

THE FEDERAL RESERVE BANK *of* KANSAS CITY
RESEARCH WORKING PAPERS

Monetary Policy Regime Switches and Macroeconomic Dynamics

Andrew T. Foerster

June 2013; Revised November 2014

RWP 13-04

Monetary Policy Regime Switches and Macroeconomic Dynamics*

Andrew T. Foerster[†]

November 5, 2014

Abstract

This paper considers the determinacy and distributional consequences of regime switching in monetary policy. While switching in the inflation target does not affect determinacy, switches in the inflation response can cause indeterminacy. Satisfying the Taylor Principle period-by-period is neither necessary nor sufficient for determinacy when inflation responses switch; indeterminacy can arise if monetary policy responds too aggressively to inflation in the active regime. Inflation target switches primarily impact the level of inflation, whereas inflation response switches primarily impact the volatility. Expecting a switch in the inflation target has minor effects on volatility, whereas expecting a switch in the inflation response raises volatility more substantially.

Keywords: Regime switching, Taylor Rule, Inflation Targeting, Determinacy

JEL Codes: C63, E31, E52

*The views expressed herein are solely those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System. I thank the editor and two anonymous referees, as well as Francesco Bianchi, Troy Davig, Jim Nason, Chris Otrok, and seminar participants at the Federal Reserve Bank of Philadelphia, the Midwest Macroeconomics, SED, CEF, and Missouri Economics Conferences for helpful comments.

[†]Research Department, Federal Reserve Bank of Kansas City, 1 Memorial Drive, Kansas City, MO 64198, andrew.foerster@kc.frb.org.

1 Introduction

During the 1970s, the US economy experienced significant macroeconomic volatility and a relatively high average inflation rate. In the early 1980s, the Federal Reserve under Volcker raised interest rates in an attempt to reduce the average inflation rate and lower volatility. During the recent financial crisis and ensuing slow recovery, economists renewed debate about monetary policy objectives and the desirability of the Federal Reserve either relaxing its inflation response or changing its inflation target. Federal Reserve officials responded by suggesting some tolerance for inflation above its 2% target, but without changing that target. Other economists suggested a temporary increase in the inflation target to a value in the 4-6% range (Rogoff (2008), Blanchard et al. (2010), and Ball (2013)). In Japan, after more than a decade of deflation and low growth, the Bank of Japan responded in 2013 by raising its inflation target to 2% from its previous 1% inflation "goal."

In the Volcker disinflation example, a monetary policy switch possibly occurred, either to a lower inflation target, an increased willingness to fight inflation deviations from target, or both. In the recent US example, a policy switch could be to a higher inflation target, a decreased willingness to fight inflation deviations from target, or both. In the Japan example, the change in a stated inflation goal serves as an explicit switch (Romer (2013)). To the extent that policy switches occurred in the past and may occur again in the future, economic agents expect that changes can occur, and these expectations may affect macroeconomic outcomes.

This paper examines what two different monetary policy switching assumptions – changing inflation targets and inflation responses – imply for macroeconomic dynamics. It allows for discrete changes in the monetary policy rule, both in the inflation target and how strongly the monetary authority responds to inflation deviations from target, and examines the economy's behavior when neither, one, or both policy parameters switch. It studies how policy switches affect existence and uniqueness of the economy's equilibrium, and how the distributions of macroeconomic variables change depending upon which parameters switch.

Much recent research considers how monetary policy impacts macroeconomic stability, including Woodford (2003), Christiano et al. (2005), and Smets and Wouters (2007). Changes in

the conduct of monetary policy and changes in macroeconomic performance led to debate over whether monetary policy remained fixed or changed over time. Using a Markov-switching vector autoregression (MS-VAR), Sims and Zha (2006) find support for fixed monetary policy with stochastic volatility rather than switching monetary policy. In a rational expectations framework, some research supports switches in inflation targets (Schorfheide (2005)), while some supports no switching inflation target (Liu et al. (2011)). In addition, several authors (Davig and Doh (2008), Bianchi (2013), and Chib et al. (2011)) provide evidence of switches in the inflation response. While these papers only allow for one monetary policy switching type, this paper describes the differences in macroeconomic behavior generated by these different assumptions.

The different monetary policy switching types have different determinacy implications. Determinacy – the existence and uniqueness of a stable equilibrium – represents an important consideration for the conduct of monetary policy. Failure to achieve determinacy, Clarida et al. (2000) and Lubik and Schorfheide (2004) argue, explains the higher macroeconomic volatility experienced during the 1970s. However, these two papers ignore the potential for repeated policy changes and the effects of expectations. When the monetary policy rule switches over time, Davig and Leeper (2007), Farmer et al. (2009), and Cho (2011) show the determinacy properties change relative to the case when the policy rule remains fixed.

This paper makes two contributions regarding determinacy properties of models with switching parameters: first, it considers inflation target switching, and second, it considers a model with predetermined variables rather than a purely forward looking model. Investigating determinacy previously required a forward looking model (Davig and Leeper (2007), Farmer et al. (2009)); models with predetermined variables either couldn't address determinacy (Svensson and Williams (2007), Farmer et al. (2011)), or had to disregard certain solution types (Cho (2011)). This paper shows that inflation target switching does not impact determinacy. It also shows that satisfying the Taylor Principle period-by-period is neither necessary nor sufficient for determinacy when inflation responses switch, and that indeterminacy can arise if the central bank responds too aggressively to inflation in the active regime.

In addition to determinacy, this paper shows that different monetary policy switching as-

assumptions imply different macroeconomic variable distributions. Switching in the inflation target primarily affects the average inflation rate in the economy, whereas inflation response switching primarily affects inflation’s volatility. Both realizations of switches in the policy rule affect the distributions, but also expectations about future switches. Different outcomes can result even in periods when the monetary policy rule is fixed, these outcomes depend on the type of policy switch that agents expect in the future. If agents expect a switch in the inflation target, the level of inflation changes, whereas agents expecting a switch in the inflation response causes the volatility of inflation to change.

The remainder of the paper proceeds as follows. Section 2 describes a New Keynesian dynamic stochastic general equilibrium model with regime switching, and the different switching types considered. Section 3 discusses determinacy, examining the effects of different switching assumptions on the existence and uniqueness of the equilibrium. Section 4 discusses the effects of monetary policy switches on macroeconomic outcomes, considering both the long-run distributions and the role of expectations. Finally, Section 5 concludes.

2 Model with Monetary Policy Regimes

This Section presents a New Keynesian dynamic stochastic general equilibrium model where parameters governing the inflation target and the inflation response change over time. The following subsections describe the model’s several parts: households, producers, fiscal policy, the monetary authority, the stationary equilibrium, followed by the calibration and discussion of the solution method.

2.1 Households

A representative household chooses consumption of a set of differentiated goods $C_{j,t}$, hours worked H_t , and nominal bonds B_t to maximize lifetime expected discounted utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t/A_t)^{1-\tau} - 1}{1-\tau} - H_t \right), \text{ with } C_t = \left(\int_0^1 C_{j,t}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \quad (1)$$

where \mathbb{E}_0 denotes the expectations operator conditional on information at time $t = 0$. Preferences depend on the discount factor β , the degree of risk aversion τ , the elasticity of substitution η , and the technology level A_t . Households purchase bonds B_t that pay out a nominal rate R_t at $t + 1$, pay nominal lump sum taxes T_t , earn a real wage W_t , and receive real profits from firms D_t . Consequently, given a price level P_t , they face the budget constraint

$$C_t + \frac{B_t}{P_t} + \frac{T_t}{P_t} = W_t H_t + R_{t-1} \frac{B_{t-1}}{P_t} + D_t. \quad (2)$$

2.2 Producers

Intermediate goods producers, indexed by j on the unit interval, produce differentiated products $Y_{j,t}$ using hours $H_{j,t}$ according to the linear technology

$$Y_{j,t} = A_t H_{j,t}, \quad (3)$$

where total factor productivity (TFP) A_t follows a unit root process with drift ω

$$\log A_t = \omega + \log A_{t-1} + a_t, \quad (4)$$

and the disturbance a_t follows an autoregressive process

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t} \quad (5)$$

where $\varepsilon_{a,t} \sim N(0, 1)$ denotes a TFP shock.

Firms hire labor $H_{j,t}$ in a competitive market at wage W_t , and minimize labor costs subject to meeting demand at their posted price $P_{j,t}$. Firms are subject to a Calvo friction when setting prices, so a firm re-optimizes its price with probability $1 - \gamma$, and with probability γ it re-indexes its price according to steady-state inflation: $P_{j,t} = \Pi_{ss} P_{j,t-1}$. If a firm re-optimizes in period $t = 0$, it chooses P_0 to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\gamma)^t \frac{\lambda_t}{\lambda_0} \left\{ \left(\left(\frac{\Pi_{ss}^t P_0}{P_t} \right)^{1-\eta} - \left(\frac{\Pi_{ss}^t P_0}{P_t} \right)^{-\eta} mc_t \right) Y_t \right\}, \quad (6)$$

where λ_t is the household's marginal utility of consumption at time t , so $\beta^t \lambda_t / \lambda_0$ denotes the stochastic discount factor from the household discounting profits t periods into the future.

