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# When Does the Cost Channel Pose a Challenge to Inflation Targeting Central Banks?

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## Abstract

In a sticky-price model where firms finance their production inputs, there is both a lower and an upper bound on the central bank's inflation response necessary to rule out the possibility of self-fulfilling inflation expectations. This paper shows that real wage rigidities decrease this upper bound, but coefficients in the range of those on the Taylor rule place the economy well within the determinacy region. However, when there is time-variation in the share of firms who finance their inputs (i.e. Markov-Switching) then inflation targeting interest rate rules frequently result in indeterminacy, even if the central bank also targets output. Adding a nominal growth target to the policy rule can often alleviate this indeterminacy and therefore anchor inflation expectations.

**Keywords:** Cost Channel, Taylor Rule, Determinacy, Regime Switching, Nominal Growth  
**JEL codes:** E3, E4, E5, C62

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# 1 Introduction

The advice to central banks that a well designed interest rate reaction function mechanically adjusts the policy rate more than one for one to deviations of inflation from target (c.f. Taylor (1993)) is one of the most robust policy prescriptions in monetary theory. However, the Taylor principle as described above is not without its caveats. Importantly, Bruckner and Schabert (2003), Surico (2008) and Christiano, Trabandt, and Walentin (2010) show that an upper bound may need to be placed on the central bank’s reaction to inflation in order to ensure expectations remain well anchored.<sup>1</sup> This constraint on the central bank arises when there is a timing mismatch between when the firm produces its product and when it gets paid for the product. Firms in this situation will typically have to finance inputs with short-term loans called working capital. This environment introduces a cost channel which works counter to the typical transmission mechanism of monetary policy in sticky-price models.

The typical transmission mechanism of monetary policy in New-Keynesian models suggests that central banks can eliminate self-fulfilling inflation episodes by increasing the nominal interest rate more than the increase in inflation. Such an aggressive interest rate hike shifts consumption to the future and therefore decreases current demand, marginal costs, and prices. Whenever the cost-channel is present, however, the increase in nominal interest rates may actually confirm the expected inflation. This could happen, for example, if the central bank tries to eliminate the inflation scare by forcefully raising nominal interest rates. The aggressive interest rate rise increases firms’ financing costs and leads to higher marginal costs and, through the Phillips curve, higher inflation.

In practice, central banks have attempted to avoid this feedback loop by targeting measures of inflation excluding debt-servicing costs. Moreover, central banks which target inflation measures that do include direct interest rate effects typically supplement their policy analysis with alternative measures of inflation. Sweden’s Riksbank, for example, officially targets CPI inflation, which includes interest payments on mortgages in the price of housing services, but publishes a forecast of CPIF inflation which measures the change in consumer prices while holding fixed mortgage financing costs. The CPIF is similar in this respect to the PCE inflation rate targeted by the Federal Reserve and the HICP inflation measure targeted by the European Central Bank. However, if price setters pass-on changes in their financing cost to customers, as the cost channel posits, then none of these price measures eliminate the indirect effects that changes in borrowing costs have on inflation.

Despite the knock-on effects of interest rate changes on consumer prices, the cost channel may still not pose a significant hurdle to inflation targeting central banks. For example, targeting current inflation instead of expected future inflation significantly enlarges the determinacy regions of interest rate rules in the presence of the cost-channel (Bruckner and Schabert, 2003). In addition, Surico (2008) shows that a “flexible” inflation target (in the sense that stabilizing prices is not the only concern of the central bank) is less likely to induce self-fulfilling equilibria from an overly-hawkish central bank. This type of central bank reaction function is consistent with a Taylor rule which includes a reaction to output as well as inflation. Therefore, the presence of the cost channel may only be a theoretical curiosity and not a serious challenge to inflation targeting central banks.

The positive appeal of the cost channel is its ability to explain the empirical regularity that an exogenous monetary policy tightening results in a lower price level only several periods after the policy disturbance (See for example Barth and Ramey, 2001). VAR evidence suggests that prices may increase in the short-run in response to a monetary policy shock before falling. This “price-puzzle” was originally identified by Sims (1992) as a robust feature across several economies, including the

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<sup>1</sup>Other apparent failures of the Taylor principle involve the interaction of the inflation response coefficient and trend-inflation (Coibion and Gorodnichenko, 2011) and the timing of the stock of money which enters the utility function (Carlstrom and Fuerst, 2001).

U.S., Germany, and France. In an effort to reconcile the puzzling, or at least inertial, response of prices following a monetary policy shock in the data with sticky-price equilibrium models, Christiano, Eichenbaum, and Evans (2005) and Henzel, Hulsewig, Mayer, and Wollmershauser (2009) use a minimum distance estimator which reveals that, on average, there is a cost-channel present in the U.S. and the Euro area which helps to explain the gradual fall in prices following a monetary contraction.<sup>2</sup>

These findings are conditional on underlying real-rigidities in addition to the cost-channel and sticky prices. Intuitively, to generate the gradual response of prices in the data to a monetary policy shock, it is necessary to have other frictions, in addition to the cost-channel, which prevent the demand channel from affecting marginal cost in the short-run. Of these frictions, sluggish wage-adjustment is key since it prevents non-financial input cost from changing for firms who rely on labor for production. Therefore, the cost channel, when paired with wage rigidities, is able to generate empirically plausible responses to monetary policy shocks. However, the aforementioned studies of equilibrium determinacy in the presence of the cost channel have typically focused on the case of no wage frictions.

In this paper, I characterize the determinacy regions of interest rate rules when real wages only gradually adjust to the marginal rate of substitution between consumption and labor. I show that for interest rate rules which satisfy the Taylor principle, the determinacy regions shrink when the degree of wage rigidity increases. Although the resulting determinacy regions are smaller, they still imply that interest rate rules with coefficients in line with the Taylor rule place the economy well within the determinacy region.

In addition to relaxing the assumption that real wages are flexible, I also allow the share of firms that need to finance their wage bill to vary over time. Empirical evidence suggests the degree of the cost channel has evolved over time. Whereas, Bruckner and Schabert (2003), Surico (2008) and Christiano et al. (2010) assume a constant fraction of firms finance their inputs, I model the share of firms who are subject to the cost-channel as a 2-state Markov-Switching process. In this Markov-Switching DSGE (MS-DSGE) model, inflation targeting regimes can induce indeterminacy even if they include a reaction to output. Therefore, the Taylor rule is indeterminate for a wide range of parameters governing the Markov-Switching process.

Other nominal targets are not subject to the same pitfalls that plague inflation targeting rules. A small coefficient on nominal money growth in the central bank's reaction function can often eliminate the multiple equilibria that appear under inflation targeting in this MS-DSGE model. Since nominal money demand is a function of interest rates and nominal output, adding nominal money growth to a Taylor rule transforms it into a Taylor rule with interest rate smoothing and a nominal GDP growth reaction. Analytic results from the constant parameter model and sensitivity analysis in the MS-DSGE model suggest that nominal GDP growth targeting plays a key role in anchoring inflation expectations independent of interest-rate smoothing. In other words, once the central bank establishes an intermediate nominal GDP growth target, which could possibly be implemented by way of targeting nominal money growth, inflation targeting can then be pursued without producing sunspot equilibria.

This finding is perhaps not surprising considering the existing literature on the global equilibrium determinacy of Taylor rules. In particular, Christiano and Rostagno (2001) and Behabib, Schmitt-Grohe, and Uribe (2002) find that the global determinacy of the Taylor rule can be restored once the central bank commits to switching to a money-growth targeting regime. The results in this paper are similar in spirit. However, this is the first study which shows that a money growth target can restore local determinacy of an otherwise indeterminate Taylor rule.<sup>3</sup> Implicit in this interpretation of the

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<sup>2</sup>These findings corroborate the single-equation estimates of Phillips Curves (Ravenna and Walsh, 2006; Chowdhury, Hoffmann, and Schabert, 2006) and multi-equation decompositions of inflation (Tillmann, 2008) which find the short-term interest rate plays a significant role in shaping inflation in the U.S. and euro area.

<sup>3</sup>Another distinction is that I assume that fiscal policy is passive in the sense of Leeper (1991).

results is the existence of a stable money demand relationship, thereby allowing the central bank to effectively influence money growth. However, if this were not the case, then the determinacy results would still apply to a Taylor rule with interest rate smoothing and a nominal GDP growth target. Throughout the paper, these two interpretations are equivalent; however, in Section 6, the former interpretation is applied to understand the European Central Bank’s “two-pillar” approach to price stability in the context of this model.

## 2 The Model

This section describes a log-linearized sticky price model in which firms have to finance their inputs prior to production. In the appendix, the model is derived in detail, however, here I present the relevant model equations to answer the questions of interest in this paper regarding local equilibrium determinacy. The non-policy block of the model, in large part, follows from Ravenna and Walsh (2006) and Blanchard and Gali (2007).

$$x_t = \mathbb{E}_t x_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1}) + \varepsilon_t^x \quad (1)$$

$$w_t = \rho w_{t-1} + (1 - \rho)(1 + \varphi)x_t + \varepsilon_t^w \quad (2)$$

$$m_t = x_t - \eta r_t + \varepsilon_t^m \quad (3)$$

$$\pi_t = \kappa(w_t + \alpha(s_t)r_t) + \beta \mathbb{E}_t \pi_{t+1} + \varepsilon_t^\pi \quad (4)$$

In the above equations,  $x_t$  is the output gap,  $\pi_t$  denotes the quarterly inflation rate,  $w_t$  is the real wage rate,  $r_t$  is the nominal interest rate on one-period bonds, and  $m_t$  denotes real money balances. All variables are expressed as percent deviations from their steady state values. The exogenous processes  $\varepsilon_t^x$ ,  $\varepsilon_t^w$ ,  $\varepsilon_t^\pi$  and  $\varepsilon_t^m$  are linear combinations of structural preference, technology, and money-demand shocks. Since the focus of the paper is on local determinacy, the exact way the structural shocks enter these exogenous processes is left in the appendix.

Equation 1 is the household’s Euler equation which relates the expected rate of output growth to the real return on a 1-period bond. Equation 2 shows the evolution of the real wage rate, where following Blanchard and Gali (2007) and Hall (2005), I assume the real wage only adjusts gradually to the marginal rate of substitution. The speed of adjustment is determined by the magnitude of  $0 \leq \rho \leq 1$ . When  $\rho = 0$ , the model collapses to the flexible wage model previously studied in the cost-channel determinacy literature. The parameter  $\varphi > 0$  is the labor supply elasticity (or inverse Frisch elasticity).

Equation 3 is the household’s money demand equation in which  $\eta > 0$  is the interest semi-elasticity. Equation 4 summarizes the pricing decision that firms face. The parameter  $0 < \beta < 1$  is the household’s discount factor. The parameter  $\kappa = (1 - \omega)(1 - \beta\omega)/\omega$  is the slope of the Phillips curve, in which  $1/(1 - \omega)$  is the average duration of prices. The strength of the (potentially time-varying) cost channel is governed by the term  $0 \leq \alpha(s_t) \leq 1$ . The micro-foundations for motivating the parameter  $\alpha$  stem from two possible modeling strategies. Rabanal (2007) interprets  $\alpha$  as the share of firms in the aggregate who must finance their wage bill, while Christiano et al. (2010) interpret  $\alpha$  as the share of each firm’s wage bill which is financed each period. In either interpretation, this parameter determines the supply-side effects of monetary policy.

One deviation from the standard models in Woodford (2003) and Gali (2008) is  $m_t$  is an aggregate of interest bearing and non-interest-bearing assets following Belongia and Ireland (2014). This adjustment is made to allow for a more direct interpretation of the use of monetary aggregates by central banks. For example, the ECB monitors the growth rate of M3 (according to their press conference

transcripts). Although the micro-foundations of monetary aggregation are more clearly spelled out with this specification, after log-linearizing, the money demand equation is isomorphic to those found in Woodford (2003) and Gali (2008). The model is closed with a specification of monetary policy which I assume can be described by an interest rate feedback rule of the type specified by Taylor (1993); however, I also allow for a reaction to the nominal growth rate of money:

$$r_t = \phi_\pi \pi_t + \phi_x x_t + \phi_\mu \mu_t + \varepsilon_t^{mp}, \quad (5)$$

$$\mu_t = m_t - m_{t-1} + \pi_t, \quad (6)$$

where  $\varepsilon_t^{mp}$  is an i.i.d. monetary policy shock.

### 3 Baseline model

Consider first a model most similar to that analyzed by Bruckner and Schabert (2003), Surico (2008) and Christiano et al. (2010) in which wages are flexible ( $\rho = 0$ ), and a constant fraction of firms must finance their inputs prior to production  $\alpha(s_t) = \alpha$ . In this section, I show that if the central bank is too aggressive in adjusting its policy rate to movements in inflation, unwanted volatility may emerge due to a multiplicity of equilibria. This is the point raised by Bruckner and Schabert (2003), Surico (2008) and Christiano et al. (2010). Therefore, the following proposition is not novel, but creates a baseline to compare the determinacy regions of inflation targeting rules with wage rigidities and a time-varying cost channel.

**Proposition 1.** *If the central bank follows the policy rule  $r_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$  then there exists a unique bounded REE if and only if*

$$1 < \phi_\pi < \begin{cases} \infty, & \text{for } \alpha \leq \frac{(1+\varphi)}{2} \\ \frac{2(1+\beta)+\kappa(1+\varphi)}{\kappa(2\alpha-(1+\varphi))}, & \text{for } \alpha > \frac{(1+\varphi)}{2}. \end{cases}$$

To get a better sense of how the cost channel alters the determinacy regions of inflation targeting rules, consider the supply and demand side effects of monetary policy separately. Higher values of  $\varphi$  imply larger demand side effects, where as higher values of  $\alpha$  imply larger supply-side effects. The demand side effects work through the household's Euler equation and labor supply curve. On the other hand, the supply-side effects work directly through the Phillips curve.

