients to forecast the prescribed interest rule forward. Blue box denotes zero lower bound period.
Sources: Hakkio and Kahn; authors' calculations.

III. Conclusion

The period in which monetary policy was constrained by the ZLB poses a challenge for researchers and policymakers who wish to measure the stance of monetary policy in a comparable way to the pre-ZLB period. Gauging the current stance of monetary policy more precisely is important for setting an appropriate path of monetary policy in the future. This article proposes a shadow interest rate that can measure the policy stance seamlessly both on and off the zero lower bound using common statistical factors that explain government and private-sector borrowing conditions. The shadow rate measure we construct is comparable to the effective federal funds rate during the pre-ZLB period and distinct from the Wu-Xia shadow rate, which imposes no-arbitrage restrictions on the Treasury yield curve. Relaxing these no-arbitrage restrictions allows us to separate out movements in long-term interest rates due to common risk factors from those due to maturity-specific, idiosyncratic factors. Our statistical analysis shows that macroeconomic variables respond to our shadow rate after the onset of the ZLB in a similar manner to how they responded to the effective federal funds rate during the pre-ZLB period.

Finally, we compare the estimated shadow rate with the policy prescription from an estimated rule based on data from the pre-ZLB period. While the shadow rate is higher than the policy prescription during the ZLB period until the beginning of QE3, continued stimulus by forward guidance and asset purchases mostly offset the past misses as of February 2016. As a result, moving the federal funds rate target closer to the policy prescription—instead of trying to compensate for past misses in monetary stimulus—may be appropriate moving forward.

Appendix

Obtaining a Shadow Rate from a No-Arbitrage Term Structure Model

Under no-arbitrage restrictions, long-term government bond yields are equal to risk-adjusted averages of short-term interest rates. Suppose that while the observed short-term interest rate (r_t) is constrained by the lower bound (\underline{r}), the shadow rate (s_t) is not constrained and driven by the same risk factors (X_t) as in the pre-lower-bound period.

$$r_t = \max\{\underline{r}, s_t\}, \quad y_{\{n,t\}} = E_t^Q \left\{ \frac{\sum_{j=1}^n s_{t+j}}{n}, \underline{r} \right\}, \quad s_t = s(X_t; \theta), \quad X_{\{t+1\}} = f(X_t, \varepsilon_{\{t+1\}}; \theta)$$

where $y_{\{n,t\}}$ stands for the government bond yield with the maturity of n -period and θ describes the vector of parameters governing the evolution of the shadow rate and risk factors. When long-term bond yields are not constrained by the lower bound and s and f are linear with respect to arguments, $y_{\{n,t\}}$ is also linear with respect to X_t . Once θ is known, we can derive the shadow rate and the model-implied yield curve in the presence of the lower-bound constraint. Krippner calibrates θ from other research to derive the yield curve for Japanese data while Wu and Xia estimate θ using Treasury bond yields data and back out the shadow rate.

Obtaining a shadow rate from a factor analysis of various interest rates

The no-arbitrage term structure models explained above impose tight restrictions on the dynamics of risk factors and the cross-section of bond yields. While these restrictions are appealing from a theoretical viewpoint, in practice it is hard to precisely model risk factors that can explain not only government bond yields but also private-sector borrowing rates. As an alternative approach to constructing the shadow rate, we rely on a statistical analysis of various interest rates.

Suppose that Y_t represents a vector of demeaned government and private-sector borrowing rates. While the dimension of Y_t may be high, most of its variation can be explained by a low-dimensional vector of statistical factors (F_t) as follows:

$$Y_t = \Lambda F_t + e_t,$$

where e_t stands for a vector of idiosyncratic components that are cross-sectionally independent. Since F_t is a purely statistical output, it does

not have any economic meaning comparable to the shadow rate in other studies. This article uses 12 variables for Y_t and extracts three principal components, F_t , out of it. We then regress the effective federal funds rate onto the constant and F_t using pre-ZLB period data (June 1976–November 2008) from the United States:

$$FFR_t = c_0 + c_1 F_t + u_t = s_t + u_t.$$

We use the estimated coefficients from this regression to obtain the measure of the shadow rate during the ZLB period. Lombardi and Zhu follow a similar factor analysis approach but use different datasets. They include short-term interest rates in during the pre-ZLB period but treat them as missing observations during the ZLB period. The authors back out missing values of short-term interest rates from the factor model by recovering the expected values conditional on the observed series. This method is conceptually similar to ours but more intensive to calculate.

Endnotes

¹In this case, investors can construct a self-financing bond portfolio with a positive payoff by shortselling short-term bonds in spot and forward markets and buying long-term bonds in spot markets.

²One explanation for their diverging paths is that the 10-year Treasury yield still reflected changing expectations of the future path of policy in response to economic news while the two-year Treasury yield became less responsive to economic news.

³“These actions [newly announced asset purchases], which together will increase the Committee’s holdings of longer-term securities by about \$85 billion each month through the end of the year, should put downward pressure on longer-term interest rates, support mortgage markets, and help to make broader financial market conditions more accommodative.”

⁴The Lombardi and Zhu shadow rate measure, which also does not impose no-arbitrage restrictions, shows a similar pattern to our measure, though the variation in their measure is somewhat larger.

⁵In addition to relaxing no-arbitrage restrictions, including private-sector borrowing conditions also makes some difference in the estimate of the shadow rate. If we obtain the shadow rate based on the pre-ZLB regression of the effective federal funds rate on the two-year and 10-year Treasury yields, this alternative shadow rate is lower than our original measure both in the beginning of the ZLB period and after liftoff. We interpret this difference as indicating that the transmission of monetary policy to private-sector borrowing conditions was attenuated in the midst of the financial crisis (late 2008–early 2009) but became stronger after late 2014 as the anticipation of a rate hike grew.

⁶Wu and Xia similarly find a much stronger response of the unemployment rate to a monetary policy shock during the ZLB period.

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