2.3 Fiscal Policy

The government purchases a fraction ζ_t of each intermediate good, $G_{j,t} = \zeta_t Y_{j,t}$, and has the same CES aggregation function as the household. The fraction of goods purchased satisfies $g_t = \frac{1}{1-\zeta_t}$, where g_t follows an autoregressive process

$$\log g_t = (1 - \rho_g) \log g_{ss} + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{g,t}, \quad (7)$$

where $\varepsilon_{g,t} \sim N(0, 1)$ denotes a government spending shock. The government collects lump-sum taxes T_t and issues bonds B_t to cover spending G_t and bond expenses $R_{t-1}B_{t-1}$:

$$\frac{T_t}{P_t} + \frac{B_t}{P_t} = G_t + R_{t-1} \frac{B_{t-1}}{P_t}. \quad (8)$$

Market clearing implies that available production of each good $Y_{j,t}$ equals consumption $C_{j,t}$ plus government spending $G_{j,t}$,

$$Y_{j,t} = C_{j,t} + G_{j,t}. \quad (9)$$

2.4 Monetary Policy

Monetary policy follows a Taylor rule, meaning nominal rates follow

$$\frac{R_t}{R_{s_t^*}^*} = \left(\frac{R_{t-1}}{R_{s_t^*}^*} \right)^{\rho_R} \left(\left(\frac{\Pi_t}{\Pi_{s_t^*}^*} \right)^{\psi_{s_t^*}^{\psi}} \left(\frac{Y_t/A_t}{\tilde{Y}_{ss}} \right)^{\phi} \right)^{1-\rho_R} \exp(\sigma_r \varepsilon_{r,t}) \quad (10)$$

where ρ_R dictates the degree of interest rate inertia, $\psi_{s_t^*}^{\psi}$ denotes the time-varying response of interest rates to the deviations of inflation Π_t from a time-varying target $\Pi_{s_t^*}^*$, and ϕ controls the constant response of interest rates to deviations of the output gap, where \tilde{Y}_{ss} indicates the steady state of detrended output. The interest rate target $R_{s_t^*}^*$ moves with the inflation target $\Pi_{s_t^*}^*$ according to $R_{s_t^*}^* = r_{ss} \Pi_{s_t^*}^*$, where r_{ss} denotes the steady state real rate. Finally, the monetary policy shock is $\varepsilon_{r,t} \sim N(0, 1)$.

The time-varying rule allows for both a changing inflation target $\Pi_{s_t^*}^*$, and changes in the inflation response $\psi_{s_t^*}^{\psi}$. These two parameters each follow independent two-state Markov processes. The Markov variable $s_t^* \in \{L, H\}$ determines the inflation target, so $\Pi_{s_t^*}^* \in \{\Pi_L^*, \Pi_H^*\}$, the

subscripts denote "low" and "high" inflation targets. The transition matrix has elements $\mathcal{Q}_{i,j} = \Pr(s_t^\pi = j | s_{t-1}^\pi = i)$:

$$\mathcal{Q} = \begin{bmatrix} \mathcal{Q}_{LL} & \mathcal{Q}_{LH} \\ \mathcal{Q}_{HL} & \mathcal{Q}_{HH} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_{LL} & 1 - \mathcal{Q}_{LL} \\ 1 - \mathcal{Q}_{HH} & \mathcal{Q}_{HH} \end{bmatrix}. \quad (11)$$

Similarly, the response of the monetary authority to the inflation gap, $\psi_{s_t^\psi}$, follows a two-state Markov process indexed by $s_t^\psi \in \{A, P\}$, so $\psi_{s_t^\psi} \in \{\psi_A, \psi_P\}$, the subscripts denote "active" and "passive" response regimes.¹ The transition matrix has elements $\mathcal{P}_{i,j} = \Pr(s_t^\psi = j | s_{t-1}^\psi = i)$:

$$\mathcal{P} = \begin{bmatrix} \mathcal{P}_{AA} & \mathcal{P}_{AP} \\ \mathcal{P}_{PA} & \mathcal{P}_{PP} \end{bmatrix} = \begin{bmatrix} \mathcal{P}_{AA} & 1 - \mathcal{P}_{AA} \\ 1 - \mathcal{P}_{PP} & \mathcal{P}_{PP} \end{bmatrix}. \quad (12)$$

Given that the two parameters have independent transitions with two regimes each, the economy has four total regimes $s_t = (s_t^\pi, s_t^\psi) \in \{L, H\} \times \{\psi_A, \psi_P\}$, with the associated transition matrix $\mathbb{P} = \mathcal{Q} \otimes \mathcal{P}$.

2.5 Stationary Equilibrium

The equilibrium consists of the first order conditions for the household and firms, the monetary authority's rule, the market clearing and aggregation conditions, and the exogenous laws of motion. Since total factor productivity follows a unit root process (4), the model has a non-stationary equilibrium. In terms of the de-trended variables $\tilde{C}_t = C_t/A_t$, $\tilde{Y}_t = Y_t/A_t$, $\tilde{W}_t = W_t/A_t$, and $\tilde{G}_t = G_t/A_t$, the economy has a stationary equilibrium and unique steady state.

2.6 Parameterization

Having established the equilibrium, now consider the parameterization and alternative regime switching models. This paper considers four alternative models nested by the framework de-

¹Typically the terms "active" and "passive" imply values greater than or less than one, respectively. Here, "active" simply refers to regimes with the stronger response and "passive" refers to a regime with the weaker response without implying values greater than or less than unity.

Table 1: Preferences, Production, and Technology Parameterization

Parameter	Description	Value
β	Discount Factor	0.9987
τ	Coefficient of Relative Risk Aversion	2.5
η	Elasticity of Substitution	10
ω	Growth Rate of TFP	0.0039
γ	Probability of No Optimization of Prices	0.66
g_{ss}	Steady State Government Spending	1.25
ρ_a	Persistence of TFP	0.80
ρ_g	Persistence of Government Spending	0.90
σ_a	Std Dev of TFP Shock	0.003
σ_g	Std Dev of Government Spending Shock	0.012
σ_r	Std Dev of Monetary Policy Shock	0.002

scribed above, which vary only in the transition matrices \mathcal{Q} and \mathcal{P} . Table 1 shows the parameters that remain fixed across models. These parameters governing preferences, production, and technology come from the estimates in Schorfheide (2005), which presents estimates of a similar model with only switching in the inflation target.

The results in Sections 3 and 4 discuss the implications of a variety of monetary policy parameterizations. However, Table 2 shows a benchmark set of parameters for monetary policy broadly consistent with estimates found in several papers. These papers typically take a "one-at-a-time" approach, and don't consider both sources of policy switching considered in this paper. Schorfheide (2005) and Liu et al. (2011) present estimates of inflation target regimes, but hold other monetary policy parameters constant. The choice of an annualized inflation target of 2% in the low regime is similar to both estimates, while the high target of 5% is slightly higher than that estimated by Liu et al. (2011) and slightly lower than that in Schorfheide (2005), and is in the 4%-6% range advocated by Rogoff (2008), Blanchard et al. (2010), and Ball (2013). The inflation response parameters of 2.45 in the active regime and 0.95 in the passive regime

Table 2: Benchmark Monetary Policy Parameterization

Parameter	Description	Value
ψ_A	Inflation Response, Active Regime	2.45
ψ_P	Inflation Response, Passive Regime	0.95
Π_L^*	Inflation Target (Annualized), Low Target Regime	2.00
Π_H^*	Inflation Target (Annualized), High Target Regime	5.00
ρ_r	Interest Rate Smoothing	0.70
ϕ	Output Gap Response	0.25

are based upon that of Bianchi (2013) and are similar to those in Davig and Doh (2008). The interest rate smoothing parameter of 0.7 and output gap response parameter of 0.25 are in line with estimates by Schorfheide (2005) and Bianchi (2013), and Davig and Doh (2008) and Bianchi (2013), respectively.

Given regime switching among four total regimes – two target and two response regimes – this paper considers four alternative models nested by the framework described above. The nesting of these alternative models depends upon the transition matrix and choosing probabilities that prevent certain regimes from occurring. The four probabilities $\{\mathcal{Q}_{LL}, \mathcal{Q}_{HH}, \mathcal{P}_{AA}, \mathcal{P}_{PP}\}$ dictate which regimes can occur. Table 3 lists the four models and the associated probabilities. In line with the estimates of Schorfheide (2005), Davig and Doh (2008), and Bianchi (2013), when switching is allowed, the each of the sources of policy switching stay the same with probability 0.95, which implies an expected duration of each regime of 20 quarters.

The first model, the "No Switching" model, has monetary policy always active and the inflation target always low. In the second model, the "Response Switching" model, the low inflation target always prevails, but the inflation response switches between active and passive. In the third model, the "Target Switching" model, the inflation target switches between high and low, but with a permanently active inflation response. Finally, the fourth model, the "Full Switching" model, has the inflation target switching between high and low, and the inflation

Table 3: Alternative Models and Transition Probabilities

Model	\mathcal{Q}_{LL}	\mathcal{Q}_{HH}	\mathcal{P}_{AA}	\mathcal{P}_{PP}
No Switching	1	0	1	0
Response Switching	1	0	0.95	0.95
Target Switching	0.95	0.95	1	0
Full Switching	0.95	0.95	0.95	0.95

response switching between active and passive.²

2.7 Solution Method

The presence of regime switching in the inflation target as well as the inflation response parameter introduces a discreteness in the model that makes typical linearization or perturbation techniques infeasible. This paper uses the perturbation method developed by Foerster et al. (2014), which allows for general regime switching environments such as the one developed here. Perturbation allows for checking the existence and uniqueness of the approximated solution; Section 3 uses this result in the discussion of determinacy. In addition, perturbation enables second-order approximations, which improve accuracy and break certainty equivalence. The simulation results in Section 4 use second-order approximations to the log policy functions.

In the presence of inflation target switching, the concept of steady state differs slightly from that in a standard model. In the framework presented above, the steady state inflation level Π_{ss} equals the steady state inflation target Π_{ss}^* , which in turn equals the mean of the ergodic distribution of $\Pi_{s_t^*}$ implied by the transition matrix \mathcal{Q} .