The demand side effects of monetary policy work through the Euler equation and the labor supply curve. An increase in real interest rates causes consumption today to decrease and, therefore, decreases the demand for labor. The extent to which the decrease in labor demand puts downward pressure on wages is calibrated by  $\varphi$ . When  $\varphi$  is large, a given change in the quantity of labor supplied results in a larger movement in the real wage. For example, setting  $\varphi = 1$  implies there is no upper bound on the inflation response since  $0 \leq \alpha \leq 1$ . At the other extreme, if  $\varphi = 0$ , then labor supply has no effect on the real wage. This acts to dampen the demand-side effects of monetary policy and in turn limits the aggressiveness with which the central bank can target inflation.

The supply-side effects of monetary policy work directly through the Phillips curve via the cost-channel parameter  $\alpha$ . When  $\alpha$  is large, increases in the nominal interest rate put more upward pressure on inflation. This happens because when a greater number of firms must borrow funds to finance their inputs, the increase in the nominal rate implies higher borrowing costs which translate into more inflationary pressure. In this sense, raising interest rates when inflation is high is akin to "throwing gasoline on a fire" to quote U.S. Congressman Wright Patman's description of the cost channel.

Combining both demand and supply effects, Proposition 1 shows that inflation targeting rules are subject to an upper bound on the inflation coefficient. The likelihood that monetary policy enters

the region of the parameter space in which the inflation coefficient is bounded above depends on the strength of these demand and supply-side effects. Once in this region of the parameter space, it is up to monetary policy to temper its aggressiveness.

Inflation targeting rules are particularly susceptible to indeterminacy with the cost channel. Rules which target the nominal growth rate of money or GDP have stable determinacy regions independent of the number of firms who need to finance their wage bill. When the cost channel is active, a money growth targeting rule prevents the central bank from raising rates too aggressively in response to higher inflation. In particular, if inflation rises above target, a central bank which attempts to stabilize the growth rate of nominal money will react by increasing interest rates (since nominal money growth includes inflation). However, the increase in nominal rates decreases the demand for money. This liquidity effect tempers the central bank’s contraction, preventing sunspot equilibria from emerging regardless of the strength of the cost channel.

**Proposition 2.** *If the central bank follows the policy rule  $r_t = \phi_\mu \mu_t + \varepsilon_t^{mp}$  then there exists a unique bounded REE if and only if  $\phi_\mu > 1$ .*

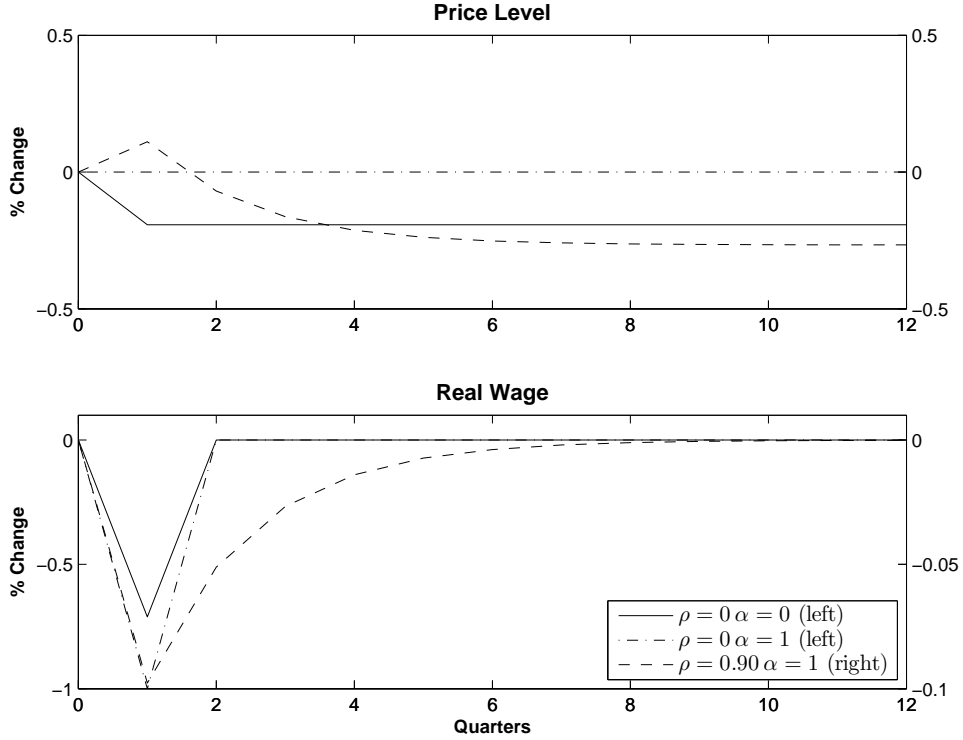
This proposition also points to a useful role for nominal GDP growth targeting in the presence of the cost channel. Since nominal money demand is a function of nominal output and the interest rate, money growth targeting rules are recoverable via nominal GDP growth targeting rules with interest rate smoothing. While Surico (2008) shows that interest rate smoothing increases determinacy regions in the presence of the cost channel, it does not eliminate the upper bound on inflation targeting rules. Proposition 2 therefore implies that nominal GDP growth targeting, implicit in nominal money growth targeting rules, anchors expectations regardless to the number of firms who rely on working capital loans. In the appendix, Proposition 4 formally shows that the policy rule  $r_t = \phi_{ngdp}(\Delta x_t + \pi_t)$  results in a unique bounded rational expectations equilibrium for all values of  $\alpha$  if, and only if,  $\phi_{ngdp} > 1$ .

This invariance result builds on the analytical results in Mitra (2003) who highlights the stability properties of strict nominal GDP growth targeting rules (i.e.  $\Delta x_t + \pi_t = 0$ ) in a sticky price model without the cost channel. Also related, Coibion and Gorodnichenko (2011) find that an output growth reaction in a Taylor rule, which could be written as a nominal GDP growth targeting Taylor rule, enlarges determinacy regions in the presence of trend inflation. However, this paper is the first to highlight the stabilizing properties of nominal growth targeting in the presence of the cost channel. These determinacy results will prove useful in finding policy rules which anchor expectations in the MS-DSGE model in Section 5 where Taylor rules are shown to be particularly susceptible to indeterminacy.

## 4 Adding Real Wage Rigidities

Wage rigidities have proven to be an important source of propagating business cycles and are considered a stock feature of DSGE models (Blanchard and Gali (2007); Smets and Wouters (2007); Christiano et al. (2005)). What is especially interesting about wage rigidities in the presence of a cost channel is the ability to generate the so called “price puzzle.” The price puzzle is said to be present when a monetary contraction leads to an initial increase in the price level/inflation. Sims (1992) first noted the puzzle as a prevalent feature of monetary vector autoregressions across multiple countries. Christiano et al. (2005) go on to show that the slow response of prices to a monetary contraction can be captured by DSGE models with both a cost channel and wage rigidities, in a larger model with capital and other real frictions. Intuitively, from the Phillips Curve in equation 4, if wages adjust slowly to a monetary contraction then inflation will increase when interest rates rise. To better understand this interaction, it is useful to analyze the model’s reaction to a monetary contraction under various assumption on the degree of wage flexibility and the cost channel.

Figure 1: Impulse response functions to an exogenous monetary contraction for different degrees of wage rigidity and the cost channel.



Note: The responses above are graphed for parameters set to the baseline U.S. calibration described in Table 4.

The impulse response functions to a monetary contraction verify this small-scale model with the cost channel and real wage rigidities is capable of generating the price puzzle. Notice, the cost channel alone is not capable of generating an initial rise in prices for the baseline calibration. In fact, the response of prices to a monetary policy shock is nil with flexible wages as the demand-side effects of higher interest rates, which pushes wages down, are canceled out by higher borrowing costs. With both partial adjustment of real wages and the cost channel, prices initially raise before falling. These impulse responses suggest that the cost channel, when coupled with wage rigidities, can produce impulse responses that are more similar to those in the data. The initial rise in prices and muted response of the real wage are all features of empirical impulse responses to a contractionary monetary policy shock, as shown in Christiano et al. (2005).

Figure 1 provides evidence that the strength of the demand-side channel of monetary policy is weakened by the sluggish response of wages. This increased relative strength of the cost channel suggests the determinacy regions for interest rate rules will be further reduced by the addition of real wage rigidities. The following proposition more formally states this conclusion.

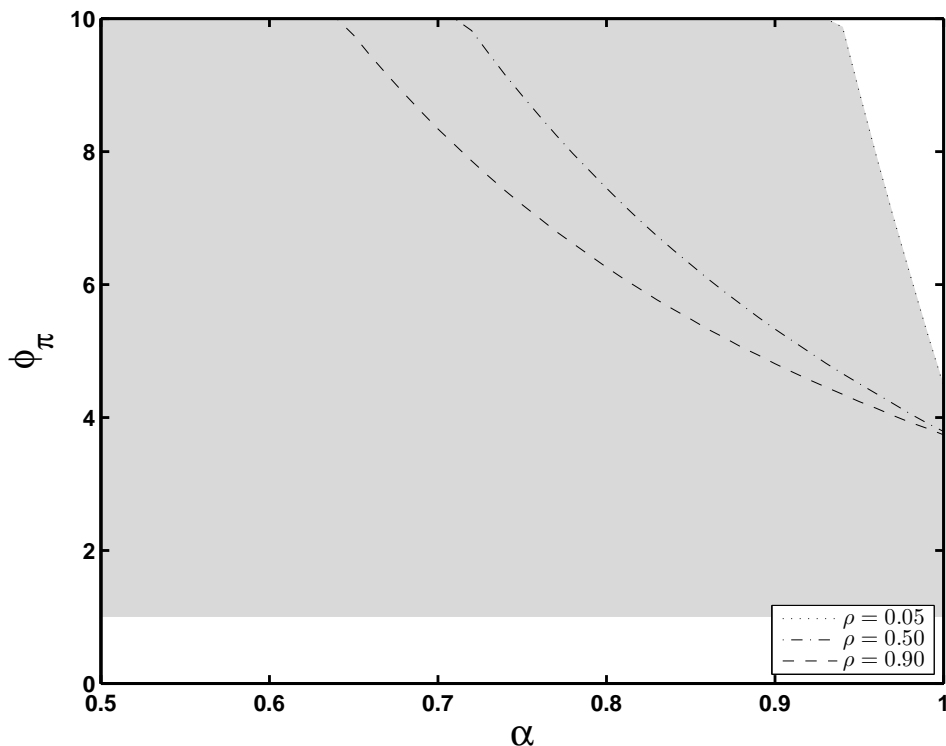
**Proposition 3.** *Suppose the central bank follows the policy rule  $r_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$  with  $\phi_\pi > 1$ . Let  $\alpha_{min}(\rho)$  denote the lower bound on the strength of the cost channel necessary to induce the upper bound  $\phi_{\pi,max}(\rho)$  on the inflation response to ensure the existence of a unique bounded rational expectations equilibrium for a given value of  $0 \leq \rho < 1$ , then:*

- i.*  $\alpha_{min}(\rho)$  is decreasing in  $\rho$ .
- ii.*  $\phi_{\pi,max}(\rho)$  is decreasing in  $\rho$ .



Real wage rigidities modify the determinacy conditions under the cost channel in two ways. First, it decreases the threshold strength of the cost channel needed to induce an upper bound on the policy response to inflation. Second, partial wage adjustment lowers this upper bound on the inflation coefficient once the cost channel is sufficiently strong. These two effects shrink the determinacy regions of inflation targeting rules that satisfy the Taylor principle relative to the case with flexible wages. Figure 2 quantifies this change in the determinacy regions. When wages are perfectly flexible the upper bound on the inflation coefficient is larger than 15 and therefore for values of  $1 < \phi_\pi < 10$  the entire region is determinate. However, introducing even a small bit of inertia into real wages lowers the upper bound to less than 5. Further increasing the wage rigidity doesn't lower the upper bound for  $\alpha = 1$  much, but it does reduce the upper bound for values of  $\alpha < 1$ .

Figure 2: Determinacy regions with a cost channel and various degrees of real wage rigidity when the central bank follows the policy rule:  $r_t = \phi_\pi \pi_t$ .



Note: The gray area to the southwest of the respective curves are determinate while the white areas are indeterminate. Parameters are set to the baseline U.S. calibration (see Table 4).

Even in the presence of wage rigidities, inflation targeting rules still have reasonably large determinacy regions. Parameters typically used to calibrate central bank reaction functions, such as the Taylor rule which sets  $\phi_\pi = 1.5$ , are well within the determinacy region. Adding an output gap reaction further increases the upper-bound on inflation targeting rules. This suggests the cost channel, even in the presence of real-wage rigidities, is unlikely to impose a binding constraint on the central bank's ability to achieve its inflation target via a Taylor rule. However, the next section shows that once a time-varying cost channel is introduced, in addition to real-wage rigidities, Taylor rules are surprisingly susceptible to multiple equilibrium. In such an environment adding an intermediate nominal growth target proves to be a useful tool to the central bank.

## 5 A Time Varying Cost Channel

Empirical estimates of the degree of the cost channel vary over time. Barth and Ramey (2001) provide indirect inference that the cost channel diminished in importance after 1983 relative to the pre-1979 period as measured by the severity of the price puzzle across several industries to a monetary policy shock.<sup>4</sup> Meanwhile, Barakchian and Crowe (2013) find a re-emergence of the price puzzle in the post 1990 sample. Tillmann (2009) provides a more direct analysis of time variation in the cost channel. The rolling windows regressions in Tillmann (2009) corroborate these split sample estimates, but also show that there is higher frequency variation in the cost channel. For example, the coefficient on interest rates is substantially more volatile than the coefficient on real marginal cost. This suggests there is parameter instability in the cost channel coefficient above and beyond that contained in other structural parameters in the Phillips curve.

There are several potential explanations as to why the cost channel coefficient varies over time. One interpretation is that the cost channel represents a financial market imperfection which is likely to fluctuate with the tightness of financial markets. This is the interpretation put forth by Tillmann (2009) who shows that the cost channel coefficient covaries with the Senior Loan Officer Opinion Survey (SLOOS) net-tightening index. Of course, if financial markets are merely responding to aggregate demand or technology shocks, then such changes in the cost channel may be endogenous. However, there is growing empirical evidence from structural models that, even after accounting for aggregate demand and supply shocks, random changes in financial market imperfections contribute to business cycles (Jermann and Quadrini, 2012; Christiano, Motto, and Rostagno, 2014). Especially relevant, Jermann and Quadrini (2012) show exogenous financial shocks that directly affect how much working-capital firms borrow explain nearly 50% of the variation in GDP at business cycle frequencies.

A second interpretation for time variation in the cost channel coefficient stems from the importance of working capital as a means for firms to smooth production through the inventory cycle (Barth and Ramey, 2001). Wen (2005) presents evidence that at high-frequencies (2-3 quarters), inventory investment is countercyclical and production is less volatile than sales in OECD countries. This suggests that, as found by Fazzari and Petersen (1993), financially constrained firms who experience a transitory demand shock may turn to working capital to finance inventories and avoid adjustment costs associated with changing production and investment plans.<sup>5</sup>

One way to capture the apparent time-variation in the cost channel is to follow Christiano and Gust (1999) and allow the degree of the cost channel to exogenously vary over time. I model the share of firms who are subject to the cost-channel as a 2-state Markov-Switching process. From the above discussion, a cost-channel regime has the interpretation of a period in which more firms are liquidity constrained to the point they must borrow to pay for their production inputs. This exogenously varying cost channel captures the aforementioned shifts in firms' liquidity positions, but does not incorporate the possible linkages between aggregate conditions that could trigger a change in firms' liquidity positions. That being said, the Markov switching cost channel does have implications for how prices endogenously evolve when the demand for liquidity is high versus low. Therefore, the model is generally consistent with the evidence in Gilchrist and Zakrajsek (2015) that liquidity conditions influence firms' price setting dynamics.

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<sup>4</sup>Although Barth and Ramey (2001) attribute these shifts to changes in financial markets, other studies have attributed these changes to shifts in monetary policy (Boivin and Giannoni, 2006; Boivin, Kiley, and Mishkin, 2010) using models without a cost channel.

<sup>5</sup>Although I have specified working capital as a means to finance firm's labor bill, if inventories enter the production function as materials inputs then Christiano et al. (2010) show that if firms finance payroll and inventories the only change in the the cost channel Phillips curve is that the upper bound on the cost channel coefficient increases above unity.

While there are positive implications of allowing the cost channel to vary over time, it is not clear how this will impact the determinacy regions of Taylor rules. In principle, introducing a time-varying cost channel could either enlarge or shrink the determinacy regions of Taylor rules in this sticky wage model. For example, in the seminal work of Davig and Leeper (2007), allowing the policy-rule coefficients to switch between active and passive can enlarge determinacy regions because the passive regime is known to be temporary and therefore inflation expectations remain anchored. Similarly here, if the cost channel is only temporarily binding, then even an aggressive inflation targeting central bank may be able to keep inflation expectations anchored during an inflation scare because firms expect, with a certain probability, that their future financing costs will decrease.

Of course, Davig and Leeper (2007) also show the cross-regime interactions can work in the other direction and shrink determinacy regions. If one regime is too passive, the passive regime can spillover and result in indeterminacy despite the presence of an active regime. In the work of Foerster (2016) for example, having both regimes active is neither necessary nor sufficient for equilibrium determinacy. In this cost-channel model the analogue could occur if policy is overly aggressive. If policy makers raise interest rates in response to an inflation scare in the no cost-channel regime, firms recognize that even though they have low financing costs today, there is a possibility that they will have to finance their inputs in the future. This expected future borrowing cost leads them to post higher prices now, making the potential inflation scare self-fulfilling.

In the numerical analysis that follows, I find this latter spillover to dominate the former. In fact, indeterminacy can arise in the MS-DSGE model for empirically plausible interest rate responses. Targeting output in addition to inflation does not alleviate the indeterminacy problem, implying that Taylor rules are indeterminate for a wide range of parameter values. As the results from the constant parameter model would suggest, this indeterminacy can be alleviated by adding a nominal growth target to the policy rule.

## 5.1 Solution Procedure and Equilibrium Refinement

I find solutions to the linearly approximated MS-DSGE model using the perturbation procedure developed by Foerster, Rubio-Ramirez, Waggoner, and Zha (2016). This model includes endogenous state variables, which precludes the use of Sims (2002) solution procedure, for example, to find the minimum state variable (MSV) solutions. Meanwhile, the solution method of Farmer, Waggoner, and Zha (2011) can analyze determinacy in such a regime-switching model in principle, but doesn't address how to perturb the non-linear model and relies on numerical analysis to find all possible equilibria. The method of Foerster et al. (2016) sidesteps both of these issues. First, the Foerster et al. (2016) method partitions the parameters of the non-linear model between those that affect the steady state and those that don't. This is particularly useful because it allows for the analysis of local determinacy in the constant parameter model and the switching parameter model from the same non-linear DSGE model. Second, their method uses Groebner Bases to find all possible equilibria.

To understand the general problem with solving rational expectations MS-DSGE models, Foerster et al. (2016) show that the regime dependent decision rules must satisfy a system of quadratic polynomial equations. In constant parameter models, this system of quadratic polynomials takes a special form that can be generally solved using a QZ decomposition as in Klein (2000) or Sims (2002). Hence, in this sense, using Groebner Bases does for rational expectation MS-DSGE models what the QZ decomposition does for rational expectation constant parameter DSGE models. However, this generalization in the solution procedure comes with a cost. While the Foerster et al. (2016) approach using Groebner Bases can find all MSV solutions of MS-DSGE models, even in this small model it is computationally intensive. To analyze determinacy regions I perform a grid search over hundreds of evenly spaced points. Even after parallelizing the computations in Mathematica, analyzing determinacy regions on a high performance cluster can take several days for one grid.

Once the set of all possible solutions to the Markov-Switching rational expectations problem are found, this large set of equilibria is further refined by selecting those which satisfy a stability concept.<sup>6</sup> The possible solution concepts are mean-squared stability (MSS) which requires the model solution to admit finite first and second moments, but allows for potentially unbounded realized paths, and both regimes stable (BRS) which requires the model's values not to wander off in a simulation if the model permanently stayed in any given regime.

The MSS refinement concept is appealing from an optimal policy perspective as policy makers who seek to maximize the welfare of the representative consumer, with or without the cost channel, minimize the expected variance of the welfare relevant output gap, inflation, and the nominal interest rate (Woodford, 2003; Ravenna and Walsh, 2006). Therefore, policy rules which bring about equilibria with explosive second moments, which would not satisfy the MSS concept, are clearly sub-optimal. That being said, equilibria which satisfy the MSS concept but nonetheless admit explosive realized paths may be difficult for policy makers to explain to the public as consistent with the central bank's mandates. This provides some motivation for focusing on BRS equilibria which eliminate some of the unbounded equilibria that are permissible under the MSS concept.

Although there is no consensus on the appropriate refinement strategy, the MSS solution refinement has been the most widely proposed concept (Foerster et al., 2016; Farmer, Waggoner, and Zha, 2009). Therefore, to align the results in this paper with those in the previous literature, I use the MSS concept here as well. However, I reference the determinacy regions implied by the BRS refinement when it is informative as to the source of equilibrium multiplicity. For example, as Foerster (2016) points out, if a calibration leads to indeterminacy under MSS but determinacy under BRS, there is an equilibrium with an explosive regime but the regime occurs rarely enough to keep the model's moments bounded.

## 5.2 Calibration

Introducing Markov switching to the DSGE model implies the switching process needs to be calibrated as well. In particular, the parameter  $\alpha(s_t)$  can take on two values:  $\alpha(s_t = 1) = 0$  (no cost channel) and  $\alpha(s_t = 2) = \alpha$  (cost channel). These different values of  $\alpha$  are determined by an exogenous 2-state Markov process with transition matrix,

$$\mathcal{P} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

where  $0 < p < 1$  is the probability of staying in the no cost-channel and  $0 < q < 1$  is the probability of staying in the cost-channel regime.

Since there are no direct estimates to guide the calibration of these parameters, in what follows I vary these parameters to illustrate how the results depend on the Markov switching process. I begin with two extreme calibrations ( $p = 0.90, q = 0.05$ ) and ( $p = 0.05, q = 0.90$ ) to highlight that Taylor rule indeterminacy can emerge even when regime switches are infrequent and short-lived so that the economy rests almost permanently in the no cost-channel or almost permanently in the cost-channel regime. I also report determinacy results for two empirically motivated calibrations of the Markov switching process. Finally, in the sensitivity analysis, I illustrate how the determinacy or indeterminacy of the Taylor rule depends on the parameterization of the Markov switching process by showing determinacy results across the  $(p, q)$  space.

In the first empirically motivated calibration, I set  $p = 0.97$  and  $q = 0.4$ . Interpreting the time variation in the cost channel at a business cycle frequency, this calibration suggests that expansions

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<sup>6</sup>The number of MSV solutions found by the Foerster et al. (2016) algorithm ranges between 36-40 depending on the exact values of the parameters.

last about 8 years on average. While the average length of the last three U.S. recessions is 10 months, Jermann and Quadrini (2012) find that random financial fluctuations contributed to the precipice of these recessions, not the entirety of the downturns. Therefore, this calibration generates a binding cost-channel for 5 months on average, which is consistent with the variance decompositions in Jermann and Quadrini (2012) that suggest shocks to firms borrowing constraints contribute to half of the variation in output.

In the second empirically motivated calibration, I set  $p = 0.6$  and  $q = 0.2$ . Interpreting the time variation in the cost channel at an inventory cycle frequency, this calibration suggests that inventory corrections occur every 2-3 quarters on average. Wen (2005) presents evidence that at this frequency, inventory investment is strongly countercyclical and production is less volatile than sales. This suggests that transitory demand shortfalls reduce firm sales more than production, driving inventories higher and reducing internal cash flows which forces firms into external financing for production and inventories.<sup>7</sup> The duration of the cost-channel regime in this calibration is consistent with the notion that within a quarter firms can typically draw down inventory build ups. For example, Fazzari and Petersen (1993) point out that finished-goods inventory stocks are typically equal to only a few days of production reinforcing Hornstein’s (1998) finding that inventory corrections contribute substantially to the quarterly variation in output, but not the business cycle component of output.

### 5.3 Taylor Rules and Indeterminacy with a MS Cost Channel

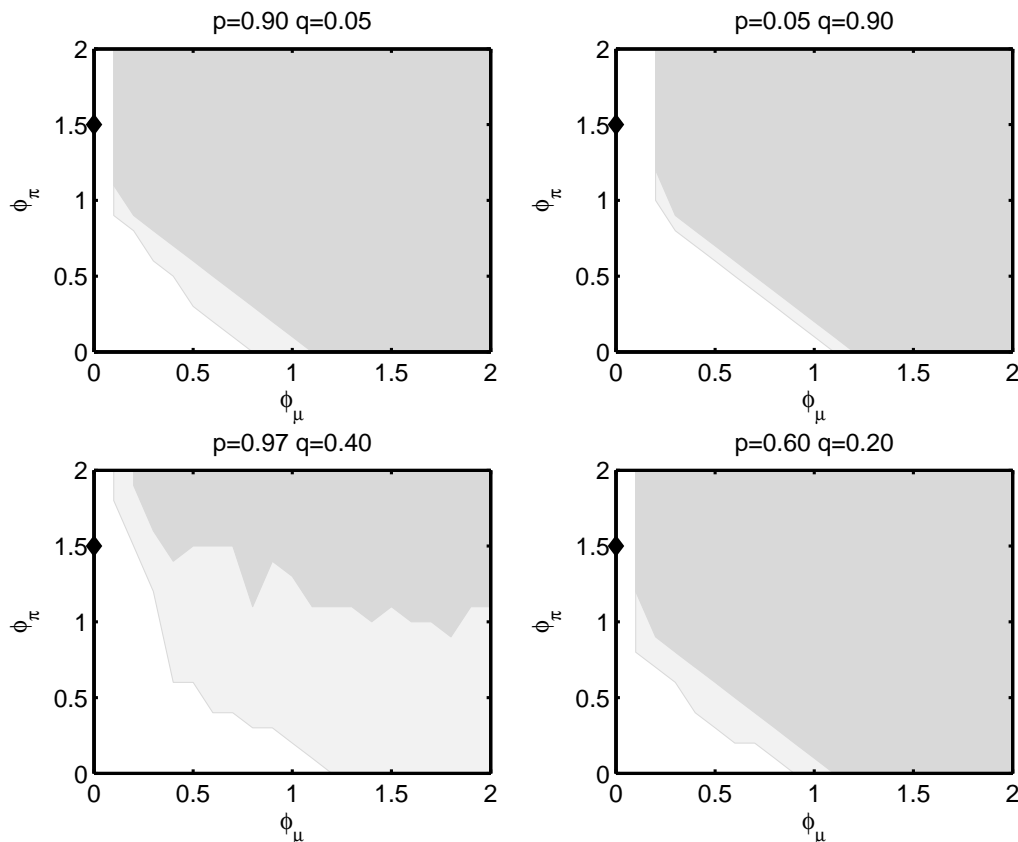
The primary finding when introducing an occasionally active cost channel is that Taylor rules can induce indeterminacy under a wide range of parameter values. Focusing first on BRS-refined equilibria, the Taylor rule leads to multiple equilibria that are stable should the economy rest permanently in each regime. This finding highlights the cross-regime spillovers introduced into the MS-DSGE model since calibrating the constant parameter model to either regime would result in a unique bounded equilibrium. But the fact that price setters internalize the possibility that they will have to finance working capital in the future leads to multiple bounded time paths under the Taylor rule even if the economy never actually experiences such a regime switch. Given this multiplicity of BRS equilibria, it is perhaps not surprising that the Taylor rule fails to isolate a unique equilibrium that is bounded in expectation. Though not true in general, in all the calibrations shown in Figure 3 determinacy under the BRS refinement is necessary for determinacy under the MSS refinement. Therefore, the inability of the Taylor rule to isolate a unique BRS time path for the economy leads to multiple time paths with finite first and second moments.