²These models each capture different possible explanations for high inflation, such as that in the 1970s. The target switching model implicitly suggests that high inflation was due to a higher inflation target, the response switching model suggests it was due to a passive inflation response, and the full switching model suggests both factors could have been at play.

3 Monetary Policy Switching and Determinacy

This Section discusses determinacy – existence and uniqueness of a stable minimum state variable (MSV) solution – of the model described in Section 2. As mentioned previously, Davig and Leeper (2007), Farmer et al. (2009), and Cho (2011) discuss the determinacy properties of models with switching inflation responses. This paper makes two contributions to this literature. First, it allows for switching in both inflation targets and inflation responses, and examines the implications for determinacy under these cases. Second, it considers determinacy from a MSV perspective in the presence of predetermined variables. As noted by Farmer et al. (2009), with regime switching, determinacy in the class of MSV solutions does not imply determinacy in a wider class of solutions; determinacy in this full set of solutions can be characterized in a forward-looking model. Farmer et al. (2011) consider MSV solutions, but cannot deal with determinacy in this class of solutions. Cho (2011) considers non-MSV solutions in a model with predetermined variables, but a solution refinement excludes multiple MSV solutions.

Using the solution method developed by Foerster et al. (2014), this paper characterizes determinacy of MSV solutions with predetermined variables. Given a determinate MSV solution, other stable non-MSV solutions may exist, and characterizing a full set of determinacy conditions in the presence of regime switching remains an interesting line of research. However, given that estimation typically uses MSV solutions, focusing on determinacy in this restricted class represents an important step.

3.1 Mean Square Stability

In order to characterize a set of parameters as producing a determinate equilibrium, there must exist a unique stable solution to the first-order approximation to the decision rules characterizing optimal behavior. Regime switching models allow for several potential stability definitions, unlike the case without switching where definitions coincide. Following Costa et al. (2005), Farmer et al. (2009), Cho (2011), and Foerster et al. (2014), among others, the determinacy results in this paper use the concept of mean square stability (MSS). The first-order solution to

a general regime switching DSGE model has a state equation of the form

$$x_t = H_x(s_t)x_{t-1} + H_\varepsilon(s_t)\varepsilon_t + H_\chi(s_t), \quad (13)$$

where x_{t-1} denotes predetermined variables for time t . Mean square stability implies the process (13) has finite first and second moments in expectation:

$$\lim_{j \rightarrow \infty} \mathbb{E}_t[x_{t+j}] = \bar{x} \text{ and } \lim_{j \rightarrow \infty} \mathbb{E}_t[x_{t+j}x'_{t+j}] = \Sigma. \quad (14)$$

Importantly, MSS allows unbounded realizations of the paths for x_t , so long as the process has finite first and second moments in expectation.

The dependence of the coefficient $H_x(s_t)$ in equation (13) on the regime s_t implies the standard stability condition – that H_x has eigenvalues in the unit circle – breaks down. Costa et al. (2005) show the process (13) satisfies mean square stability if and only if the following matrix has all eigenvalues inside the unit circle:

$$\mathbb{T} = (\mathbb{P} \otimes I_{n_x^2}) \text{diag}[H_x(s_t) \otimes H_x(s_t)], \quad (15)$$

where $n_x = \dim(x_t)$. The fact that \mathbb{T} depends upon the transition matrix \mathbb{P} and the coefficients in the state equation $H_x(s_t)$ makes analytic characterizations of determinacy conditions nearly impossible to obtain. Consequently, the following results use numerical search to characterize regions with a determinate solution.

The fact that MSS allows unbounded realizations so long as they have finite first and second moments leads to important determinacy implications. An alternative stability concept used by Davig and Leeper (2007), bounded stability, requires bounded paths and hence eliminates temporarily explosive paths. Bounded stability has no known tractable check in the current setup, which is an advantage of using MSS.³ In order to highlight the importance of explosive paths, the following results introduce a stability definition, Both Regimes Stable (BRS). This stability concept requires $H_x(s_t)$ to have eigenvalues inside the unit circle for all s_t . BRS is a different stability concept than bounded stability, but it is related. In particular, BRS is a

³As Farmer et al. (2009) highlight, bounded stability requires all possible permutations $H_x(s_T)H_x(s_{T-1}) \cdots H_x(s_0)$ to have all eigenvalues less than one in modulus, for all T .

necessary but not sufficient condition for bounded stability. To see this relationship, note that if BRS fails, then some $H_x(s_t)$ has eigenvalues outside the unit circle, and if that regime occurs repeatedly the path of x_t is unbounded, violating bounded stability. If a set of parameters produces indeterminacy under MSS but determinacy under BRS, there is a solution with an explosive regime but that regime occurs with low enough expected duration to keep the first and second moments finite in expectation.

The following subsections consider the implications of switching for determinacy. Section 3.2 demonstrates that inflation target switching is irrelevant for determinacy. In contrast, inflation response switching matters for determinacy, as Section 3.3 shows.

3.2 Irrelevance of Inflation Target Switching

First consider determinacy in the presence of inflation target switching. As noted by Woodford (2003), in standard New Keynesian models such as the No Switching model, the well known Taylor Principle holds, where responding more than one-for-one to inflation deviations from target guarantees determinacy. Further, the inflation target does not affect determinacy, since price indexation to steady state inflation undoes any effects of positive trend inflation. As a result, a model with positive trend inflation and one with zero trend inflation have the same determinacy regions.

In models with inflation target switching, the presence of price indexation to steady state inflation produces a similar result: determinacy does not depend on the target level of inflation, a result that holds with both a constant and a switching target. The following Theorem formally states this fact.

Theorem 1 *In the model described in Section 2, the steady state inflation target level Π_{ss}^* and the switching inflation targets Π_L^* and Π_H^* do not affect determinacy.*

Proof. See Appendix. ■

Theorem 1 implies that, among the switching parameters, only the inflation responses ψ_P and ψ_A and their transition matrix \mathcal{P} affect determinacy. The No Switching model has the

same determinacy properties as the Target Switching model, regardless of the inflation targets. In addition, the Response Switching and the Full Switching models have identical determinacy properties, regardless of the inflation targets. Consequently, focusing on the case where only the inflation response switches sufficiently characterizes determinacy.⁴

3.3 Importance of Inflation Response Switching

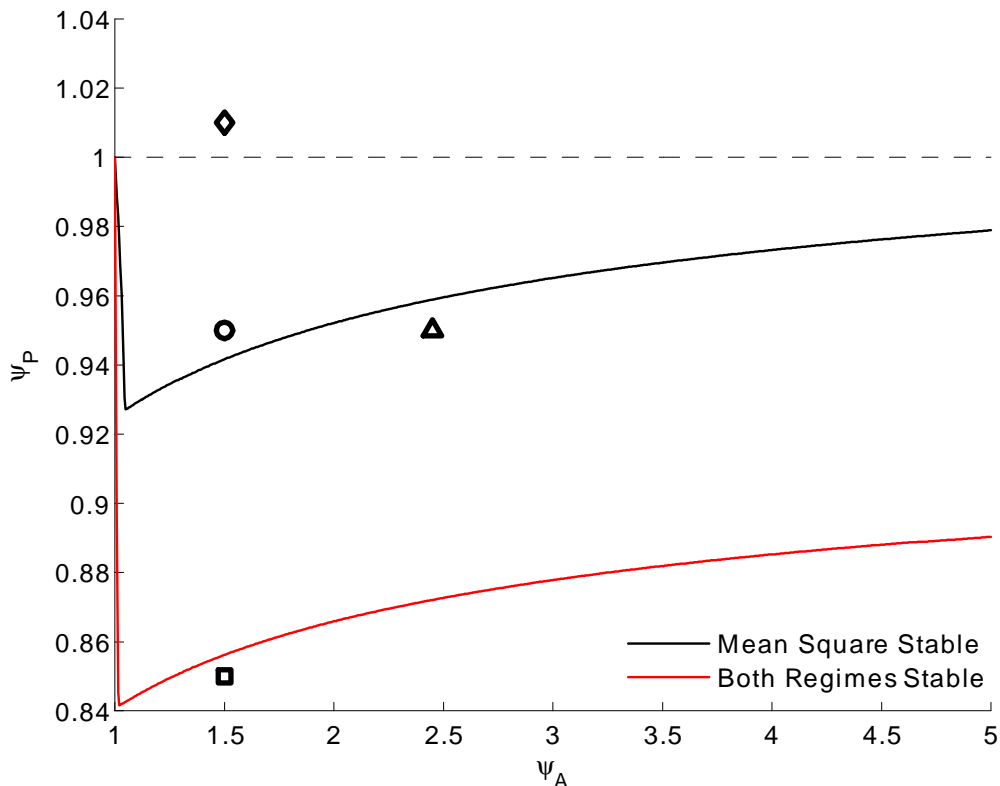
In the presence of regime switching, the transition matrix between regimes \mathbb{P} and the response parameters ψ_A and ψ_P affect determinacy. As a result, the Taylor Principle does not need to hold every period. Davig and Leeper (2007) argue for a "Long-Run Taylor Principle," where the economy can move through regimes that would imply indeterminacy if considered in isolation, but the economy overall can have a determinate solution so long as the regimes implying determinacy offset those implying indeterminacy.

Figure 1 shows boundary regions for determinacy as the inflation response parameters change given $\rho_r = 0.7$, $\phi = 0$, and $\mathcal{P}_{AA} = \mathcal{P}_{PP} = 0.95$. The plot depicts, given an inflation response for the active regime ψ_A , the minimum value of ψ_P that yields a determinate solution. Parameter combinations above a line generate a determinate equilibrium, whereas below a line imply indeterminacy. The standard Taylor Principle holds for the No Switching model, if the passive regime lasted forever determinacy requires $\psi_P > 1$. The boundary for MSS confirms the Long-Run Taylor Principle idea, since values of $\psi_A > 1$ allow some $\psi_P < 1$ while still preserving determinacy. The plot also shows the determinacy line for the BRS alternative stability concept. Points in between the MSS and BRS lines have multiple MSS solutions, but one BRS solution, implying that one of the MSS solutions has an explosive regime.