In addition to showing that Taylor rules are susceptible to indeterminacy when the cost channel varies over time, Figure 3 also highlights one way this indeterminacy can be resolved. Adding a small reaction to nominal money growth restores determinacy of the Taylor rule. In general, ensuring that  $\phi_\pi + \phi_\mu > 1$  is important for determinacy which essentially is a restatement of the Taylor principle. However, one important caveat is that  $\phi_\mu$  and  $\phi_\pi$  must each be sufficiently large. For example, while targeting only nominal money growth is sufficient to generate a unique BRS equilibria, such rules may require an independent inflation reaction to bring about a unique MSS equilibria. The calibration  $p = 0.97$  and  $q = 0.40$  is particularly insightful. While only reacting to money growth can bring about a unique BRS-bounded time path for the economy, this rule also enables explosive paths that are not BRS to maintain MSS. Adding a sufficiently large  $\phi_\pi$  coefficient to the rule causes these paths to explode fast enough to eliminate all of the MSS equilibria that are not BRS-bounded resulting in a single equilibrium that is both BRS and MSS.

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<sup>7</sup>This mechanism is clearly articulated in Barth and Ramey (2001).

Figure 3: Determinacy regions with a Markov-Switching cost channel when the central bank follows the policy rule:  $r_t = \phi_\pi \pi_t + 0.125x_t + \phi_\mu \mu_t$



Note: The light gray area is determinate under the BRS equilibrium refinement, the dark gray area is determinate under the MSS equilibrium refinement, and the white area is indeterminate. The black diamond denotes the baseline Taylor rule with  $\phi_\pi = 1.5$  and  $\phi_\mu = 0$ . Other parameters are set to the baseline U.S. calibration described in Table 4.

## 5.4 Decomposing Nominal Money Growth Targeting

This section uses an indirect approach to analyze the relative importance of adding interest rate smoothing and nominal GDP growth targeting to the Taylor rule to anchor expectations when the cost channel varies over time. Figure 3 highlights how adding a money growth target to the Taylor rules enlarges determinacy regions but, as previously discussed, a more natural interpretation of the change to the Taylor rule that occurs when equations 3, 5, and 6 are combined may be the addition of interest rate smoothing and nominal GDP growth targeting. While it would be interesting to directly analyze the relative importance of nominal GDP growth targeting versus interest-rate smoothing, computational limitations in the solution procedure limit the size of model for which determinacy regions can be analyzed. In particular, introducing lagged interest rates and output into the policy rule increases the number of state variables in the model and makes determinacy analysis computationally infeasible.<sup>8</sup>

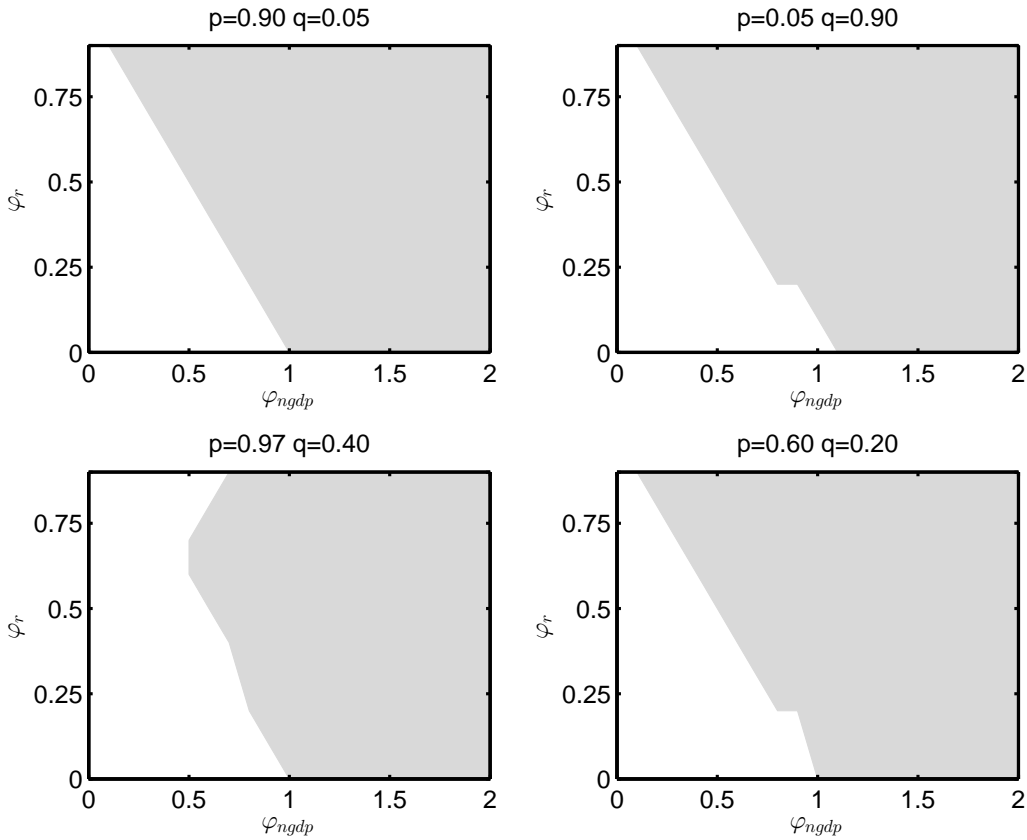
<sup>8</sup>The Groebner Bases procedure can even be sensitive to the parameterization of this small model. For some parameterizations of the baseline model (for example setting  $\omega < 0.6$ ), the Groebner-Bases procedure is unable to make progress in finding the number of solutions after more than 48 hours.

However, substituting equations 3 and 6 into equation 5, and ignoring exogenous shocks, gives:

$$\begin{aligned}
 r_t &= \frac{\eta\phi_\mu}{1 + \eta\phi_\mu}r_{t-1} + \frac{\phi_\mu}{1 + \eta\phi_\mu}(\Delta x_t + \pi_t) + \frac{\phi_\pi}{1 + \eta\phi_\mu}\pi_t + \frac{\phi_x}{1 + \eta\phi_\mu}x_t \\
 &= \varphi_r r_{t-1} + \varphi_{ngdp}(\Delta x_t + \pi_t) + \varphi_\pi \pi_t + \varphi_x x_t.
 \end{aligned}
 \tag{7}$$

In equation 7 above, for a given  $\phi_\mu$  sufficiently small, varying  $\eta$  varies the importance of the interest rate sensitivity whereas, for a given  $\eta$  sufficiently small, varying  $\phi_\mu$  varies the importance of the nominal GDP component. This allows for an indirect exploration of the roles played by interest rate smoothing versus nominal GDP growth targeting in inducing determinacy without introducing anymore state-variables into the existing model.<sup>9</sup>

Figure 4: Determinacy regions with a Markov-Switching cost channel when the central bank follows the policy rule:  $r_t = \varphi_r r_{t-1} + \varphi_{ngdp}(\Delta x_t + \pi_t)$



Note: The dark gray area is determinate under the MSS equilibrium refinement while the white area is indeterminate. Other parameters are set to the baseline U.S. calibration described in Table 4.

Figure 4 decomposes nominal money growth targeting into separate interest-rate smoothing and nominal GDP growth targeting components. In general,  $\varphi_r + \varphi_{ngdp} > 1$  should be satisfied to achieve determinacy which simply describes the long-run Taylor principle: In the long run, the central bank should react more than one for one to changes in nominal aggregates. However, a more robust prescription calls for setting  $\varphi_{ngdp} > 1$ . Consider for example the point  $(\varphi_r, \varphi_{ngdp}) = (0.9, 0.3)$ .

<sup>9</sup>In all the simulations  $\eta$  remains positive since setting  $\eta = 0$  is only possible by eliminating the shopping-time friction which generates the demand for deposits and thereby eliminating the source of funds for working capital loans.

While this point is well within the determinacy region for most calibrations of the Markov-switching parameters, it fails to achieve determinacy when  $p = 0.97$  and  $q = 0.40$ . Despite a high-degree of interest rate smoothing, the reaction to nominal GDP growth is insufficient to push the economy into the determinacy region. Similarly, when  $p = 0.97$  and  $q = 0.40$  the point  $(\varphi_r, \varphi_\pi) = (0.9, 0.3)$  is indeterminate, suggesting that satisfying the long-run Taylor principle alone is not always sufficient to anchor expectations on a unique equilibrium. Meanwhile, setting  $\varphi_{ngdp} > 1$  can isolate a unique MSS equilibrium in all these calibrations regardless of the degree of interest rate smoothing. Consequently, although interest rate smoothing can play an important role in achieving determinacy in the absence of no other form of history dependence in the monetary policy rule these examples echo the relative merits of directly targeting nominal GDP growth established in the constant parameter model.

## 5.5 Sensitivity Analysis

Shifting the baseline parameters in ways which increase the demand-side effects of monetary policy do not alleviate the indeterminacy of the Taylor rule. In particular, lowering the degree of wage rigidity, increasing the nominal rigidities (which flattens the Phillips curve), and increasing the value of the inverse Frisch elasticity of the labor supply still leads to Taylor rule indeterminacy (See Table 1). This highlights that the indeterminacy is not driven by an infinitely elastic labor supply curve, a mechanism which limits wage movements in response to interest rate changes and drives indeterminacy of forward looking interest rate rules (Bruckner and Schabert, 2003). Additionally, shifting the baseline parameters in ways which diminish the supply-side effects of monetary policy doesn't lead to determinate Taylor rules either. For example, assuming there is incomplete pass-through from policy rates to borrowing costs, as in Chowdhury et al. (2006), by setting  $0 < \alpha < 1$ , still results in indeterminacy under the Taylor rule.

Adjusting the policy rule, either by increasing the reaction to inflation or the output gap also fails to eliminate the indeterminacy of Taylor rules. While it is not surprising based on results from the constant parameter model that a higher inflation reaction fails to eliminate the indeterminacy, these results do raise the question as to why an output response doesn't alleviate the indeterminacy of the Taylor rule. Surico (2008) shows the equilibrium effect of reacting to output in the presence of the cost channel causes the output gap to fall less in response to a monetary contraction, therefore dampening the inflation response to a given movement in interest rates. Larger reactions to output therefore require larger reactions to inflation to counteract this effect. In the presence of real-wage rigidities, this effect is working to increase the lower-bound on the inflation response at the same time the wage-rigidity is working to decrease the upper-bound on the inflation response. Consequently, Table 1 indicates that a more aggressive output gap reaction not only fails to counteract indeterminacy, but it actually requires an even larger  $\phi_\mu$  coefficient to induce determinacy.

To understand how the calibration of the Markov switching process affects the determinacy properties of Taylor rules, Figure 5 shows determinacy results across the  $(p, q)$  space for  $0.05 \leq p, q \leq 0.95$  under the MSS equilibrium refinement. Several features stand out. First of all, when  $p = 1 - q$  both policy rules are determinate. This is because this calibration induces a constant cost channel in expectation, eliminating all the cross-regime spillovers.<sup>10</sup> Second, away from this diagonal, the Taylor rule is more likely to be indeterminate than not. Augmenting the Taylor rule with interest rate smoothing and a nominal GDP growth target, as implemented in this model by way of a money growth target, enlarges the determinacy region across the  $(p, q)$  space. Finally, money-growth targeting is not a panacea for Taylor rule indeterminacy. For several parameter combinations a stronger reaction to nominal GDP growth than is provided by money growth targeting is required. For example, when

<sup>10</sup>To see this, consider the one-step ahead forecast of  $\alpha(s_t)$ . If the economy is currently in regime 1 then:  $\mathbb{E}_t \alpha(s_{t+1}) = p\alpha(s_{t+1} = 1) + (1 - p)\alpha(s_{t+1} = 2)$ . Similarly, if the economy is currently in regime 2 then:  $\mathbb{E}_t \alpha(s_{t+1}) = (1 - q)\alpha(s_{t+1} = 1) + q\alpha(s_{t+1} = 2) = p\alpha(s_{t+1} = 1) + (1 - p)\alpha(s_{t+1} = 2)$  because  $p = 1 - q$ .