First, consider the point $(\psi_A, \psi_P) = (1.5, 1.01)$, given by the diamond in Figure 1, which has both regimes satisfying the Taylor Principle. In this case, if the passive regime occurred in isolation, since $\psi_P > 1$, a determinate equilibrium would result. With regime switching, only

⁴Theorem 1 depends on full price indexation to steady state inflation along with a response to an output gap defined as deviations from steady state. For implications of other assumptions in environments without regime switching, see, for example, Woodford (2003), Ascari and Ropele (2009), and Coibion and Gorodnichenko (2011).

Figure 1: Determinacy Regions, Mean Square Stability



one solution satisfies MSS, and it also satisfies BRS, so the economy returns to steady state after shocks in both the active and passive regimes.

Now, consider a move to a weaker passive regime, $(\psi_A, \psi_P) = (1.5, 0.95)$, given by the circle in Figure 1, which has determinacy under both MSS and BRS. If the passive regime occurred in isolation, the Taylor Principle dictates that since $\psi_P < 1$, indeterminacy would result. With regime switching, consistent with the Long-Run Taylor Principle, a unique equilibrium occurs even though the interest rate responds less than one-for-one to inflation in the passive regime since the probability of a switch to an active regime aligns expectations and keeps inflation from exploding. In this case, there is only one solution satisfying MSS. In addition, this solution is BRS, so the economy returns to steady state after shocks in both the active and passive regimes.

If the passive regime becomes even weaker, moving to the point $(\psi_A, \psi_P) = (1.5, 0.85)$ given by the square in Figure 1, indeterminacy arises under both MSS and BRS. In this case, there

are two MSS solutions, each also satisfying BRS, so both the active and passive regimes are stable. While BRS does not necessarily guarantee bounded stability, this indeterminacy result is consistent with typical indeterminacy in models without regime switching: after shocks, multiple paths returning the economy to steady state are possible, leading the way for sunspots.

Now, if instead the active regime becomes stronger, moving to the point $(\psi_A, \psi_P) = (2.45, 0.95)$ given by the triangle in Figure 1, indeterminacy arises under MSS, but not under BRS. In this case, the economy has two solutions that satisfy MSS, only one of which satisfies BRS. In the solution that satisfies BRS, the economy returns to steady state after shocks in both regimes. The solution that does not satisfy BRS has an explosive passive regime, where shocks generate movements away from steady state. If the economy happened to produce a sequence of regimes where the passive regime occurred forever, the economy would be unbounded, clearly a violation of the bounded stability concept. In this case, the added strength of the active regime keeps the first and second moments finite in expectation, as arbitrarily long sequences of passive regimes occurs with correspondingly low probabilities.

As a result, having a strong active regime can produce indeterminacy by enabling explosive paths in the passive regime while maintaining MSS. In these circumstances, having a weaker active response to inflation could actually produce determinacy, since a switch from passive to active in that case would not necessarily have a strong enough response to reign in explosive behavior.⁵ While the exact determinacy region depends on the parameterization in Table 1, deviations from this calibration produce qualitatively similar regions. However, the determinacy regions are highly sensitive to the values in Tables 2 and 3, including the transition probabilities,

⁵The fact that high inflation responses can cause indeterminacy is not new in the literature with no regime switching. For example, Carlstrom and Fuerst (2001) show that, in a standard NK model with sticky prices, determinacy of equilibrium puts both a lower bound and an upper bound on the aggressiveness of the interest rate responses to inflation. De Fiore and Liu (2005) obtain a similar result in a small open economy. In these cases, different determinacy regions depend upon different assumptions on preferences or technology. In the model considered in this paper, it is not a feature of the No Switching model that higher inflation responses can cause indeterminacy. Instead, the interaction of the regimes along with a too strong active regime generates indeterminacy behavior not seen in the absence of switching.

degree of interest rate smoothing, and response to the output gap. As the following subsections highlight, these other parameters affect determinacy regions primarily through whether they eliminate or support explosive passive regimes that are still MSS.

3.3.1 Indeterminacy and Transition Probabilities

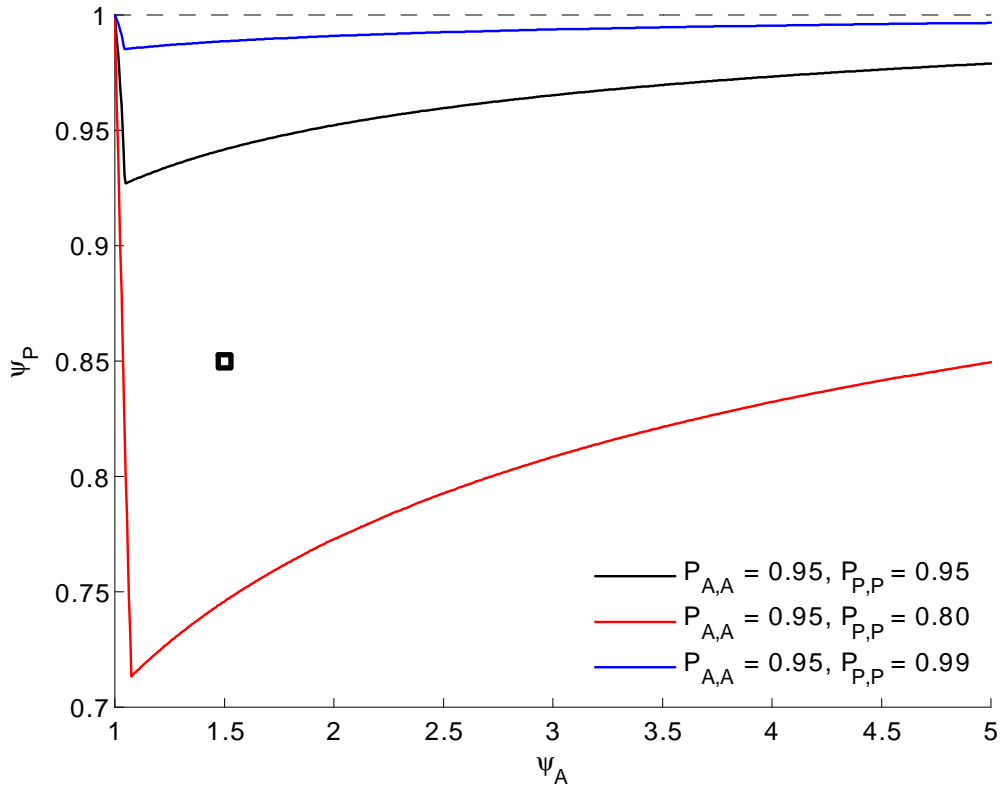
How relatively strong the active and passive regimes can be while still producing determinacy depends critically on the transition probabilities. Davig and Leeper (2007) show the passive regime can be weak if it has low expected duration. Figure 2 shows this principle extends to a model with interest rate inertia and MSS as a stability concept. Figure 2 shows the determinacy boundary of the previously considered $\mathcal{P}_{PP} = 0.95$, as well as for $\mathcal{P}_{PP} = 0.99$ and $\mathcal{P}_{PP} = 0.80$, all while $\mathcal{P}_{AA} = 0.95$, $\rho_r = 0.7$, and $\phi = 0$. When $\mathcal{P}_{PP} = 0.99$, the passive regime has a long expected duration, and so the response ψ_P needs to be relatively strong to ensure determinacy. When $\mathcal{P}_{PP} = 0.80$, the passive regime has shorter expected duration, and the response ψ_P can be weaker than when $\mathcal{P}_{PP} = 0.95$ for a given ψ_A , as seen in the downward shift in the determinacy boundary.

The point $(\psi_A, \psi_P) = (1.5, 0.85)$ given by the square in Figure 2, leads to indeterminacy when $\mathcal{P}_{PP} = 0.95$ under both MSS and BRS, as seen out in Figure 1. When the passive regime has lower expected duration, $\mathcal{P}_{PP} = 0.80$, the economy leads to determinacy. As in the Long Run Taylor Principle, the low persistence of the passive regime helps coordinate expectations on a single path back to steady state, leading to a determinate solution.

3.3.2 Indeterminacy and Interest Rate Smoothing

A second factor that affects determinacy is the interest rate smoothing parameter ρ_r . Davig and Leeper (2007) consider a model without interest rate inertia, $\rho_r = 0$, and determinacy depends on the transition matrix \mathcal{P} the responses in each regime ψ_A and ψ_P . With $\rho_r > 0$, the dependence of policy on the previous nominal rate potentially exacerbates the dynamics induced by regime switching. When interest rates depend only on contemporaneous variables, switches in inflation responses take hold quickly, since the nominal rate is unencumbered by its history.