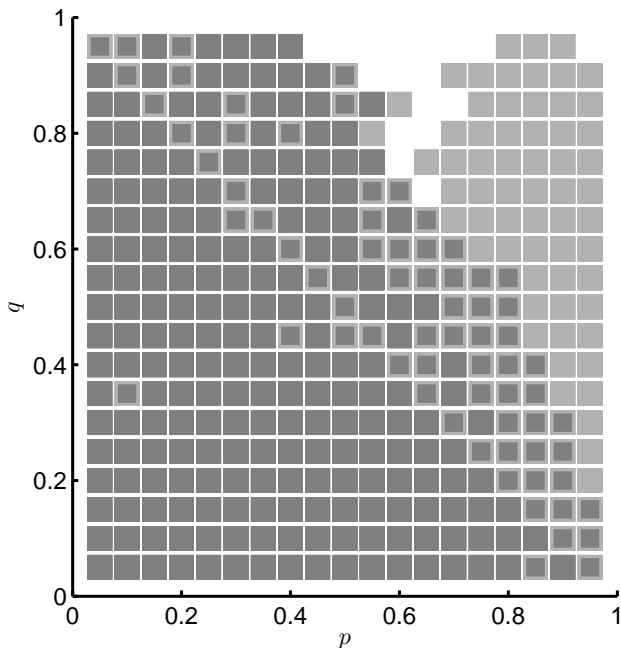
Table 1: Sensitivity Analysis

Calibration		Minimum $\phi_\mu$
$p = 0.90$ and $q = 0.05$		
Baseline		0.1
Less Real Wage Rigidity	$\rho = 0.5$	0.1
Small Frisch Elasticity	$\varphi = 1$	0.1
Flat Phillips Curve	$\omega = 0.9$	0.1
Fewer Cost Channel Firms	$\alpha = 0.5$	0.1
Aggressive Inflation Reaction in Policy Rule	$\phi_\pi = 5$	0.1
Large Output Gap Reaction in Policy Rule	$\phi_x = 0.5$	0.1
$p = 0.05$ and $q = 0.90$		
Baseline		0.2
Less Real Wage Rigidity	$\rho = 0.5$	0.1
Small Frisch Elasticity	$\varphi = 1$	0.1
Flat Phillips Curve	$\omega = 0.9$	0.2
Fewer Cost Channel Firms	$\alpha = 0.5$	0.1
Aggressive Inflation Reaction in Policy Rule	$\phi_\pi = 5$	0.3
Large Output Gap Reaction in Policy Rule	$\phi_x = 0.5$	0.3
$p = 0.97$ and $q = 0.40$		
Baseline		0.4
Less Real Wage Rigidity	$\rho = 0.5$	0.1
Small Frisch Elasticity	$\varphi = 1$	0.3
Flat Phillips Curve	$\omega = 0.9$	0.2
Fewer Cost Channel Firms	$\alpha = 0.5$	1.2
Aggressive Inflation Reaction in Policy Rule	$\phi_\pi = 5$	0.2
Large Output Gap Reaction in Policy Rule	$\phi_x = 0.5$	1.5
$p = 0.60$ and $q = 0.20$		
Baseline		0.1
Less Real Wage Rigidity	$\rho = 0.5$	0.1
Small Frisch Elasticity	$\varphi = 1$	0.2
Flat Phillips Curve	$\omega = 0.9$	0.1
Fewer Cost Channel Firms	$\alpha = 0.5$	0.1
Aggressive Inflation Reaction in Policy Rule	$\phi_\pi = 5$	0.1
Large Output Gap Reaction in Policy Rule	$\phi_x = 0.5$	0.2

Notes: This table is constructed by fixing  $\phi_\mu = 1.5$ ,  $\phi_x = 0.125$ , and searching a grid of values of  $\phi_\mu$  starting from 0 and increasing by 0.1 steps. The first value of  $\phi_\mu$  which yields a unique MSS equilibrium is reported in the third column.

$p = 0.95$  and  $q = 0.95$  the Taylor rule is indeterminate but, in the spirit of equation 7, adding an aggressive nominal GDP target with  $\varphi_{ngdp}$  around 6 can anchor inflation expectations. Similar rules are effective in delivering a unique MSS equilibrium for the other combinations of  $(p, q)$  where multiple MSS equilibrium arise under the Taylor rule.

Figure 5: Determinacy regions with a Markov-Switching cost channel when the central bank follows the inflation targeting rule:  $r_t = 1.5\pi_t + 0.125x_t + \phi_\mu\mu_t$



Note: The light gray squares are determinate under the Taylor rule with  $\phi_\mu = 0$ , the dark squares are determinate under the Taylor rule with  $\phi_\mu = 1.5$ , the light and dark gray squares are determinate under both rules, and the white area is indeterminate under both rules. Other parameters are set to the baseline U.S. calibration described in Table 4.

## 6 Implementing Determinate Policy Rules

The limited ability of Taylor rules to anchor expectations when the cost channel varies over time raises the question of how central banks can achieve determinacy in practice. While interest rate smoothing is helpful, nominal GDP growth targeting seems the more effective option for keeping inflation expectations anchored. In practice, nominal money growth targeting is often suggested as a means of achieving a nominal GDP growth target. For example, Feldstein and Stock (1994) and Belongia and Ireland (2015) put forth equation of exchange based approaches to targeting nominal GDP using monetary aggregates. These approaches rely on the relative stability of trend money velocity which, in other words, implies a stable money demand relationship – a nontrivial assumption. In such cases the central bank is able to target nominal money growth to achieve a nominal GDP growth target and the policy rule in Equation 5 can be interpreted as an interest rate rule containing an intermediate nominal money growth target.

The European Central Bank (ECB) in particular has designated a role to monetary aggregates in its institutional design to exploit the long-run link between money and prices. In particular, the governing council uses a two-pillar approach to achieve price stability. The first pillar rests on economic analysis of dynamics and shocks which may affect short and medium-term developments in prices. The second pillar rests on monetary analysis with a focus on the medium to long-run prospects for price stability implied by the growth rate of a broad monetary aggregate (M3). The governing council takes into consideration information from both pillars, using one another to “cross-check” the risks to the ECB’s objective of price-stability over the medium term.

Table 2: Econometric Estimation of the ECB Reaction Function

$r_t = \phi_0 + \phi_\pi \pi_t + \phi_\mu \mu_t + \phi_y \Delta y_t + u_t$			
	$\hat{\phi}_\pi$	$\hat{\phi}_\mu$	$\hat{\phi}_y$
<i>Baseline</i>			
GMM Estimates	1.490***	0.164***	
Standard-Error	0.100	0.044	
<i>With Alternative Instrument Set</i>			
GMM Estimates	1.512***	0.157***	
Standard-Error	0.096	0.052	
<i>With Output Growth</i>			
GMM Estimates	1.486***	0.170***	0.074
Standard-Error	0.097	0.044	0.081

Note 1: The instrument set includes a constant and 1 to 4 lags of: real unit labor cost, HP-filtered output gap, GDP deflator inflation, commodity price inflation, the term-spread. The instrument set follows closely from Ravenna and Walsh (2006).

Note 2: The alternative instrument set includes a constant and 1 to 4 lags of detrended output, GDP deflator inflation, the short-term interest rate, M3 growth, commodity price inflation, and the term-spread. The instrument set follows closely from Clarida, Gali, and Gertler (2000).

Note 3: One, two, or three asterisks denote statistical significance at the 10%, 5%, or 1% level respectively.

Although ECB press conferences always feature an overview of both the economic and monetary analysis, it is unclear how much weight the ECB has historically placed on money growth when setting interest rates. For the purposes of calibrating their policy rule, I estimate a simple reaction function for the ECB using the Area-Wide Model (AWM) dataset constructed by Fagan, Henry, and Mestre (2001). The estimation strategy uses two-step GMM to efficiently deal with the endogeneity inherent in central bank reaction functions. The data used covers the sample from 1981-Q1 to 2013-Q4. The short-term interest rate is used to measure  $r_t$ , the GDP deflator is used to measure  $\pi_t$  and the growth rate of M3 is used to measure  $\mu_t$ . All standard errors are Newey-West HAC to correct for serial correlation.<sup>11</sup>

The estimated rule has a similar coefficient on inflation as in the Taylor rule (Taylor, 1993) and a statistically significant reaction to M3 money growth. The magnitude of the response to nominal money growth is similar to that found in Andres, Lopez-Salido, and Nelson (2009) who estimate the ECB's policy rule as part of a DSGE model. The coefficient estimates are rather robust to varying the instrument sets and including a reaction to real GDP growth. One concern is that by specifying the rule without any inclusion of real economic activity, the coefficient estimate on M3 growth is biased as a result of an omitted variable. However, including real GDP growth in the reaction function yields an insignificant coefficient on real activity and has no significant effect on the other parameters.<sup>12</sup>

I now explore the merits of the ECB's two-pillar approach to price stability by asking whether it is likely to anchor expectations on a unique MSS equilibrium. The approach to analyzing the determinacy properties of the ECB's policy rule follows the spirit of the analysis in Coibion and Gorodnichenko (2011). In particular, determinacy is viewed as a probabilistic outcome which is quantified by drawing policy parameters from the joint asymptotic Normal distribution of the estimated rule many times

<sup>11</sup>All of the GMM estimates fail to reject the null that the over-identifying restrictions are satisfied.

<sup>12</sup>Including an interest rate smoothing-term in the policy rule yielded discouraging results for the inflation reaction parameter. In particular, the estimates of  $\phi_\pi$  were small and insignificant when the policy rule was specified as  $r_t = \phi_0 + \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_\mu \mu_t + \phi_y \Delta y_t) + u_t$ .

and calculating the percentage of parameter draws which induce determinacy.<sup>13</sup> One complication arises in this setting though; it is not computationally feasible to check determinacy at each point drawn during the simulation. To sidestep this issue, I calculate determinacy on an evenly spaced grid of  $(\phi_\pi, \phi_\mu)$  points and then use bilinear interpolation to calculate the probability that any given draw is determinate. The average probability across all the draws then represents the likelihood that the estimated policy rule results in a unique MSS equilibrium.

Table 3: Probably of Determinacy Under the Estimated ECB Policy Rule

Calibration		Estimated Rule	$\phi_\mu = 0$
$p = 0.90$ and $q = 0.05$			
Baseline		98.60%	0.00%
Less Real Wage Rigidity	$\rho = 0.5$	98.62%	0.00%
Small Frisch Elasticity	$\varphi = 1$	98.69%	0.00%
Flat Phillips Curve	$\omega = 0.9$	98.71%	0.00%
Fewer Cost Channel Firms	$\alpha = 0.5$	98.64%	0.00%
$p = 0.05$ and $q = 0.90$			
Baseline		93.76%	0.00%
Less Real Wage Rigidity	$\rho = 0.5$	98.54%	0.00%
Small Frisch Elasticity	$\varphi = 1$	87.07%	0.00%
Flat Phillips Curve	$\omega = 0.9$	66.76%	0.00%
Fewer Cost Channel Firms	$\alpha = 0.5$	85.65%	0.00%
$p = 0.97$ and $q = 0.40$			
Baseline		0.00%	0.00%
Less Real Wage Rigidity	$\rho = 0.5$	60.41%	0.00%
Small Frisch Elasticity	$\varphi = 1$	0.00%	0.00%
Flat Phillips Curve	$\omega = 0.9$	0.00%	0.00%
Fewer Cost Channel Firms	$\alpha = 0.5$	0.00%	0.00%
$p = 0.60$ and $q = 0.20$			
Baseline		65.41%	0.00%
Less Real Wage Rigidity	$\rho = 0.5$	98.62%	0.00%
Small Frisch Elasticity	$\varphi = 1$	98.59%	0.00%
Flat Phillips Curve	$\omega = 0.9$	80.17%	0.00%
Fewer Cost Channel Firms	$\alpha = 0.5$	98.62%	0.00%

Notes: This table is constructed by making 50,000 draws from a multi-variate normal distribution with a mean equal to the baseline point estimates in Table 2 and with a covariance matrix equal to the corresponding estimated covariance matrix. The average probability across all draws is reported in the table. All other parameters are fixed at the baseline euro area calibration from Table 4.

The estimated reaction function of the ECB is likely to deliver a unique MSS equilibrium for the majority of parameter values considered in Table 3. Lowering the degree of real-wage rigidity increases the demand side effects of monetary policy and increases the probability of determinacy. Since real wage rigidity is adjusted while holding fixed the level of price rigidity, the most natural

<sup>13</sup>Under assumptions laid out in Hansen (1982), the limiting distribution of the GMM estimator will be distributed Normally. This exercise can also be given a Bayesian interpretation of performing a posterior predictive analysis if a non-informative Normal prior is specified over the policy parameters so that the posterior distribution is also Normal (Chernozhukov and Hong, 2003).

interpretation of this comparative static is one of reducing nominal wage rigidity.<sup>14</sup> These results suggest that if structural reforms were implemented to make wages more flexible in the euro area, as recently proposed by the ECB, the IMF, and the OECD, the ECB’s policy rule would be more likely to result in determinacy.<sup>15</sup> Interestingly, if the ECB were to drop the second pillar of its price stability framework by setting  $\phi_\mu = 0$  then the economy would be pushed into the indeterminate region of the parameter space for all of the parameter variations considered.

This doesn’t however imply that there isn’t scope for altering the policy rule to further improve the prospects of achieving determinacy. For example, the calibration of the Markov-switching parameters  $p = 0.97$  and  $q = 0.4$  reveals that the ECB’s estimated rule can, for some calibrations, result in indeterminacy. In these cases, even when  $\rho = 0.5$ , the ECB’s historical policy rule lies near the boundary of the determinacy region. The primary shortcoming of the ECB’s rule in these cases is an insufficient reaction to nominal growth. Simply increasing  $\phi_\mu = 0.5$  would make determinacy much more likely. However, for some calibrations, such a rule would still likely lead to multiple equilibria. In such cases the central bank could increase the response to nominal GDP growth, which is governed in the money growth targeting rule by the interest semi-elasticity of money demand. This highlights a primary shortcoming of targeting nominal GDP growth by targeting nominal money growth; the relative weights on lagged interest rates and nominal GDP growth depend on private sector parameters, such as the interest semi-elasticity of money demand. More directly targeting nominal GDP growth would likely increase the probability of determinacy under these calibrations according to Figure 4.

## 7 Conclusion

Real wage rigidities, when combined with the cost channel, shrink the determinacy regions of interest rate rules targeting inflation. These smaller determinacy regions pose no challenge to central banks who seek to stabilize inflation by following interest rate rules with coefficients of the magnitude specified in the Taylor (1993) rule. However, when the cost channels varies according to a two-state Markov-Switching process, these interest rate rules often result in multiple equilibria. Adding interest rate smoothing and a nominal GDP growth target, perhaps implemented by way of targeting nominal money growth, provides a practical way to alleviate this indeterminacy in many instances. In light of these results, I estimate a reaction function for the ECB and find evidence of their “two-pillar” approach to price stability which uses monetary analysis as a cross-check to achieve its inflation target. The small coefficient on money-growth in the ECB’s reaction function increases the probability of determinacy of otherwise indeterminate Taylor rules in the MS-DSGE model.