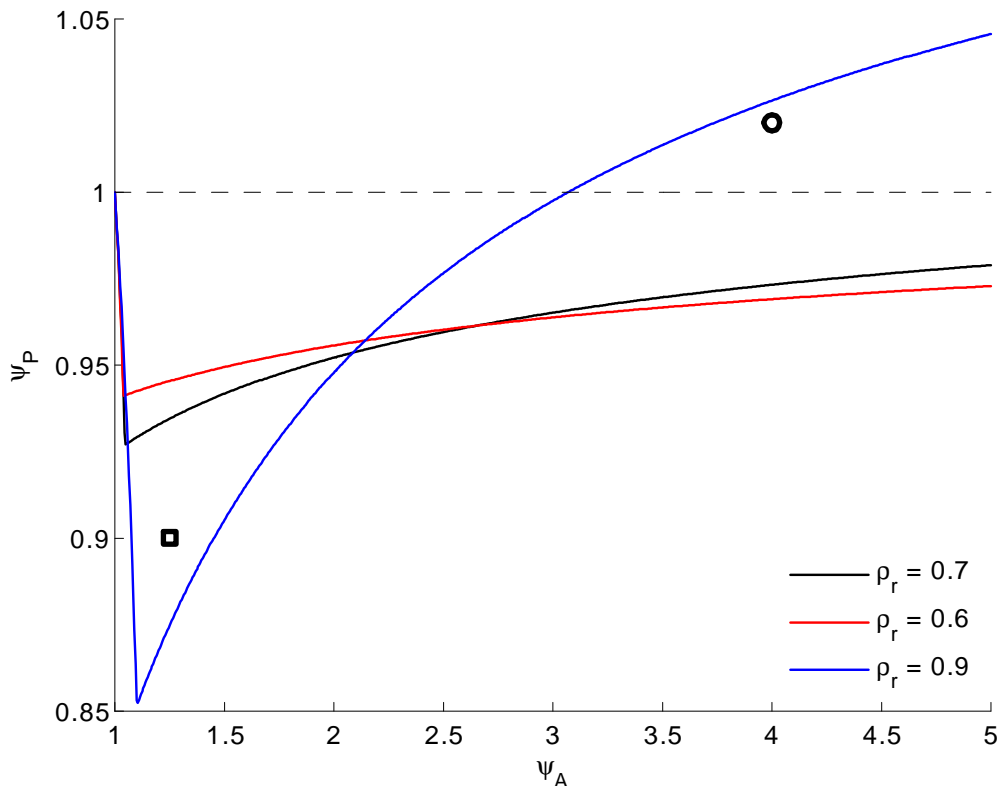
Figure 2: Determinacy Regions, Changing Probabilities



With inertia in nominal rates, changes in the nominal rate occur much more slowly, so changing the smoothing parameter affects the distribution across future nominal rates, and hence what produces a determinate solution.

Figure 3 shows the shape of the determinacy boundary depends significantly on the interest rate smoothing parameter. As ρ_r increases from 0.6 to 0.7 to 0.9, given $\phi = 0$ and $\mathcal{P}_{AA} = \mathcal{P}_{PP} = 0.95$, the slope of the boundary becomes more pronounced, expanding the determinacy region for lower values of ψ_A and diminishing the region for higher values. The reason is that with higher smoothing, deviations of the interest rate from target become more persistent, exacerbating the effects of regime switching. In particular, when ψ_A is low, a higher degree of smoothing makes explosive regimes even more explosive, so they no longer satisfy MSS, expanding the determinacy region. When ψ_A is high, a higher degree of smoothing makes explosive regimes less explosive so they satisfy MSS, restricting the determinacy region.

Figure 3: Determinacy Regions, Varying Interest Rate Smoothing



To highlight how the determinacy region is expanded for low values of ψ_A , consider $(\psi_A, \psi_P) = (1.25, 0.90)$, given by the square in Figure 3. As seen in Figure 1, the economy has two MSS solutions when $\rho_r = 0.7$, one of which is explosive when in the passive regime. Under this solution where the passive regime is explosive, the moderate degree of interest rate smoothing helps maintain MSS, since a switch from the passive to the active regime leads to interest rates responding relatively quickly. The result is that expectations about a switch from the passive regime to an active regime that responds quickly help keep inflation from exploding too fast in the passive regime. When $\rho_r = 0.9$, on the other hand, if the passive regime is experiencing explosive inflation, a switch to an active regime that responds with a lot of inertia takes much longer for interest rates to rise. These slowly rising interest rates do not contain increasing inflation, so expectations in the passive regime does not keep inflation from rising too quickly, and the explosive solution no longer maintains stability under MSS. In other words, when inter-

est rates have higher degrees of inertia, explosive passive regimes become even more unstable, thereby eliminating solutions under the MSS concept and expanding the region of determinacy.

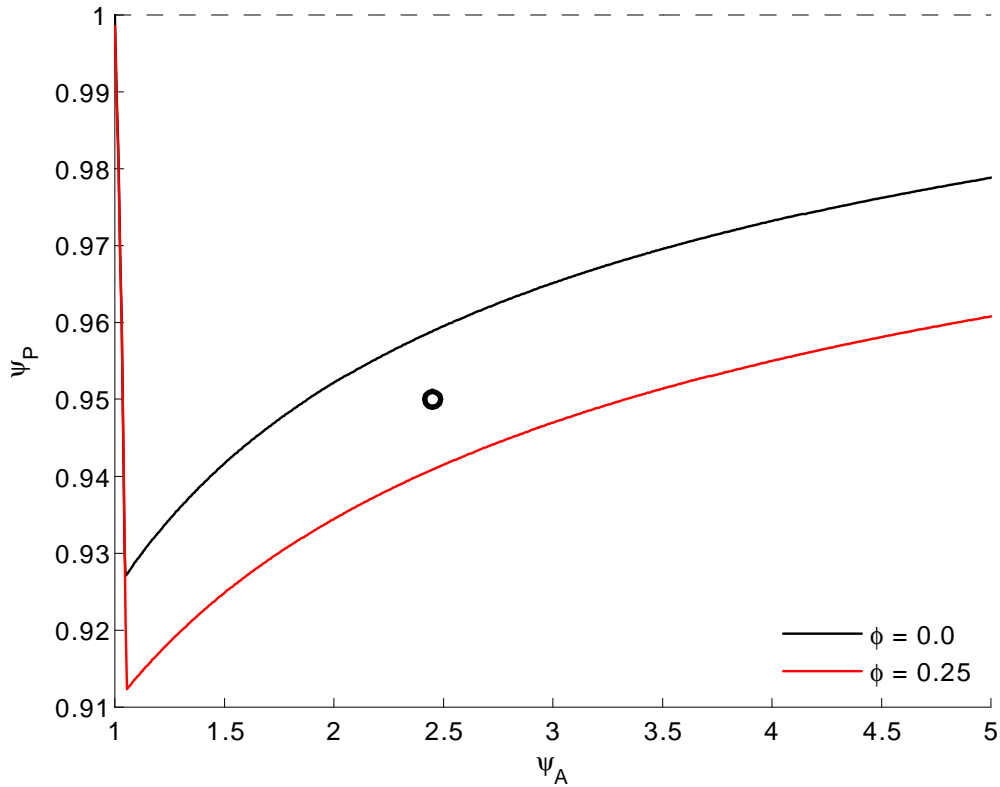
The same mechanism ends up restricting the determinacy region for high values of ψ_A . Consider $(\psi_A, \psi_P) = (4.0, 1.02)$, which is given by the circle in Figure 3. Under these parameters, both regimes would be stable when considered in isolation. A passive regime with $\psi_P = 1.02$ is large enough to guarantee determinacy if considered in isolation, but there is a second, slightly explosive solution where the economy gradually moves away from steady state. When considering a model without regime switching, this solution is eliminated under standard definitions of stability. With regime switching and a smoothing parameter of $\rho_r = 0.7$, the active regime doesn't alter the implications of the explosive solution as it is not part of a MSS solution. However, when $\rho_r = 0.9$, a slightly explosive passive regime becomes possible under MSS, since a switch to a very strong active regime leads to only slow interest rate increases. The smooth interest rate increases keep inflation from exploding too quickly, leading to a second MSS solution and indeterminacy. Consequently, when interest rates have a large smoothing component, explosive passive regimes that are still MSS become possible for large ψ_A , increasing the region of indeterminacy.

3.3.3 Indeterminacy and Output Gap Response

A third factor that impacts the determinacy region is the output gap response ϕ . In the No Switching model, the output gap response does not have an impact. As with interest rate smoothing, regime switching affects the distribution of the output gap, and hence interest rates and determinacy. Figure 4 shows the determinacy boundaries when $\phi = 0$ as previously considered, as well as the benchmark parameterization value of $\phi = 0.25$. Having a positive output gap response shifts the boundary downward, increasing the determinacy region.

The point $(\psi_A, \psi_P) = (2.45, 0.95)$, denoted by a circle in Figure 4 generates indeterminacy with no output gap response, but determinacy with an output gap response. As discussed, the indeterminacy when $\phi = 0$ depends on a second solution with an explosive passive regime. When $\phi = 0.25$, as inflation increases, the output gap becomes increasingly negative, and the response

Figure 4: Determinacy Regions, Effect of Output Gap Response



to the output gap mitigates the increase in interest rates that would otherwise occur. As a result, interest rates increase less, providing less of a damper on inflation, making the passive regime even more explosive and no longer MSS. As the output gap response increases, the interest rate becomes slower to tame explosive passive regimes, making it harder to support a second MSS solution with an explosive regime. The result of this effect is that the indeterminacy area shrinks as ϕ increases.

4 Monetary Switching and Macroeconomic Outcomes

After discussing how different monetary policy switching assumptions affect determinacy, this Section examines how the different assumptions impact economic outcomes. Monetary policy switches affect outcomes through two channels: the realization of policy regimes and expecta-

tions about future regimes. In order to disentangle effects of each channel, this section presents simulations of the ergodic long-run distributions of each economy and counterfactual simulations where the regime remains fixed. In the ergodic long-run distributions, differences between the models reflect both channels, as monetary policy switches among the regimes and agents' behavior reflects their expectations about future regimes. In the counterfactual distributions, agents expect switching but monetary policy remains fixed in the low target, active response regime. Consequently, in the counterfactuals, differences between the models are generated purely by differences in agents' expectations about possible future regimes that never materialize. Leeper and Zha (2003) call these differences expectation formation effects, and Liu et al. (2009) show these effects vary across regimes.

Using the perturbation solution method developed by Foerster et al. (2014), the simulations use second-order approximations, which improve accuracy and break certainty equivalence. Without regime switching, first-order approximations satisfy certainty equivalence, as pointed out by Schmitt-Grohe and Uribe (2004), meaning the scale of the exogenous shocks does not impact the solution; second-order approximations break certainty equivalence, so the approximated decision rules reflect the shocks' variances. With regime switching, Foerster et al. (2014) show first-order approximations can break certainty equivalence if the switching affects the steady state of the economy, such as with inflation target switching; other switching that doesn't affect the steady state, such as inflation response switching, requires a second-order approximation to break certainty equivalence. The lack of certainty equivalence allows switching in the inflation response to have level effects.

For each model, the following results consider the means and standard deviations across models and policy parameters for annualized inflation, the annualized nominal rate, and normalized output. Given the two nonlinearity types – the regime switching and the second-order approximation – characterizing these distributions requires simulation.⁶ For each model, the

⁶Bianchi (2012) shows that, with regime switching but not a second-order approximation, closed-form expressions for first and second moments of distributions follows from the first-order approximation. Andreasen et al. (2013) show that, with up to a third-order approximation but not regime switching, the first and second moments of distributions also have closed-form expressions.

distributions use 50000 simulations each lasting 10000 periods; a 1000 period burn-in eliminates any impact from initial conditions. The long-run distributions simulations allow regime switches to occur according to the relevant transition matrices. The counterfactual distributions do not allow switches: agents expect switching but switches do not occur along the simulated paths *ex-post*, meaning expectations produce different outcomes despite a fixed policy rule.

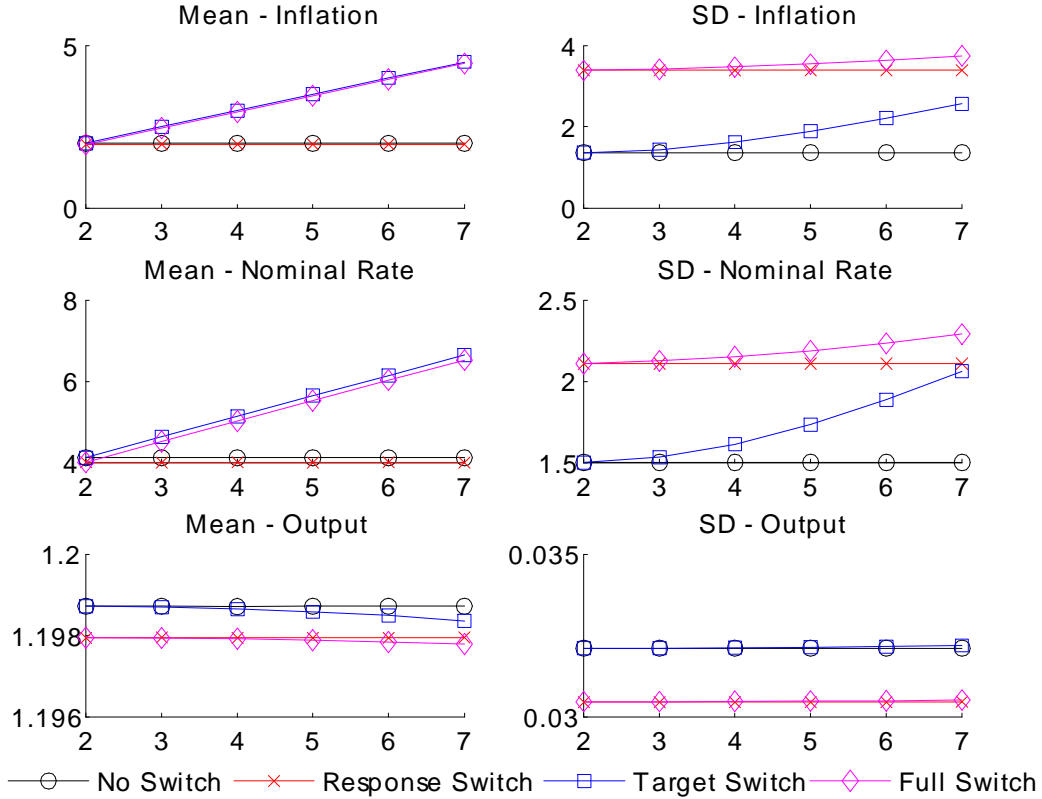
The results first examine the effects of inflation target switching and then inflation response switching. Section 4.1 focuses on the effects of changing the inflation target, and how it primarily affects the level of inflation rather than the volatility. This result holds true for both the long-run and the counterfactual simulations, highlighting how both the realized switches and expectations of future switches channels play a role. Section 4.2 focuses on the effects of changing the inflation response, which primarily affects the volatility of inflation rather than the level. Examining the long-run and counterfactual distributions show that both realized and expectations of switches produce this result.

4.1 Level Effects of Inflation Target Switching

First consider the effects of inflation target switching on the distribution of inflation, the nominal rate, and output in each of the four models. For concreteness, the simulations fix the policy parameters at their baseline calibration in Table 2. The exception is Π_H^* , which varies from an annual target of 2% to 7%, which spans a fixed regime parameterization and estimates in the literature (Liu et al. (2011), Schorfheide (2005)).

Figure 5 shows how the means and standard deviations of the long run ergodic distribution change for the four models as Π_H^* changes. Both the No Switching and Response Switching models show zero change as Π_H^* changes. This fact follows from the fact that these models each have a transition matrix with $\mathcal{Q}_{LL} = 1 - \mathcal{Q}_{HH} = 1$, which dictates that the economy never visits the high inflation target regime, and agents know this fact. In the No Switching Model, the monetary authority keeps the standard deviation of inflation low and around the 2% target. Low inflation and low inflation volatility help lead to a relatively high output level. In the Response Switching model, expectations and periods of passive monetary policy produce a

Figure 5: Long-Run Distributions as Π_H^* Changes



higher inflation volatility, but still centered around a 2% target. The higher volatility leads to a lower output level through certainty non-equivalence of the second-order approximation.

Both the Target Switching and Full Switching, by contrast, have $Q_{LL} = Q_{HH} = 0.95$, so changing Π_H^* alters outcomes because it occurs in the ergodic distribution and because agents know it will occur. As the high inflation target varies between 2% and 7%, the Target Switching and Full Switching models exhibit a larger level effect and a smaller volatility effect in the long run. For both models, as the high target increases, so does the average level of inflation in the economy, as a result the nominal rate increases in a similar manner. The difference between optimal price setting behavior and simple indexation to steady state inflation also increases, leading to higher price distortions in both regimes. As a result, the level of output falls as the inflation target increases. The higher degree of price distortions also lead to higher responses to shocks, so the standard deviation of inflation increases. The magnitude of this increase is

relatively small: even in the case with a high inflation target of 7%, the volatility of inflation in the Target Switching model is well below that in the Response Switching Model. The Full Switching model combines aspects of the Response and Target Switching models, producing the highest average inflation and volatility.

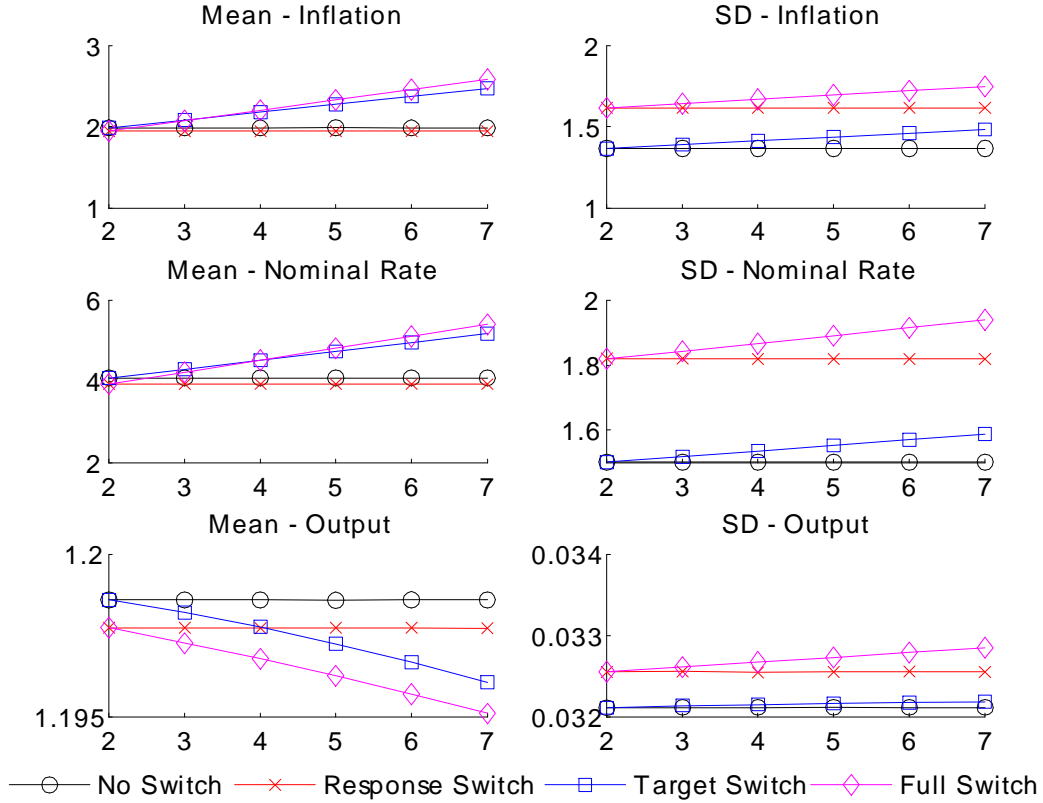
Figure 6 displays the means and standard deviations as Π_H^* changes for the counterfactual simulation that restricts monetary policy to the active response, low target regime. Since monetary policy is fixed and identical across these models, only the expectations channel generates differences in outcomes. As in the long-run ergodic simulations, the No Switching and Response Switching models are invariant to changes in Π_H^* , since agents know the high target regime can never occur. In the No Switching model, the counterfactual simulations and the long-run ergodic distribution are identical, since policy never changes. The Response Switching model, on the other hand, has agents that expect a switch to the passive inflation response regime that never materializes, leading to higher volatility, a result that is explored in depth in the next subsection.