Central banks can realize the stabilization benefits of targeting nominal money growth without exposing the economy to shifts in money demand by adding interest rate smoothing and nominal GDP growth to their policy reactions. Such rules tend to have more stable determinacy regions than typical Taylor rules regardless of the strength of, or variation in, the cost channel. Among these two components, targeting nominal GDP growth is relatively more effective than adding lagged interest rates to the policy rule in a number of situations according to analytic results from the constant parameter cost channel model and numerical analysis in the Markov-switching cost channel model. From a broader perspective, this paper finds that history dependence, by way of nominal money growth targeting, interest rate smoothing, or nominal GDP growth targeting is a common feature of policy rules which anchor inflation expectations at the central bank’s target and hence lead to better macroeconomic outcomes in the presence of the cost channel.

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<sup>14</sup>Knell (2013) shows in a model with nominal wage and price rigidity, the degree of real wage rigidity is influenced primarily by the nominal rigidity parameters.

<sup>15</sup>See [https://www.ecb.europa.eu/pub/pdf/other/art01\\_eb201502.en.pdf](https://www.ecb.europa.eu/pub/pdf/other/art01_eb201502.en.pdf), <https://www.imf.org/external/np/seminars/eng/2014/EurBook/pdf/7.pdf>, and [https://www.oecd.org/about/publishing/Euro\\_Area\\_Brochure\\_EN.pdf](https://www.oecd.org/about/publishing/Euro_Area_Brochure_EN.pdf)

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# A DSGE Model

This section describes the DSGE model used in the paper in detail. The model motivates the cost channel through a timing mismatch between when firms pay their wage bill and receive payment for their output. This timing can be described by dividing period  $t$  into 2 separate sub-periods: first a production and trading period and then a settlement period.

## Sub-Period 1: Production and Trading Period

- All shocks are realized.
- The intermediate goods firms hire labor to produce their differentiated output. The final goods firm purchases inputs from the intermediate goods firms. A fraction of these purchases are paid for on the spot, the remaining fraction are bought on zero-interest firm credit. The intermediate goods producers finance a portion of their wage bill  $\alpha(s_t)$  with a working-capital loan from the bank. All wages are paid. The household buys consumption goods and carries out financial transactions.

## Sub-Period 2: Settlement Period

- The fraction of the intermediate goods that haven't yet been paid for by the final goods producers, receive payment from the final goods producing firms allowing these firms to pay-off their working-capital loans with interest. The household receives their deposits with interest from the bank and dividend payments from the intermediate goods firms.

## A.1 The Household

The representative household enters any period  $t = 0, 1, 2, \dots$  with a portfolio consisting of maturing bonds  $B_{t-1}$  and monetary assets totaling  $A_{t-1}$ . The household faces a sequence of budget constraints in any given period. In the securities trading session the household can buy and sell bonds, receives wages  $W_t$  for hours worked  $H_t$  during the period, purchases consumption goods  $C_t$  and allocates their monetary assets between currency  $N_t$  and deposits  $D_t$ . Any loans  $L_t^{hh}$  needed to finance these transactions are made at this time. This is summarized in the constraint below.

$$N_t + D_t = \frac{B_{t-1}}{\Pi_t} + \frac{A_{t-1}}{\Pi_t} - \frac{B_t}{R_t} + W_t H_t - C_t + L_t^{hh} \quad (\text{A.1})$$

At the end of the period, the household receives dividends  $F_{i,t}$  from intermediate goods firms and receives interest on deposits  $R_t^D D_t$ , repays loans  $R_t^L L_t^{hh}$  and receives any residual assets of bank  $F_t^b$ . Any remaining funds are combined with central bank transfers  $\tau_t$  and are carried over in the form of monetary assets  $A_t$  into the next period.

$$A_t = N_t + \int_0^1 F_{i,t} di + R_t^D D_t - R_t^L L_t^{hh} + F_t^b + \tau_t. \quad (\text{A.2})$$

The household seeks to maximize their lifetime utility, discounted at rate  $\beta$ , subject to these constraints. The period flow utility of the household takes the following form.

$$U_t = \zeta_t \left[ \ln(C_t) - \xi \frac{H_t^{1+\varphi}}{1+\varphi} - H_t^s \right]$$

The household receives utility from consumption and dis-utility from working and shopping. Time spent shopping increases with aggregate consumption  $C_t^A$  (i.e. long lines) but is reduced with higher

liquidity services.<sup>16</sup> Therefore the time spent shopping takes the following form:

$$H_t^s = \frac{1}{\chi} \left( \frac{v_t C_t^A}{M_t} \right)^\chi. \quad (\text{A.3})$$

The time-varying preference parameter  $\zeta_t$  enters the linearized Euler equation as an IS shock and similarly,  $v_t$  enters the linearized money demand equation as a money demand shock. Both of these processes are assumed to follow an AR(1) (in logs).

$$\ln(\zeta_t) = \rho_\zeta \ln(\zeta_{t-1}) + \varepsilon_t^\zeta \quad \varepsilon_t^\zeta \sim \mathcal{N}(0, \sigma_\zeta) \quad (\text{A.4})$$

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \varepsilon_t^v \quad \varepsilon_t^v \sim \mathcal{N}(0, \sigma_v) \quad (\text{A.5})$$

The monetary aggregate,  $M_t$ , which enters the shopping-time function takes a rather general CES form,

$$M_t = \left[ \nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (\text{A.6})$$

where  $\nu$  calibrates the relative expenditure shares on currency and deposits and  $\omega$  calibrates the elasticity of substitution between the two monetary assets. Given these parameters,  $\chi$  is left free to calibrate the interest semi-elasticity of money demand.

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. Letting  $\mathcal{C}_t = [C_t, H_t, M_t, N_t, D_t, L_t^{hh}, B_t, A_t]$  denote the vector of choice variables, the household's problem can be recursively defined using Bellman's method:

$$\begin{aligned} V_t(B_{t-1}, A_{t-1}) = \max_{\mathcal{C}_t} & \left\{ \zeta_t \left[ \ln(C_t) - \xi \frac{H_t^{1+\varphi}}{1+\varphi} - \frac{1}{\chi} \left( \frac{v_t C_t^A}{M_t} \right)^\chi \right] \right. \\ & - \lambda_t^1 \left( D_t + N_t + C_t - L_t^{hh} - W_t H_t - A_{t-1}/\Pi_t + B_t/R_t - B_{t-1}/\Pi_t - \tau_t \right) \\ & - \lambda_t^2 \left( M_t - \left[ \nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right) \\ & \left. - \lambda_t^3 \left( A_t - N_t - \int_0^1 F_{i,t} di - R_t^D D_t + R_t^L L_t^{hh} - F_t^b \right) + \beta \mathbb{E}_t [V_{t+1}(B_t, A_t)] \right\}. \end{aligned}$$

The first order necessary conditions are given by the following equations:

$$\frac{\zeta_t}{C_t} = \beta \mathbb{E}_t \left[ \frac{\zeta_{t+1}}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right] \quad (\text{A.7})$$

$$MRS_t = \xi H_t^\varphi C_t \quad (\text{A.8})$$

$$W_t = W_{t-1}^{\rho_W} MRS_t^{(1-\rho_W)} \quad (\text{A.9})$$

$$\frac{v_t C_t}{M_t} = \left( \frac{\lambda_t^2}{\lambda_t^1} v_t \right)^{\frac{1}{\chi+1}} \quad (\text{A.10})$$

$$N_t = \nu M_t \left[ \frac{\lambda_t^2/\lambda_t^1}{(R_t - 1)/R_t} \right]^\omega \quad (\text{A.11})$$

$$D_t = (1-\nu) M_t \left[ \frac{\lambda_t^2/\lambda_t^1}{(R_t - R_t^D)/R_t} \right]^\omega \quad (\text{A.12})$$

$$R_t = R_t^L. \quad (\text{A.13})$$

<sup>16</sup>The assumption that aggregate consumption enters the shopping time specification as opposed to household consumption prevents the real-balances effect (i.e. real balances appearing in the log-linearized IS and Phillips Curve) which is not well supported by either U.S. or European Data (See for example Ireland (2004); Andres et al. (2009).)

The above equations are quite standard with a few exceptions. Notice that equations A.8 and A.9 can be combined to yield the typical condition that the real wage equals the marginal rate of substitution when  $\rho_w = 0$ . However, when  $0 < \rho_w < 1$  this condition only holds in steady state and there may be short-run deviations from this optimality condition due to real wage rigidity as in Hall (2005) and Blanchard and Gali (2007). Also, I have imposed  $C_t^A = C_t$  in the shopping-time function after optimizing.

## A.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There are a unit measure of intermediate goods producing firms indexed by  $i \in [0, 1]$  who produce a differentiated product. The final goods firm produces  $Y_t$  combining inputs  $Y_{i,t}$  using the production technology,

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

in which  $\theta > 1$  governs the elasticity of substitution between inputs. The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem:

$$\max_{Y_{i,t} \in [0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di,$$

subject to the above constant returns to scale technology. The resulting first order condition defines the demand curve for each intermediate goods producing firm's product:

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t. \quad (\text{A.14})$$

## Intermediate Goods Producing Firm

Given the downward sloping demand for its product in A.14, the intermediate goods producing firm has the ability to set the price of its product above marginal cost. To permit aggregation and allow for the consideration of a representative firm, I assume all such firms have the same constant returns to scale technology:

$$Y_{i,t} = z_t H_{i,t}. \quad (\text{A.15})$$

The  $z_t$  term in A.15 is an aggregate technology shock that follows an AR(1),

$$\ln(z_t) = \rho_z \ln(z_{t-1}) + \varepsilon_t^z \quad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z). \quad (\text{A.16})$$

The term  $H_{i,t}$  in the production function denotes the level of employment chosen by the intermediate goods firm. Given the linear production function, the intermediate goods producing firm's real marginal cost takes the same functional form in the effective factor prices:

$$MC_t = v \left( \frac{R_t^L}{\bar{R}^L} \right)^{\alpha(s_t)} \frac{W_t}{P_t z_t},$$

which shows the effective wage rate differs from the real wage according to the share of the wage bill which must be financed. Reasons for using this functional form to specify the cost channel in the non-linear model are discussed further in Section A.6. A production subsidy,  $v$ , is introduced to make the steady state price of goods equal to the social marginal cost of production. Without the subsidy, the monopolist would set prices higher than the marginal social benefit.

The price setting ability of each firm is constrained as in Calvo (1983). In this staggered price-setting framework, the price level  $P_t$  is determined in each period as a weighted average of a fraction of firms  $1 - \omega$  are able to re-optimize their price and a fraction  $\omega$  must leave their prices unchanged. Therefore, each firm maximizes the present value of its current and future discounted profits, taking into account the possibility that the firm may not be able to re-optimize for sometime:

$$\max_{\{P_{i,t}^*\}} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega)^j \frac{\lambda_{t+j}^3}{\lambda_t^1} [P_{i,t}^* Y_{i,t+j} - MC_{t+j} P_{t+j} Y_{i,t+j}]$$

subject to

$$Y_{i,t+j} = \left( \frac{P_{i,t}^*}{P_{t+j}} \right)^{-\theta} Y_{t+j}.$$

The firm's first order condition is given by:

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega)^j \frac{\lambda_{t+j}^3}{\lambda_t^1} Y_{i,t+j} \left( \frac{P_{i,t}^*}{P_{t-1}} - \frac{\theta}{\theta-1} MC_{t+j} \prod_{k=0}^j \Pi_{t+k} \right) = 0. \quad (\text{A.17})$$

Finally, in the symmetric equilibrium, the aggregate price dynamics are determined by the following price aggregate:

$$P_t = \left[ \omega (P_{t-1})^{1-\theta} + (1-\omega) (P_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (\text{A.18})$$

where  $P_t^*$  is the optimal price firms choose who re-optimize in period  $t$ .

### A.3 The Financial Firm

The financial firm performs the intermediation process of accepting household's deposits and in turn loaning these funds to firms and households. The financial firm must satisfy the accounting identity which specifies assets (loans to firms plus reserves) equal liabilities (deposits),

$$L_t^{hh} + L_t^f + rr D_t = D_t. \quad (\text{A.19})$$

Although changes in banking regulation have effectively eliminated reserve requirements, banks may often choose to hold reserves in lieu of making loans. Therefore, instead of assuming the central bank controls the reserve ratio  $rr$ , I assume it is exogenously fixed and represents the average ratio of deposits banks hold for regulatory and liquidity purposes.

The financial firm chooses  $L_t = L_t^{hh} + L_t^f$  and  $D_t$  in order to maximize period profits

$$\max_{L_t, D_t} R_t^L L_t - R_t^D D_t - L_t + D_t$$

subject to the balance sheet constraint A.19. Since the loan and deposits markets are perfectly competitive, substituting the balance-sheet constraint into the profit function and imposing zero profits results in the loan-deposit spread,

$$R_t^L - R_t^D = (R_t^L - 1)rr. \quad (\text{A.20})$$

This expression describes the loan deposit spread as a function of the foregone revenue of making loans when deposits are held as reserves instead of being loaned out.