In the Target and Full Switching models, expectations about a switch to the high target regime generate differences as Π_H^* increases. The results in Figure 6 confirm that the main effect of inflation target switching is through the level of inflation rather than the volatility. While the monetary authority targets 2% inflation, price indexation to steady state inflation generates higher-than-target inflation, which in turn produces higher nominal rates and lower output. As in the long-run simulations, the standard deviations increase as the high inflation target increases. Relative to the changes in the means, again, the increase in volatility generated by inflation target switching is relatively minor. For example, the Target Switching model fails to generate the volatility of the Response Switching model even when the high inflation target is 7%.

4.2 Volatility Effects of Inflation Response Switching

Now consider how, in each of the four models, switching in inflation responses impacts the distribution of inflation, the nominal rate, and output. In these simulations, the policy parameters

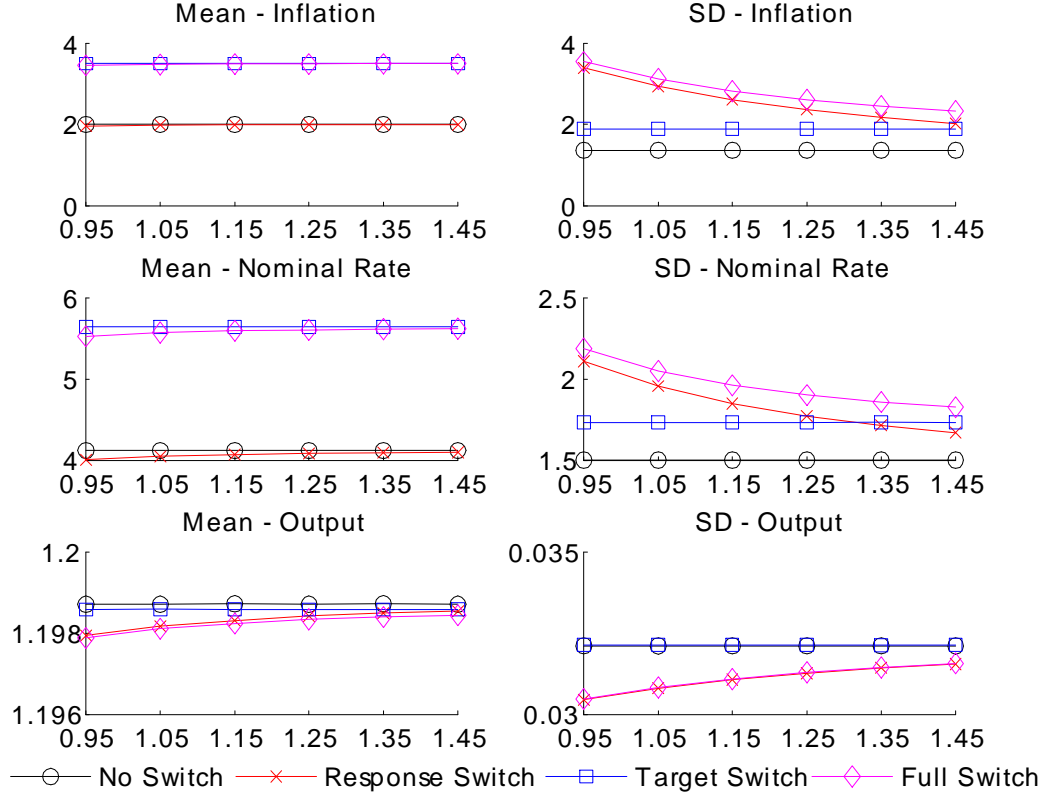
Figure 6: Counterfactual Distributions as Π_H^* Changes



are fixed at their benchmark calibration in Table 2, with the exception of the passive response parameter ψ_P , which varies from 0.95 to 1.45. These values cover a range of estimates, including Bianchi (2013) and Davig and Doh (2008), while still respecting the determinacy conditions discussed in Section 3.

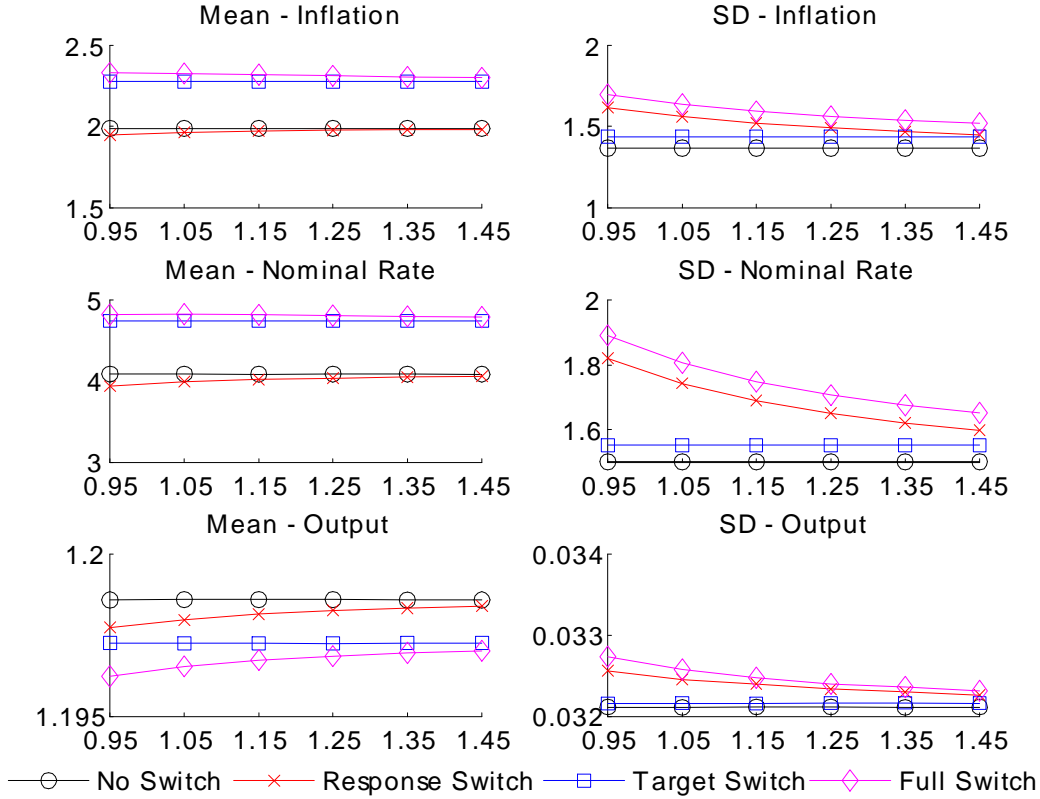
Figure 7 plots the means and standard deviations of the long run ergodic distribution change for the four models as ψ_P changes. Mirroring the results for changing inflation targets, in the No Switching and Target Switching models changing ψ_P has no effect. Since $\mathcal{P}_{AA} = 1 - \mathcal{P}_{PP} = 1$, these models never experience the passive response regime, a fact that agents know. In the No Switching Model, the monetary authority again keeps the standard deviation of inflation low and around the 2% target, leading to a high output level. As previously described, in the Target Switching model, expectations and periods of high inflation targets produce a higher level of inflation, but only slightly more volatility, which have a slight effect on the level of output.

Figure 7: Long-Run Distributions as ψ_P Changes



Changing ψ_P has effects in the Response Switching and Full Switching models, since $\mathcal{P}_{AA} = \mathcal{P}_{PP} = 0.95$, implying that the passive response parameter impacts outcomes because it occurs in the ergodic distribution and because agents know it will occur. When the passive response parameter declines from 1.45 to 0.95, the Response Switching and Full Switching models show increased inflation volatility, and a smaller level effect. In the long-run simulations for these two models, the economy experiences passive response regimes in which the monetary authority does not respond strongly to inflation deviations from target. As the passive regime becomes weaker, inflation volatility increases, which is accompanied by higher interest rate volatility. The higher volatility has a slight impact on average output through the lack of certainty equivalence. The average inflation rate exhibits negligible effects from the passive response parameter. In the Response Switching model, when ψ_P is the relatively high value of 1.45, the standard deviation of inflation slightly exceeds that in the Target Switching model, yet the average inflation rate

Figure 8: Counterfactual Distributions as ψ_P Changes



shows little change.

Figure 8 displays the means and standard deviations in the counterfactual simulation that restricts monetary policy to the active response, low target regime, but varies the passive response parameter ψ_P . In these simulations, only expectations can generate differences in outcomes, since monetary policy is fixed and identical across models. Since the No Switching and Target Switching models do not visit the passive response regime, they are unaffected by changes in ψ_P . Again, the No Switching model has identical long-run and counterfactual simulations, since policy never changes. The Target Switching model, on the other hand, has agents that expect a switch to the high target regime that never materializes, leading to higher average inflation, as pointed out in Section 4.1.