## A.4 Equilibrium and the Output Gap

Here I define the equilibrium conditions which close the model. Equilibrium in the final goods market requires that the accounting identity

$$Y_t = C_t \quad (\text{A.21})$$

holds. Equilibrium in the money market, bond market and loan market requires that at all times:

$$\begin{aligned} A_t &= \frac{A_{t-1}}{\Pi_t} + \tau_t \\ B_t &= B_{t-1} = 0 \\ L_t &= L_t^{hh} + \int_0^1 L_{i,t}^f di \end{aligned}$$

respectively. Market clearing in the labor market requires that labor supply equals labor demand:

$$H_t = \int_0^1 H_{i,t} di = \int_0^1 \frac{Y_{i,t}}{z_t} di = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} di \frac{Y_t}{z_t}$$

where the second equality uses the firm's production function A.15 and the third equality uses the demand for the intermediate goods product A.14. Therefore, aggregate output is related to aggregate labor supply and technology by:

$$Y_t = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} di H_t z_t. \quad (\text{A.22})$$

To facilitate the analysis of monetary policy rules which feature a reaction to the real economy, I will log-linearize the model in terms of the efficient, or first-best, output gap  $X_t = \frac{Y_t}{Y_t^*}$ . In this economy,  $Y_t^*$  this is the level of output arising from the frictionless problem:

$$\max_{C_t, H_{i,t}} \zeta_t \left[ \ln(C_t) - \xi \frac{\left( \int_0^1 H_{i,t} di \right)^{1+\varphi}}{1+\varphi} \right] \quad \text{subject to} \quad C_t = z_t \left[ \int_0^1 H_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}.$$

The resulting first order condition yields the efficient level of output:  $Y_t^* = C_t^* = z_t / (\xi)^{1/(1+\varphi)}$ , which yields the following expression for the efficient output gap, denoted by  $X_t$ :

$$X_t = \frac{Y_t}{Y_t^*} = \xi^{1/(1+\varphi)} \frac{Y_t}{z_t}. \quad (\text{A.23})$$

Finally, the production subsidy  $v$  to the intermediate goods producers is set so that in steady state the price of goods equals the social marginal cost of production. The monopolistically competitive firm in the economy sets their price in steady-state:

$$P = (1 - v) \frac{\theta}{\theta - 1} PW.$$

The social marginal cost of production is the marginal rate of substitution between labor and consumption for the household, which in steady state, equals the real wage rate. Therefore, setting  $W = 1$  and solving for  $1 - v$  implies:

$$1 - v = \frac{\theta - 1}{\theta}. \quad (\text{A.24})$$

## A.5 The Non-Linear Model

This section lists the full set of equilibrium conditions from the non-linear model.

$$\frac{\zeta_t}{C_t} = \beta \mathbb{E}_t \left[ \frac{\zeta_{t+1} R_t}{C_{t+1} \Pi_{t+1}} \right] \quad (\text{A.25})$$

$$MRS_t = \xi C_t H_t^\varphi \quad (\text{A.26})$$

$$W_t = W_{t-1}^{\rho_w} MRS_t^{(1-\rho_w)} \quad (\text{A.27})$$

$$\frac{v_t C_t}{M_t} = (u_t v_t)^{\frac{1}{\chi+1}} \quad (\text{A.28})$$

$$N_t = \nu M_t \left[ \frac{u_t}{(R_t - 1)/R_t} \right]^\omega \quad (\text{A.29})$$

$$D_t = (1 - \nu) M_t \left[ \frac{u_t}{(R_t - R_t^D)/R_t} \right]^\omega \quad (\text{A.30})$$

$$M_t = \left[ \nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1 - \nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (\text{A.31})$$

$$\Lambda_t^1 = \frac{\zeta_t}{C_t} \quad (\text{A.32})$$

$$u_t = \Lambda_t^2 / \Lambda_t^1 \quad (\text{A.33})$$

$$\Lambda_t^1 = \Lambda_t^3 R_t^L \quad (\text{A.34})$$

$$R_t^L = R_t \quad (\text{A.35})$$

$$Y_t = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} di z_t H_t \quad (\text{A.36})$$

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \omega)^j \frac{\Lambda_{t+j}^3}{\Lambda_t^1} Y_{i,t+j} \left( \Pi_t^* - \frac{\theta}{\theta-1} MC_{t+j} \prod_{k=0}^j \Pi_{t+k} \right) \quad (\text{A.37})$$

$$\Pi_t^{1-\theta} = \omega + (1 - \omega) (\Pi_t^*)^{1-\theta} \quad (\text{A.38})$$

$$MC_t = \frac{\theta-1}{\theta} \left( \frac{R_t^L}{\bar{R}^L} \right)^{\alpha(s_t)} W_t / z_t \quad (\text{A.39})$$

$$L_t = D_t (1 - rr) \quad (\text{A.40})$$

$$R_t^L - R_t^D = (R_t^L - 1) rr \quad (\text{A.41})$$

$$X_t = \xi^{1/(1+\varphi)} \frac{Y_t}{z_t} \quad (\text{A.42})$$

$$Y_t = C_t \quad (\text{A.43})$$

$$\ln(\zeta_t) = \rho_\zeta \ln(\zeta_{t-1}) + \varepsilon_t^\zeta \quad (\text{A.44})$$

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \varepsilon_t^v \quad (\text{A.45})$$

$$\ln(z_t) = \rho_z \ln(z_{t-1}) + \varepsilon_t^z \quad (\text{A.46})$$

## A.6 The Log-Linear Model

In this section I provide a linear representation of the model by taking a first order Taylor expansion of the relevant equations around the the symmetric equilibrium with no trend in inflation or technology. All lower case variables denote log deviations from the steady-state:  $g_t = \ln(\hat{g}_t) - \ln(\bar{g})$ , where  $\bar{g}$  is the steady state value of  $g_t$ .

The log-linear Euler equation, expressed in terms of the efficient output gap, can be derived from combining A.25, A.42 and A.43:

$$x_t = \mathbb{E}_t x_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1}) + (1 - \rho_\zeta) \zeta_t - (1 - \rho_z) z_t, \quad (\text{A.47})$$

so that in equation 1  $\varepsilon_t^x = (1 - \rho_\zeta) \zeta_t - (1 - \rho_z) z_t$ .

The evolution of the real wage is determined by A.27 which can be combined with A.26, A.36, A.42 and A.43 to express the real wage as a function of last periods real wage and the output gap (and technology):

$$w_t = \rho w_{t-1} + (1 - \rho)(1 + \varphi)(x_t + z_t). \quad (\text{A.48})$$

The above expression eliminates  $H_t$  from  $MRS_t$  using the relationship between aggregate output and hours supplied, technology and price dispersion. The price dispersion term is zero to a first-order approximation around the zero inflation steady state. This expression shows that in equation 2  $\varepsilon_t^w = (1 - \rho)(1 + \varphi) z_t$ .

The log-linear money demand equation can be derived in two steps. First, I combine equations A.29, A.30 and A.31 to show that in equilibrium:

$$\begin{aligned} u_t &= \left[ \nu \left( \frac{R_t - 1}{R_t} \right)^{1-\sigma} + (1 - \nu) \left( \frac{R_t - R_t^D}{R_t} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\ &= \frac{R_t - 1}{R_t} \left[ \nu + (1 - \nu) (rr)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \end{aligned}$$

where the second equality follows from A.41. Then I use this expression for  $u_t$  in A.28 to arrive at the following expression for real money balances:

$$m_t = x_t - \frac{\beta^2}{1 - \beta} \frac{1}{1 + \chi} r_t + \frac{\chi}{1 + \chi} v_t + z_t, \quad (\text{A.49})$$

so that in equation 3  $\eta = \beta^2 / (1 - \beta)(1 + \chi)$  and  $\varepsilon_t^m = \frac{\chi}{1 + \chi} v_t + z_t$ .

Finally, the Phillips Curve can be derived in two steps. First, log-linearizing A.37:

$$\begin{aligned} \pi_t^* &= (1 - \beta\omega) \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega) (w_{t+j} - z_{t+j} + \alpha(s_{t+j}) r_{t+j}) + \sum_{k=0}^j \pi_{t+k} \\ &= (1 - \beta\omega) \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega) (w_{t+j} - z_{t+j} + \alpha(s_{t+j}) r_{t+j}) + \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\omega) \pi_{t+j} \\ &= \beta\omega \pi_{t+1}^* + (1 - \beta\omega) (w_t - z_t + \alpha(s_t) r_t) + \pi_t. \end{aligned}$$

The first relationship follows from linearizing the firms pricing decision around the zero-inflation steady-state. The second equality follows after some algebra and the third equality rewrites the infinite sum as a recursive formula. In this derivation, I follow the *Partition Principle* of Foerster et al. (2016), and therefore I do not linearize around  $\alpha(s_t)$  so that the log-linear regime switching model maintains



the inherent non-linearity in the switching parameters. The fact that I don't linearize around  $\alpha(s_t)$  and that it doesn't affect the model's steady state, allows me to use this same linearization to study local determinacy in the regime switching model. Therefore, this specification of the cost channel produces a log-linear approximation in the constant parameter model that is isomorphic to standard log-linear cost channel model like those specified by Rabanal (2007) and Christiano et al. (2010) and estimated in Ravenna and Walsh (2006) while, at the same time, this specification introduces switching in the coefficient matrices in the MS-DSGE model. Next, I linearize equation A.38, around the zero inflation steady state and use the resulting expression  $\pi_t = (1 - \omega)\pi_t^*$  to eliminate  $\pi_t^*$  above:

$$\pi_t = \frac{(1 - \omega)(1 - \beta\omega)}{\omega}(w_t - z_t + \alpha(s_t)r_t) + \beta\mathbb{E}_t\pi_{t+1}, \quad (\text{A.50})$$

so that in equation 4  $\kappa = (1 - \omega)(1 - \beta\omega)/\omega$  and  $\varepsilon_t^\pi = -\kappa z_t$ .

## A.7 Baseline Calibration

Table 4: Baseline Model Calibration

Parameter		U.S	Euro Area
Discount Rate	$\beta$	0.99	0.99
Real Wage Adjustment	$\rho$	0.90	0.90
Inverse Frisch Elasticity	$\varphi$	0	0
Interest Semi-Elasticity of Money Demand	$\eta$	2	3.2
Calvo Probability	$\omega$	0.6	0.75
Share of Cost Channel Firms	$\alpha$	1	1
Inflation Reaction in Policy Rule	$\phi_\pi$	1.5	1.49
Output Gap Reaction in Policy Rule	$\phi_x$	0.125	0
Money Growth Reaction in Policy Rule	$\phi_\mu$	0	0.16

The model is calibrated so that each period represents one quarter. I set  $\beta = 0.99$  which implies an annualized nominal bond rate equal to 4%. There is little empirical evidence on the real wage adjustment parameter, so I follow the baseline calibration in Blanchard and Gali (2007) and set  $\rho = 0.9$ . Also supporting this calibration, this parameter is large enough to generate an inertial inflation response to a monetary policy shock. I set the elasticity of hours worked,  $\varphi = 0$ , following much of the macro literature (Hansen, 1985; Rogerson, 1988; Ireland, 2004). Ravenna and Walsh (2006) estimate  $\alpha$  to be statistically indistinguishable from 1 using U.S. data. Furthermore, Chowdhury et al. (2006) find similar evidence for many euro area countries. The multi-equation decompositions of inflation by (Tillmann, 2008) also finds that  $\alpha = 1$  is difficult to statistically reject for the U.S. and euro area.

For the U.S. model, I set  $\omega = 0.6$  so the average duration of a price is about 7 months, as found by Bils and Klenow (2004) from micro level data. Meanwhile, for the euro area calibration I set  $\omega = 0.75$  as found by Dhyne, Alvarez, Le Bihan, Veronese, Dias, Hoffmann, Jonker, Lunnemann, Rumler, and Vilmunen (2006) who also use micro level data. Following Ireland (2009), I set  $\eta = 2$  for the U.S. Meanwhile, for the euro area, I use the estimate from Andres et al. (2009) of  $\eta = 3.2$ . Both of these studies also find support for the unit income elasticity of money demand specified in this equilibrium model. The policy rule parameters for the U.S. are taken from the benchmark work of Taylor (1993). for the euro area I use the estimates from Section 6.

## B Proofs

In this section I present the proofs to the results stated in the paper. All proofs for determinacy omit the possibility that an eigenvalue is exactly equal to one. In such a case, a log-linear approximation to the non-linear model can not pin down the question of local equilibrium existence and uniqueness. The proofs rely on the results from Woodford (2003) regarding determinacy in 2-dimensional forward looking models and 3-dimensional models with 2 forward-looking variables and 1 predetermined variable. For ease of exposition, I restate the propositions from Woodford (2003) before providing proofs to the propositions in the paper.

### Proposition C.1, Woodford (2003)

Consider a linear rational-expectations model of the form

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A} \mathcal{F}_t + \mathcal{B} \epsilon_t$$

where  $\mathcal{F}_t$  is a  $2 \times 1$  vector of forward-looking variables,  $\epsilon_t$  is a vector of exogenous disturbance terms, and  $\mathcal{A}$  is a  $2 \times 2$  matrix of coefficients. The rational expectations equilibrium is determinate if and only if the matrix  $\mathcal{A}$  has both eigenvalues outside the unit circle. This condition in turn is satisfied if and only if either Case I or Case II below are true.

#### Case I

$$\det \mathcal{A} > 1 \tag{B.1}$$

$$\det \mathcal{A} - \text{tr} \mathcal{A} > -1 \tag{B.2}$$

$$\det \mathcal{A} + \text{tr} \mathcal{A} > -1 \tag{B.3}$$

#### Case II

$$\det \mathcal{A} - \text{tr} \mathcal{A} < -1 \tag{B.4}$$

$$\det \mathcal{A} + \text{tr} \mathcal{A} < -1 \tag{B.5}$$

### Proposition C.2, Woodford (2003)

Consider a linear rational-expectations model of the form

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A} \mathcal{F}_t + \mathcal{B} \epsilon_t$$

where  $\mathcal{F}_t$  is a  $3 \times 1$  vector with 2 forward-looking variables and 1 predetermined variable,  $\epsilon_t$  is a vector of exogenous disturbance terms, and  $\mathcal{A}$  is a  $3 \times 3$  matrix of coefficients. The rational expectations equilibrium is determinate if and only if the matrix  $\mathcal{A}$  has exactly 2 eigenvalues outside the unit circle. This condition in turn is satisfied if and only if either Case I, Case II and/or Case III below are true, in which the characteristic equation of the matrix  $\mathcal{A}$  is written in the form:

$$\mathcal{P}(\lambda) = \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0. \tag{B.6}$$

#### Case I

$$A_2 + A_1 + A_0 + 1 < 0 \tag{B.7}$$

$$A_2 - A_1 + A_0 - 1 > 0 \tag{B.8}$$

## Case II

$$A_2 + A_1 + A_0 + 1 > 0 \quad (\text{B.9})$$

$$A_2 - A_1 + A_0 - 1 < 0 \quad (\text{B.10})$$

$$A_0^2 + A_1 - A_2 A_0 - 1 > 0 \quad (\text{B.11})$$

## Case III

$$A_2 + A_1 + A_0 + 1 > 0 \quad (\text{B.12})$$

$$A_2 - A_1 + A_0 - 1 < 0 \quad (\text{B.13})$$

$$|A_2| > 3 \quad (\text{B.14})$$

## Proposition 1

Consider the dynamic system defined by equations 1, 2, 4 and the policy rule  $r_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$ ,

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A} \mathcal{F}_t + \mathcal{B} \varepsilon_t$$

$$\mathcal{A} = \begin{bmatrix} 1 + \frac{\kappa(1+\varphi)}{\beta} & \frac{\phi_\pi(\beta+\alpha\kappa)-1}{\beta} \\ -\frac{\kappa(1+\varphi)}{\beta} & \frac{1-\phi_\pi\alpha\kappa}{\beta} \end{bmatrix}$$

where  $\mathcal{F}_t = \begin{bmatrix} x_t & \pi_t \end{bmatrix}^T$ . The system has two non-predetermined variables and therefore the system will have a unique rational expectations equilibrium if, and only if, Case I or Case II is satisfied in Proposition C.1. I will show the conditions in the theorem are necessary and sufficient for Case I to hold.

The conditions for Case I to hold are as follows:

$$\det \mathcal{A} > 1 \iff 1 + \kappa \phi_\pi (1 + \varphi - \alpha) > \beta \quad (\text{B.15})$$

$$\det \mathcal{A} - \text{tr} \mathcal{A} > -1 \iff \kappa(1 + \varphi)(1 - \phi_\pi) + \beta < \beta \quad (\text{B.16})$$

$$\det \mathcal{A} + \text{tr} \mathcal{A} > -1 \iff \phi_\pi \kappa (2\alpha - (1 + \varphi)) < 2(1 + \beta) + \kappa(1 + \varphi). \quad (\text{B.17})$$

Condition B.15 holds so long as  $\phi_\pi > 0$ , condition B.16 places the lower bound restriction that  $\phi_\pi > 1$  and condition B.17 always holds when  $\alpha \leq (1 + \varphi)/2$ ; but when  $\alpha > (1 + \varphi)/2$  then condition B.17 imposes the upper-bound that  $\phi_\pi < \frac{2(1+\beta)+\kappa(1+\varphi)}{\kappa(2\alpha-(1+\varphi))}$ .

## Proposition 2

Consider the dynamic system defined by equations 1 - 4 and the policy rule  $r_t = \phi_\mu m_t + \varepsilon_t^{mp}$ ,

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A} \mathcal{F}_t + \mathcal{B} \varepsilon_t$$

$$\mathcal{A} = \begin{bmatrix} 1 + \frac{\kappa(1+\varphi)}{\beta} + \phi_\mu \frac{\alpha\kappa+\beta}{\beta(\phi_\mu\eta+1)} & \phi_\mu + \frac{\phi_\mu\alpha\kappa-1}{\beta} - \frac{\phi_\mu^2(\alpha\kappa+\beta)\eta}{\beta(\phi_\mu\eta+1)} & -\phi_\mu - \frac{\phi_\mu\alpha\kappa}{\beta} + \frac{\phi_\mu^2(\alpha\kappa+\beta)\eta}{\beta(\phi_\mu\eta+1)} \\ -\frac{\kappa(1+\varphi)}{\beta} - \frac{\phi_\mu\alpha\kappa}{\beta(\phi_\mu\eta+1)} & \frac{1-\phi_\mu\alpha\kappa}{\beta} + \frac{\phi_\mu^2\alpha\kappa\eta}{\beta(\phi_\mu\eta+1)} & \frac{\phi_\mu\alpha\kappa}{\beta} - \frac{\phi_\mu^2\alpha\kappa\eta}{\beta(\phi_\mu\eta+1)} \\ \frac{1}{\phi_\mu\eta+1} & -\frac{\phi_\mu\eta}{\phi_\mu\eta+1} & \frac{\phi_\mu\eta}{\phi_\mu\eta+1} \end{bmatrix}$$

where  $\mathcal{F}_t = \begin{bmatrix} x_t & \pi_t & m_t \end{bmatrix}^T$ . The system has two non-predetermined variables and one predetermined variable and therefore the system will have a unique rational expectations equilibrium if,

and only if, Case I, Case II, and/or Case III is satisfied in Proposition C.2. The coefficients of the characteristic polynomial are given by:

$$\begin{aligned} A_2 &= -\frac{(\phi_\mu\eta + 1)(1 + \beta + \kappa(1 + \varphi)) + \beta(\phi_\mu\eta + \phi_\mu)}{\beta(\phi_\mu\eta + 1)} \\ A_1 &= \frac{(\phi_\mu\eta + \phi_\mu)(1 + \beta + \kappa(1 + \varphi)) + (\phi_\mu\eta + 1)}{\beta(\phi_\mu\eta + 1)} \\ A_0 &= \frac{(\phi_\mu\eta + \phi_\mu)}{\beta(\phi_\mu\eta + 1)}. \end{aligned}$$

Here, I show that the conditions in Case III are always satisfied if, and only if,  $\phi_\mu > 1$ . The conditions for Case III to hold are as follows:

$$A_2 + A_1 + A_0 + 1 > 0 \iff \frac{\kappa(1 + \varphi)(\phi_\mu - 1)}{\beta(\phi_\mu\eta + 1)} > 0 \quad (\text{B.18})$$

$$A_2 - A_1 + A_0 - 1 < 0 \iff -\frac{((\phi_\mu\eta + 1) + (\phi_\mu\eta + \phi_\mu))(2(1 + \beta) + \kappa(1 + \varphi))}{\beta(\phi_\mu\eta + 1)} < 0 \quad (\text{B.19})$$

$$|A_2| > 3 \iff (\phi_\mu\eta + 1)((1 - \beta) + \kappa(1 + \varphi)) + \beta(\phi_\mu - 1) > 0. \quad (\text{B.20})$$

Condition B.18 holds if, and only if,  $\phi_\mu > 1$ , condition B.19 is always true for positive parameters, and condition B.20 always holds, assuming  $\phi_\mu > 1$  as required by condition B.18.

### Proposition 3

Consider the dynamic system defined by equations 1, 2, 4 and the policy rule  $r_t = \phi_\pi\pi_t + \varepsilon_t^{mp}$ ,

$$\begin{aligned} \mathbb{E}_t\mathcal{F}_{t+1} &= \mathcal{A}\mathcal{F}_t + \mathcal{B}\varepsilon_t \\ \mathcal{A} &= \begin{bmatrix} 1 + \frac{\kappa(1-\rho)(1+\varphi)}{\beta} & \frac{\phi_\pi(1+\alpha\kappa)+1}{\beta} & \frac{\kappa\rho}{\beta} \\ -\frac{\kappa(1-\rho)(1+\varphi)}{\beta} & \frac{1-\phi_\pi\alpha\kappa}{\beta} & -\frac{\kappa\rho}{\beta} \\ (1-\rho)(1+\varphi) & 0 & \rho \end{bmatrix} \end{aligned}$$

where  $\mathcal{F}_t = [x_t, \pi_t, w_t]^T$ . The system has two non-predetermined variables and one predetermined variable and therefore the system will have a unique rational expectations equilibrium (REE) only if Case I, Case II, and/or Case III is satisfied in Proposition C.2. The coefficients of the characteristic polynomial are given by:

$$\begin{aligned} A_2 &= \frac{\phi_\pi\alpha\kappa - \kappa(1 + \varphi)(1 - \rho) - (1 + \beta)(1 + \rho) + \rho}{\beta} \\ A_1 &= \frac{\phi_\pi\kappa(1 + \varphi)(1 - \rho) - \phi_\pi\alpha\kappa(1 + \rho) + (1 + \beta)(1 + \rho) - \beta}{\beta} \\ A_0 &= \frac{\phi_\pi\alpha\kappa\rho - \rho}{\beta}. \end{aligned}$$

Here, I show that the conditions in Case I are never satisfied when  $\phi_\pi > 1$ . Case I requires that:

$$A_2 + A_1 + A_0 + 1 < 0 \iff \frac{\kappa(1 + \varphi)(1 - \rho)(\phi_\pi - 1)}{\beta} < 0$$

Therefore, active monetary policy equilibria only exist in Case II or Case III. In either Case II or Case III case, a necessary condition for a unique equilibrium to exist is that:

$$\begin{aligned} A_2 - A_1 + A_0 - 1 < 0 \\ \iff \\ \phi_\pi\kappa(2\alpha(1 + \rho) - (1 + \varphi)(1 - \rho)) - 2(1 + \beta)(1 + \rho) - \kappa(1 + \varphi)(1 - \rho) < 0 \end{aligned} \quad (\text{B.21})$$

If  $\alpha \leq \frac{(1+\varphi)(1-\rho)}{2(1+\rho)} \equiv \alpha_{min}(\rho)$ , then condition B.21 always holds. However, when  $\alpha > \alpha_{min}(\rho)$ , condition B.21 requires that in any unique rational expectations equilibrium:

$$\phi_\pi < \frac{2(1+\beta)(1+\rho) + \kappa(1+\varphi)(1-\rho)}{\kappa(2\alpha(1+\rho) - (1+\varphi)(1-\rho))} \equiv \phi_{\pi,max}(\rho).$$

To establish the first claim in Proposition 3, notice that:

$$\frac{d\alpha_{min}(\rho)}{d\rho} = -\frac{(1+\varphi)}{(1+\rho)^2} < 0.$$

Therefore, in the presence of partial real wage adjustment, the minimum strength of the cost channel necessary to induce an upper bound on the inflation response is decreasing in  $\rho$ . As for the second claim, notice that:

$$\frac{d\phi_{\pi,max}(\rho)}{d\rho} = -\frac{2\kappa(1+\varphi)(\alpha\kappa + (1+\beta))}{(\kappa(2\alpha(1+\rho) - (1+\varphi)(1-\rho)))^2} < 0.$$

Therefore, the upper bound on the inflation response necessary for a unique bounded rational expectations equilibrium to exist is decreasing in  $\rho$ .

## Proposition 4

Consider the dynamic system defined by equations 1, 2, 4 and the policy rule  $r_t = \phi_{ngdp}(\Delta y_t + \pi_t) + \varepsilon_t^{mp}$ , where  $\Delta y_t = y_t - y_{t-1}$  and  $y_t = x_t + \varepsilon_t^y$ ,

$$\mathbb{E}_t \mathcal{F}_{t+1} = \mathcal{A} \mathcal{F}_t + \mathcal{B} \varepsilon_t$$

$$\mathcal{A} = \begin{bmatrix} 1 + \frac{\kappa(1+\varphi)}{\beta} + \phi_{ngdp} \frac{\alpha\kappa + \beta}{\beta} & \phi_{ngdp} + \frac{\phi_{ngdp}\alpha\kappa - 1}{\beta} & -\phi_{ngdp} - \frac{\phi_{ngdp}\alpha\kappa}{\beta} \\ -\frac{\kappa(1+\varphi)}{\beta} - \frac{\phi_{ngdp}\alpha\kappa}{\beta} & \frac{1 - \phi_{ngdp}\alpha\kappa}{\beta} & \frac{\phi_{ngdp}\alpha\kappa}{\beta} \\ 1 & 0 & 0 \end{bmatrix}$$

where  $\mathcal{F}_t = [x_t, \pi_t, y_t]^T$ . The system has two non-predetermined variables and one predetermined variable and therefore the system will have a unique rational expectations equilibrium if, and only if, Case I, Case II, and/or Case III is satisfied in Proposition C.2. The coefficients of the characteristic polynomial are given by:

$$A_2 = -\frac{\beta(\phi_{ngdp} + 1) + \kappa(1 + \varphi) + 1}{\beta}$$

$$A_1 = \frac{\phi_{ngdp}(1 + \beta + \kappa(1 + \varphi)) + 1}{\beta}$$

$$A_0 = -\frac{\phi_{ngdp}}{\beta}.$$

Here, I show that the conditions in Case III are always satisfied if, and only if,  $\phi_{ngdp} > 1$ . The conditions for Case III to hold are as follows:

$$A_2 + A_1 + A_0 + 1 > 0 \iff \frac{\kappa(1 + \varphi)(\phi_{ngdp} - 1)}{\beta} > 0 \quad (\text{B.22})$$

$$A_2 - A_1 + A_0 - 1 < 0 \iff -\frac{(\phi_{ngdp} + 1)(2(1 + \beta) + \kappa(1 + \varphi))}{\beta} < 0 \quad (\text{B.23})$$

$$|A_2| > 3 \iff (1 - \beta) + \kappa(1 + \varphi) + \beta(\phi_{ngdp} - 1) > 0. \quad (\text{B.24})$$

Condition B.22 holds if, and only if,  $\phi_{ngdp} > 1$ , condition B.23 is always true for positive parameters, and condition B.24 always holds, assuming  $\phi_{ngdp} > 1$  as required by condition B.22.