Expectations of a switch to a passive regime in the Response Switching and Full Switching models changes behavior. The plots in Figure 8 highlight that the main effect of response

switching is through the volatility of inflation rather than the mean. Even though the simulations have the monetary authority reacting to inflation deviations in an active way, expectations of a switch to the passive response regime produce higher inflation volatility. After shocks that drive inflation away from target, price-optimizing firms factor in a possible switch that will produce a slower return to the inflation target, and so adjust their prices by a larger amount, producing higher inflation volatility. The nominal rate responds with higher volatility as well, and more distorted prices with a fixed rule produce higher output volatility. Expectations of a weaker passive regime hence produce higher volatility, but the impact on the means is limited. Similar to the long-run distributions, the Response Switching Model exhibits higher volatility of inflation for all values of ψ_P , but there is little impact on the level of inflation.

5 Conclusion

Monetary policy rules tend to change over time; using a new Keynesian model, this paper discussed the implications for different types of monetary policy switches on determinacy and the distributions of outcomes in the economy. While switches in the inflation target do not affect determinacy, switching in the inflation response potentially causes indeterminacy. This indeterminacy can arise if the monetary authority responds too aggressively to inflation in the active regime. In the distributions of outcomes, inflation target switches have a strong effect on the level of inflation, while inflation response switches primarily impact the volatility.

In the presence of response switches, standard Taylor Principle results fail to hold; determinacy can result even when one regime implies indeterminacy, and indeterminacy can result even when both regimes imply determinacy. The long-run distributions depend upon the type of switches: varying inflation responses change inflation volatility, whereas varying inflation targets move the level of inflation. This result also holds for the effects of expectations, since agents with rational expectations about future regime changes alter their behavior based upon these expectations, leading to different outcomes based upon the type of expected switches.

Several open questions remain for future investigation. The model considered here assumes

constant probabilities of changing regimes. However, these probabilities may actually depend upon economic outcomes, such as the threshold model of Davig and Leeper (2008). In addition, determining whether inflation target or inflation response switches occurred in US history ultimately represents an empirical question. Finally, a framework where optimal policy generates a switching rule remains an interesting consideration.

References

- Andreasen, M., J. Fernandez-Villaverde, and J. Rubio-Ramirez (2013). The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications. Working Paper 18983, NBER.
- Ascari, G. and T. Ropele (2009). Trend Inflation, Taylor Principle, and Indeterminacy. *Journal of Money, Credit, and Banking* 41(8), 1557–1584.
- Ball, L. (2013). The Case for Four Percent Inflation. *Central Bank Review* 13(2), 17–31.
- Bianchi, F. (2012). Methods for Markov-Switching Models. Working Paper.
- Bianchi, F. (2013). Regime Switches, Agents’ Beliefs, and Post-World War II US Macroeconomic Dynamics. *Review of Economic Studies* 80(2), 463–490.
- Blanchard, O., G. Dell’Ariccia, and P. Mauro (2010). Rethinking Macroeconomic Policy. *Journal of Money, Credit, and Banking* 42(s1), 199–215.
- Carlstrom, C. T. and T. S. Fuerst (2001). Timing and Real Indeterminacy in Monetary Models. *Journal of Monetary Economics* 47(2), 285–298.
- Chib, S., K. Kang, and S. Ramamurth (2011). Monetary Policy Regime Changes and the Term Structure: Evidence from a DSGE Model. Working Paper.
- Cho, S. (2011). Characterizing Markov-Switching Rational Expectation Models. Working Paper.

- Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy* 113(1), 1–45.
- Clarida, R., J. Gali, and M. Gertler (2000). Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics* 115(1), 147–180.
- Coibion, O. and Y. Gorodnichenko (2011). Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation. *American Economic Review* 101(1), 341–370.
- Costa, O., M. Fragoso, and R. Marques (2005). *Discrete-Time Markov Jump Linear Systems*. Springer.
- Davig, T. and T. Doh (2008). Monetary Policy Regime Shifts and Inflation Persistence. Research Working Paper 08-16, Federal Reserve Bank of Kansas City.
- Davig, T. and E. Leeper (2007). Generalizing the Taylor Principle. *American Economic Review* 97(3), 607–635.
- Davig, T. and E. Leeper (2008). Endogenous Monetary Policy Regime Change. In *NBER International Seminar on Macroeconomics 2006*, NBER Chapters, pp. 345–391. National Bureau of Economic Research, Inc.
- De Fiore, F. and Z. Liu (2005). Does Trade Openness Matter for Aggregate Instability? *Journal of Economic Dynamics and Control* 29(7), 1165–1192.
- Farmer, R., D. Waggoner, and T. Zha (2009). Understanding Markov-Switching Rational Expectations Models. *Journal of Economic Theory* 144(5), 1849–1867.
- Farmer, R., D. Waggoner, and T. Zha (2011). Minimal State Variable Solutions to Markov-Switching Rational Expectations Models. *Journal of Economic Dynamics and Control* 35(12), 2150–2166.
- Foerster, A., J. Rubio-Ramirez, D. Waggoner, and T. Zha (2014). Perturbation Methods for Markov Switching DSGE Models. Working Paper 13-01, Federal Reserve Bank of Kansas City.

- Leeper, E. and T. Zha (2003). Modest Policy Interventions. *Journal of Monetary Economics* 50(8), 1673–1700.
- Liu, Z., D. Waggoner, and T. Zha (2009). Asymmetric Expectation Effects of Regime Shifts in Monetary Policy. *Review of Economic Dynamics* 12(2), 284–303.
- Liu, Z., D. Waggoner, and T. Zha (2011). Sources of Macroeconomic Fluctuations: A Regime-Switching DSGE Approach. *Quantitative Economics* 2(2), 251–301.
- Lubik, T. and F. Schorfheide (2004). Testing for Indeterminacy: An Application to US Monetary Policy. *American Economic Review* 94(1), 190–217.
- Rogoff, K. (2008). Inflation is Now the Lesser Evil. Project Syndicate. December 2.
- Romer, C. (2013). It Takes a Regime Shift: Recent Developments in Japanese Monetary Policy Through the Lens of the Great Depression. Working Paper.
- Schmitt-Grohe, S. and M. Uribe (2004). Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control* 28(4), 755–775.
- Schorfheide, F. (2005). Learning and Monetary Policy Shifts. *Review of Economic Dynamics* 8(2), 392–419.
- Sims, C. and T. Zha (2006). Were There Regime Switches in US Monetary Policy? *American Economic Review* 96(1), 54–81.
- Smets, R. and F. Wouters (2007). Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach. *American Economic Review* 97(3), 586–606.
- Svensson, L. and N. Williams (2007). Monetary Policy with Model Uncertainty: Distribution Forecast Targeting. Discussion Paper 6331, CEPR.
- Woodford, M. (2003). *Interest and Prices*. Princeton UP.

6 Appendix

This Appendix sketches the proof to Theorem 1. Foerster et al. (2014) show that determinacy – existence and uniqueness of a mean square stable, minimum state variable solution – depends on the solution to a set of quadratic equations. To derive these quadratic equations, the equilibrium conditions for each regime are stacked:

$$\mathbb{F}(\mathbf{Y}_t, \mathbf{X}_t, x_{t-1}, \varepsilon_t, \chi) = \begin{bmatrix} \mathbb{F}_1(\mathbf{Y}_t, \mathbf{X}_t, x_{t-1}, \varepsilon_t, \chi) \\ \mathbb{F}_{n_s}(\mathbf{Y}_t, \mathbf{X}_t, x_{t-1}, \varepsilon_t, \chi) \end{bmatrix} = 0, \quad (16)$$

where $\mathbb{F}_{s_t}(\mathbf{Y}_t, \mathbf{X}_t, x_{t-1}, \varepsilon_t, \chi)$ denotes the equilibrium conditions conditional on regime s_t , \mathbf{Y}_t denotes the stacked non-predetermined variables for each regime, \mathbf{X}_t denotes the stacked predetermined variables for each regime, x_{t-1} denotes the fixed predetermined variables, ε_t the shocks, and χ the perturbation parameter. The *Partition Principle* in Foerster et al. (2014) dictates that since the steady state of the model depends on the inflation target but not the inflation response, the Markov Switching parameters are written as

$$\begin{bmatrix} \Pi_{s_t}^* & \psi_{s_t} \end{bmatrix}' = (1 - \chi) \begin{bmatrix} \Pi_{ss}^* & \psi_{s_t} \end{bmatrix}' + \chi \begin{bmatrix} \Pi_{s_t}^* - \Pi_{ss}^* & \psi_{s_t} \end{bmatrix} \quad (17)$$

where Π_{ss}^* denotes the ergodic mean of $\Pi_{s_t}^*$. The quadratic equation relevant for determinacy is

$$\mathbb{F}_x(\mathbf{Y}_{ss}, \mathbf{X}_{ss}, x_{ss}, 0, 0) = \frac{\partial \mathbb{F}(\mathbf{Y}_t, \mathbf{X}_t, x_{t-1}, \varepsilon_t, \chi)}{\partial x_{t-1}} \Bigg|_{\mathbf{Y}_t = \mathbf{Y}_{ss}, \mathbf{X}_t = \mathbf{X}_{ss}, x_{t-1} = x_{ss}, \varepsilon_t = 0, \chi = 0}. \quad (18)$$

Note that since $\chi = 0$ then $\Pi_{s_t}^*$ does not appear, which implies that $\mathbb{F}_x(\mathbf{Y}_{ss}, \mathbf{X}_{ss}, x_{ss}, 0, 0)$ is independent of Π_L^* and Π_H^* . In addition, due to the indexation of prices to $\Pi_{ss} = \Pi_{ss}^*$, steady state real variables \tilde{Y}_{ss} , \tilde{C}_{ss} , \tilde{G}_{ss} , and \tilde{W}_{ss} are independent of Π_{ss}^* , and hence $\mathbb{F}_x(\mathbf{Y}_{ss}, \mathbf{X}_{ss}, x_{ss}, 0, 0)$ is as well.